Q.7 $\frac{x^2}{a^2}+\frac{(mx+c)^2}{b^2}=1$

$b^2x^2+a^2(m^2x^2+c^2+2mxc)=a^2b^2$

$x^2(b^2+a^2m^2)+a^2c^2+(2a^2mc)x-a^2b^2=0$

$(b^2+a^2m^2)x^2+(2a^2mc)x+a^2(c^2-b^2)=0$

$A=b^2+a^2m^2 \quad B=2a^2mc \quad C=a^2(c^2-b^2)$

$B^2-4AC=0$ for Equal roots

$\Rightarrow (2a^2mc)^2-4(b^2+a^2m^2)(a^2c^2-a^2b^2)=0$

$\Rightarrow 4a^4m^2c^2-4(a^2b^2c^2-a^2b^4+a^4m^2c^2-a^4m^2b^2)=0$

$\Rightarrow 4a^4m^2c^2-4a^2b^2c^2+4a^2b^4-4a^4m^2c^2+4a^4m^2b^2=0$

$\Rightarrow 4a^4m^2b^2-4a^2b^2c^2+4a^2b^4=0$

$\Rightarrow 4a^2b^2(a^2m^2-c^2+b^2)=0$

$\Rightarrow a^2m^2-c^2+b^2=0 \qquad 4a^2b^2\neq 0$

$c^2=a^2m^2+b^2$ (Proved).

Q.8 $(a^2-bc)x^2+2(b^2-ca)x+(c^2-ab)=0$

$A=a^2-bc \quad B=2(b^2-ca) \quad C=c^2-ab$

$B^2-4AC=0$ for equal roots.

$\Rightarrow [2(b^2-ca)]^2-4[(a^2-bc)(c^2-ab)]=0$

$\Rightarrow 4(b^4+c^2a^2-2b^2ca)-4(a^2c^2-a^3b-bc^3+ab^2c)=0$

$4b^4+4c^2a^2-8b^2ca-4a^2c^2+4a^3b+4bc^3-4ab^2c=0$

$\Rightarrow 4b^4-12ab^2c+4a^3b+4bc^3=0$

$4b(b^3-3abc+a^3+c^3)=0$

$a^3+b^3+c^3-3abc=0 \ \wedge \ 4b=0 \quad 4\neq 0$

$a^3+b^3+c^3=3abc \ \wedge \ b=0$

Thus roots will be equal if

$b=0$ or $a^3+b^3+c^3=3abc$

TAHIR

Simultaneous Equations:

"The equations which have same solution for whole system are called Simultaneous Equations."

Exercise 4.8

Solve System of Equations:

(i) $2x-y=4$ — (i)

$2x^2-4xy-y^2=6$ — (ii)

From (i) $y=2x-4$

Putting in (ii), we have

$2x^2-4x(2x-4)-(2x-4)^2=6$

$2x^2-8x^2+16x-(4x^2+16-16x)=6$

$2x^2-8x^2+16x-4x^2-16+16x-6=0$

$-10x^2+32x-22=0$

$-2\{5x^2-16x+11\}=0 \qquad -2\neq 0$

$5x^2-16x+11=0$

$5x^2-11x-5x+11=0$

$(5x-11)(x-1)=0$

$x-1=0 \ \wedge \ 5x-11=0$

$x=1$	$x=11/5$
$y=2x-4$	$y=2x-4$
$y=2(1)-4$	$y=2(11/5)-4$
$y=2-4=-2$	$y=\frac{22}{5}-4=\frac{22-20}{5}$
$y=-2$	$y=\frac{2}{5}$

Thus Solution Set is

$\{(1,-2),(11/5,2/5)\}$

Q.2 $x+y=5$ —— (i)

$x^2+2y^2=17$ —— (iii)

From (i) $y=5-x$

Putting in (iii), we have

$x^2+2(5-x)^2=17$

$x^2+2(25+x^2-10x)=17$

$x^2+50+2x^2-20x-17=0$

$3x^2-20x+33=0$

$3x^2-11x-9x+33=0$

$(3x-11)(x-3)=0$

$3x-11=0 \quad \wedge \quad x-3=0$

$x=\frac{11}{3}$	$x=3$
$y=5-x$	$y=5-x$
$y=5-\frac{11}{3}=\frac{15-11}{3}$	$y=5-3$
$y=\frac{4}{3}$	$y=2$

The Solution Set $\{(3,2),(11/3,4/3)\}$.

