

Exercise 4.9

(51)

Solve the System of Equations

Q.1 $2x^2 = 6 + 3y^2$ — (1)

$3x^2 - 5y^2 = 7$ — (2)

From (1) $x^2 = \frac{6+3y^2}{2}$

Putting in (2), we have

$3\left(\frac{6+3y^2}{2}\right) - 5y^2 = 7$

$3(6+3y^2) - 10y^2 = 14$

$18 + 9y^2 - 10y^2 - 14 = 0$

$4 - y^2 = 0 \Rightarrow y^2 = 4$

$y = \pm 2$

$y = 2$
 $x^2 = \frac{6+3(2)^2}{2}$

$x^2 = \frac{6+3(4)}{2}$

$x^2 = \frac{6+12}{2} = \frac{18}{2}$

$x = 3$

$x = \pm 3$

$y = -2$
 $x^2 = \frac{6+3(-2)^2}{2}$

$x^2 = \frac{6+3(4)}{2}$

$x^2 = \frac{6+12}{2} = \frac{18}{2}$

$x = 3$

$x = \pm 3$

The Solution Set is

$\{(3, 2), (-3, 2), (3, -2), (-3, -2)\}$

Q.2 $8x^2 = y^2$ — (1)

$x^2 + 2y^2 = 19$ — (2)

Putting y^2 from (1) in (2), we have

$x^2 + 2(8x^2) = 19$

$x^2 + 16x^2 = 19$

$17x^2 = 19 \Rightarrow x^2 = \frac{19}{17}$

$y^2 = 8x^2$

$y^2 = 8\left(\frac{19}{17}\right) = \frac{152}{17}$

$x^2 = \frac{19}{17}$ $y^2 = \frac{152}{17}$

$x = \pm \sqrt{\frac{19}{17}}$ $y = \pm \sqrt{\frac{152}{17}}$

Thus Solution Set is

$\left\{ \left(\pm \sqrt{\frac{19}{17}}, \pm \sqrt{\frac{152}{17}} \right) \right\}$

Q.3 $2x^2 - 8 = 5y^2$ — (1)

$x^2 - 13 = -2y^2$ — (2)

From (1) $x^2 = \frac{5y^2 + 8}{2}$

Putting in (2), we have

$\frac{5y^2 + 8}{2} - 13 = -2y^2$

$5y^2 + 8 - 26 = -4y^2$

$5y^2 + 4y^2 - 18 = 0$

$9y^2 = 18 \Rightarrow y^2 = 2$

$x^2 = \frac{5(2) + 8}{2} = \frac{10 + 8}{2} = 9$

$x = 3$

$x = \pm 3$

$y^2 = 2$

$y = \pm \sqrt{2}$

The Solution Set is

$\{(\pm 3, \pm \sqrt{2})\}$

Q.4 $x^2 - 5xy + 6y^2 = 0$ (i)

$x^2 + y^2 = 45$ (ii)

(i) $\Rightarrow x^2 - 3xy - 2xy + 6y^2 = 0$

$x(x-3y) - 2y(x-3y) = 0$

$(x-2y)(x-3y) = 0$

$x-2y=0$

$x=2y$

Putting in (ii)

$(2y)^2 + y^2 = 45$

$4y^2 + y^2 = 45$

$5y^2 = 45$

$y^2 = 9$

$y = \pm 3$

$\Rightarrow x = 2y$

$x = 2(\pm 3)$

$x = \pm 6$

The Solution Set is

$\{(\pm 6, \pm 3), (\pm \frac{9}{\sqrt{2}}, \pm \frac{3}{\sqrt{2}})\}$

Q.5 $12x^2 - 25xy + 12y^2 = 0$ (i)

$4x^2 + 7y^2 = 148$ (ii)

(i) $\Rightarrow 12x^2 - 16xy - 9xy + 12y^2 = 0$

$4x(3x-4y) - 3y(3x-4y) = 0$

$(3x-4y)(4x-3y) = 0$

$3x-4y=0 \quad \wedge \quad 4x-3y=0$

$x = \frac{4y}{3}$

$x = \frac{3y}{4}$

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Putting in (2)