Q.3 $3x+2y=7$ —— (i)

$3x^2=25+2y^2$ —— (ii)

From (i) $y=\frac{7-3x}{2}$

Putting in (ii), we have

$3x^2=25+2\left(\frac{7-3x}{2}\right)^2$

$3x^2=25+2\left(\frac{49+9x^2-42x}{4}\right)$

$3x^2=\frac{50+49+9x^2-42x}{2}$

$6x^2=99+9x^2-42x$

$9x^2-6x^2-42x+99=0$

$3x^2-42x+99=0$

$3\{x^2-14x+33\}=0 \quad 3\neq 0$

$x^2-14x+33=0$

$x^2-11x-3x+33=0$

$(x-11)(x-3)=0$

$x-11=0 \quad \wedge \quad x-3=0$

$x=11$	$x=3$
$y=\frac{7-3x}{2}$	$y=\frac{7-3x}{2}$
$y=\frac{7-33}{2}=\frac{-26}{2}$	$y=\frac{7-9}{2}=\frac{-2}{2}$
$y=-13$	$y=-1$

The Solution Set is $\{(11,-13),(3,-1)\}$

Q.4 $x+y=5$ —— (i)

$\frac{2}{x}+\frac{3}{y}=2$ —— (ii)

From (i) $5-x=y$

Putting in (ii), we have

$\frac{2}{x}+\frac{3}{(5-x)}=2$

$\frac{2(5-x)+3(x)}{x(5-x)}=2$

$10-2x+3x=2x(5-x)$

$10+x=10x-2x^2$

$2x^2-10x+x+10=0$

$2x^2-9x+10=0$

$2x^2-4x-5x+10=0$

$(2x-5)(x-2)=0$

$2x-5=0 \quad \wedge \quad x-2=0$

$x=5/2 \quad \wedge \quad x=2$

$x = 5/2$

$y = 5 - x$

$y = 5 - \frac{5}{2}$

$y = \frac{10-5}{2} = \frac{5}{2}$

$y = 5/2$

$x = 2$

$y = 5 - x$

$y = 5 - 2 = 3$

$y = 3$

The solution set is

$\left\{(2, 3), \left(\frac{5}{2}, \frac{5}{2}\right)\right\}$

$x = \frac{3a+b \pm (a-b)}{4}$

$x = \frac{3a+b+(a+b)}{4}$

$x = \frac{3a+b+a-b}{4}$

$x = \frac{4a}{4} = a$

$x = a$

$y = a+b-x$

$y = a+b-a$

$y = b$

$x = \frac{3a+b-(a-b)}{4}$

$x = \frac{3a+b-a+b}{4}$

$x = \frac{2a+2b}{4} = \frac{(a+b)}{2}$

$x = \frac{a+b}{2}$

$y = a+b-x$

$y = a+b-\frac{a+b}{2}$

$y = \frac{2a+2b-a-b}{2}$

$y = \frac{a+b}{2}$

The solution set

$\left\{(a, b), \left(\frac{a+b}{2}, \frac{a+b}{2}\right)\right\}$

Q.5 $x + y = a + b$ ___ (i)

$\frac{a}{x} + \frac{b}{y} = 2$ ___ (ii)

From (i) $y = a + b - x$

Putting in (ii), we have

$\frac{a}{x} + \frac{b}{a+b-x} = 2$

$\frac{a(a+b-x) + bx}{x(a+b-x)} = 2$

$a^2 + ab - ax + bx = 2x(a+b-x)$

$a^2 + ab - ax + bx = 2ax + 2bx - 2x^2$

$2x^2 - ax + bx - 2ax - 2bx + a^2 + ab = 0$

$2x^2 - 3ax - bx + (a^2 + ab) = 0$

$2x^2 - (3a+b)x + (a^2 + ab) = 0$

$x = \frac{+(3a+b) \pm \sqrt{(3a+b)^2 - 4(2)(a^2+ab)}}{2(2)}$

$x = \frac{3a+b \pm \sqrt{9a^2+b^2+6ab-8a^2-8ab}}{4}$

$x = \frac{3a+b \pm \sqrt{a^2+b^2-2ab}}{4}$

$x = \frac{(3a+b) \pm \sqrt{(a-b)^2}}{4}$

Q.6 $3x + 4y = 25$ ___ (i)

$\frac{3}{x} + \frac{4}{y} = 2$ ___ (ii)