$4(\frac{4y}{3})^2 + 7y^2 = 148$

$4(\frac{16}{9}y^2) + 7y^2 = 148$

$\frac{64y^2 + 63y^2}{9} = 148$

$127y^2 = 148 \times 9$

$127y^2 = 1332$

$y^2 = \frac{1332}{127}$

$y = \pm \sqrt{\frac{1332}{127}}$

$y = \pm 2\sqrt{\frac{333}{127}}$

$\Rightarrow x = \frac{4}{3}(\pm 2\sqrt{\frac{333}{127}})$

$x = \pm \frac{8}{3}\sqrt{\frac{333}{127}}$

The solution set is

$\{(\pm 3, \pm 4), (\pm \frac{8}{3}\sqrt{\frac{333}{127}}, \pm 2\sqrt{\frac{333}{127}})\}$

Putting in (i)

$4(\frac{3}{4}y)^2 + 7y^2 = 148$

$4(\frac{9}{16}y^2) + 7y^2 = 148$

$\frac{9y^2 + 28y^2}{4} = 148$

$37y^2 = 148 \times 4$

$37y^2 = 592$

$y^2 = \frac{592}{37} = 16$

$y = \pm 4$

$\Rightarrow x = \frac{3}{4}(\pm 4)$

$x = \pm 3$

Q.6 $12x^2 - 11xy + 2y^2 = 0$ (i)

$2x^2 + 7xy = 60$ (2)

(i) $\Rightarrow 12x^2 - 8xy - 3xy + 2y^2 = 0$

$4x(3x-2y) - y(3x-2y) = 0$

$(4x-y)(3x-2y) = 0$

$4x-y=0$

$y=4x$

Putting in (2)

$2x^2 + 7x(4x) = 60$

$2x^2 + 28x^2 = 60$

$30x^2 = 60$

$x^2 = 2$

$3x-2y=0$

$y = \frac{3x}{2}$

Putting in (2)

$2x^2 + 7x(\frac{3x}{2}) = 60$

$4x^2 + 21x^2 = 120$

$25x^2 = 120$

$x^2 = \frac{24}{5}$

$$x^2 = 2$$

$$x = \pm \sqrt{2}$$

$$\Rightarrow y = 4x$$

$$y = 4(\pm \sqrt{2})$$

$$y = \pm 4\sqrt{2}$$

The Solution Set is

$$\{(\pm \sqrt{2}, \pm 4\sqrt{2}), (\pm 2\sqrt{6/5}, \pm 3\sqrt{6/5})\}$$

$$\text{Q.7 } x^2 - y^2 = 16 \quad \text{--- (1)}$$

$$xy = 15 \quad \text{--- (2)}$$

Multiply (1) by 15 and (2) by 16 and

$$15x^2 - 15y^2 = 240$$

$$16xy = 240$$

$$15x^2 - 16xy - 15y^2 = 0$$

$$15x^2 - 25xy + 7xy - 15y^2 = 0$$

$$5x(3x - 5y) + 3y(3x - 5y) = 0$$

$$(3x - 5y)(5x + 3y) = 0$$

$$3x - 5y = 0$$

$$x = \frac{5}{3}y$$

Putting in (2)

$$\left(\frac{5}{3}y\right) \cdot y = 15$$

$$5y^2 = 45$$

$$y^2 = 9$$

$$y = \pm 3$$

$$\Rightarrow x = \frac{5}{3}(\pm 3)$$

$$x = \pm 5$$

$$\text{S.S.} = \{(\pm 5, \pm 3),$$

$$x^2 = 24/5$$

$$x = \pm 2\sqrt{6/5}$$

$$\Rightarrow y = \frac{3}{2}x$$

$$\Rightarrow y = \frac{3}{2}(\pm 2\sqrt{6/5})$$

$$y = \pm 3\sqrt{6/5}$$

Subtracting

$$5x + 3y = 0$$

$$x = -\frac{3}{5}y$$

Putting in (2)

$$\left(-\frac{3}{5}y\right)y = 15$$

$$-3y^2 = 75$$

$$y^2 = -25 = 25i^2$$

$$y = \pm 5i$$

$$\Rightarrow x = -\frac{3}{5}(\pm 5i)$$

$$x = \mp 3i$$

$$\{(\pm 3i, \mp 5i)\}$$

$$\text{Q.8 } x^2 + xy = 9 \quad \text{--- (1)}$$

$$x^2 - y^2 = 2 \quad \text{--- (2)}$$

(53)

Multiplying (1) by 2 and (2) by 7 and

$$2x^2 + 2xy = 18$$

$$-9x^2 - 9y^2 = -18$$

$$-7x^2 + 2xy + 9y^2 = 0$$

$$7x^2 - 2xy - 9y^2 = 0$$

$$7x^2 - 9xy + 7xy - 9y^2 = 0$$

$$x(7x - 9y) + y(7x - 9y) = 0$$

$$(x + y)(7x - 9y) = 0$$

$$x + y = 0$$

$$x = -y$$

Putting in (2)

$$(-y)^2 - y^2 = 2$$

$$y^2 - y^2 = 2$$

$$0 = 2$$

Impossible to calculate root

$$7x - 9y = 0$$

$$x = \frac{9}{7}y$$

Putting in (2)

$$\left(\frac{9}{7}y\right)^2 - y^2 = 2$$

$$\frac{81}{49}y^2 - y^2 = 2$$

$$81y^2 - 49y^2 = 98$$

$$32y^2 = 98$$

$$y = \sqrt{\frac{49}{16}}$$

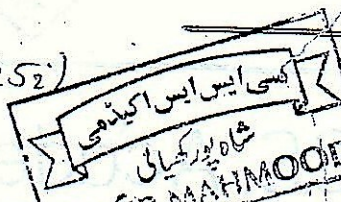
$$y =$$

$$\Rightarrow :$$

$$x = \pm$$

Thus Solution Set is

$$\left\{\left(\pm \frac{9}{4}, \pm \frac{7}{4}\right)\right\}$$



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Q.9 $y^2 - 7 = 2xy \Rightarrow y^2 - 2xy = -7$ — (1) Q.10 $x^2 + y^2 = 5$ — (1)

$2x^2 + 3 = -xy \Rightarrow 2x^2 - xy = -3$ — (2)

$xy = 2$ — (2) (54)

Multiplying (1) by 3 and (2) by 7 and adding

Multiplying (1) by 2 and (2) by 5 and subtracting

$3y^2 - 6xy = 21$

$2x^2 + 2y^2 = 10$

$+ 14x^2 - 7xy = 21$

$- 5xy = 10$

$3y^2 - 13xy + 14x^2 = 0$

$2x^2 - 5xy + 2y^2 = 0$

$14x^2 + 13xy + 3y^2 = 0$

$2x^2 - 4xy - xy + 2y^2 = 0$

$14x^2 - 6xy - 2xy + 3y^2 = 0$

$2x(x - 2y) - y(x - 2y) = 0$

$2x(7x - 3y) - y(7x - 3y) = 0$

$(2x - y)(x - 2y) = 0$

$(7x - 3y)(2x - y) = 0$

$2x - y = 0$

$x - 2y = 0$

$7x - 3y = 0$

$2x - y = 0$

$y = 2x$

$x = 2y$

$x = \frac{3}{7}y$

$x = \frac{y}{2}$

Putting in (2)

Putting in (2)

Putting in (1)

Putting in (1)

$x(2x) = 2$

$(2y)y = 2$

$y^2 - 7 = 2(\frac{3}{7}y)y$

$y^2 - 7 = 2(\frac{y}{2})y$

$2x^2 = 2$

$2y^2 = 2$

$y^2 - 7 = \frac{6y^2}{7}$

$y^2 - 7 = y^2$

$x^2 = 1$

$y^2 = 1$

$7y^2 - 49 = 6y^2$

$y^2 - y^2 = 7$

$x = \pm 1$

$y = \pm 1$

$7y^2 - 6y^2 = 49$

$0 = 7$

$\Rightarrow y = 2(\pm 1)$

$\Rightarrow x = 2(\pm 1)$

7

Not possible so

$y = \pm 2$

$x = \pm 2$

7

no answer

Thus solution set is

Set is

(± 7)

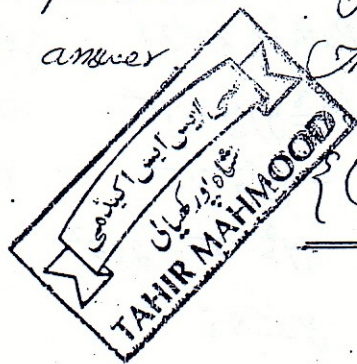
$\{(\pm 1, \pm 2), (\pm 2, \pm 1)\}$

± 3

Solution Set is

$\{(\pm 3, \pm 7)\}$

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The versatile
competative