From (i) $y = \frac{25-3x}{4}$

Putting in (ii), we have

$\frac{3}{x} + \frac{4}{\left(\frac{25-3x}{4}\right)} = 2$

$\frac{3}{x} + \frac{4.4}{(25-3x)} = 2$

$\frac{3(25-3x) + 16x}{x(25-3x)} = 2$

$75 - 9x + 16x = 2x(25-3x)$

$75 + 7x = 50x - 6x^2$

$6x^2 - 50x + 7x + 75 = 0$

$6x^2 - 43x + 75 = 0$

$6x^2 - 25x - 18x + 75 = 0$

$(6x - 25)(x - 3) = 0$

$6x-25=0$ | $x-3=0$ | The Solution set is

$x=\frac{25}{6}$ | $x=3$

$y=\frac{25-3x}{4}$ | $y=\frac{25-3x}{4}$

$y=\frac{25-3(\frac{25}{6})}{4}$ | $y=\frac{25-3(3)}{4}$

$y=\frac{25-\frac{75}{6}}{4}$ | $y=\frac{25-9}{4}$

$y=\frac{150-75}{4\cdot 6}$ | $y=\frac{16}{4}=4$

$y=\frac{75}{24}=\frac{25}{8}$ | $y=4$

$y=\frac{25}{8}$

The Solution set is

$\{(3,4),(\frac{25}{6},\frac{25}{8})\}$

Q.7 $(x-3)^2+y^2=5$ —(i)

$2x=y+6$ —(ii)

From (ii) $y=2x-6$

Putting in (i), we have

$(x-3)^2+(2x-6)^2=5$

$x^2+9-6x+4x^2+36-24x=5$

$5x^2-30x+45-5=0$

$5x^2-30x+40=0$

$5\{x^2-6x+8\}=0 \quad 5\neq 0$

$x^2-6x+8=0$

$x^2-4x-2x+8=0$

$(x-4)(x-2)=0$

$x-4=0$ | $x-2=0$

$x=4$ | $x=2$

$y=2x-6$ | $y=2x-6$

$y=2(4)-6=8-6$ | $y=2(2)-6=4-6$

$y=2$ | $y=-2$

The Solution set is

$\{(2,-2),(4,2)\}$

Q.8 $(x+3)^2+(y-1)^2=5$ —(i)

$x^2+y^2+2x=9$ —(ii)

$(x+3)^2+(y-1)^2=5$

$x^2+9+6x+y^2+1-2y=5$

$x^2+y^2+6x-2y=5-9-1$

$x^2+y^2+6x-2y=-5$ —(iii)

By Eq (ii) − Eq (iii)

$x^2+y^2+2x=9$

$-x^2-y^2-6x+2y=+5$

$-4x+2y=14$

$2y=14+4x$

$y=7+2x$

Putting in (ii), we have

$x^2+(7+2x)^2+2x=9$

$x^2+49+4x^2+28x+2x-9=0$

$5x^2+30x+40=0$

$5\{x^2+6x+8\}=0 \quad 5\neq 0$

$x^2+6x+8=0$

$x^2+4x+2x+8=0$

$(x+4)(x+2)=0$

$x+4=0$ | $x+2=0$

$x=-4$ | $x=-2$

$y=7+2x$ | $y=7+2x$

$y=7+2(-4)=7-8$ | $y=7+2(-2)=7-4$

$y=-1$ | $y=3$

$S.S.=\{(-4,-1),(-2,3)\}$

Q.9 $x^2+(y+1)^2=18$ ——(i)

$(x+2)^2+y^2=21$ ——(ii)

$x^2+y^2+2y+1=18$

$x^2+y^2+2y=18-1=17$

$x^2+y^2+2y=17$ —— ③

$x^2+4+4x+y^2=21$

$x^2+y^2+4x=21-4=17$

$x^2+y^2+4x=17$ —— ④

By Eq ③ − Eq ④

$x^2+y^2+2y=17$

$-\ x^2+y^2\pm 4x=17$

$2y-4x=0 \Rightarrow 2y=4x$

$y=2x$

Putting in ④, we have

$x^2+(2x)^2+4x-17=0$

$x^2+4x^2+4x-17=0$

$5x^2+4x-17=0$

$$x=\frac{-4\pm\sqrt{16+4(5)(17)}}{2(5)}$$

$$x=\frac{-4\pm\sqrt{16+340}}{10}=\frac{-4\pm\sqrt{356}}{10}$$

$$x=\frac{-4\pm 2\sqrt{89}}{10}=\frac{-2\pm\sqrt{89}}{5}$$

$x=\frac{-2+\sqrt{89}}{5}$ | $x=\frac{-2-\sqrt{89}}{5}$

$y=2x$ | $y=2x$

$y=2\left[\frac{-2+\sqrt{89}}{5}\right]$ | $y=2\left[\frac{-2-\sqrt{89}}{5}\right]$

$y=\frac{-4+2\sqrt{89}}{5}$ | $y=\frac{-4-2\sqrt{89}}{5}$

Thus

$$S.S=\left\{\left(\frac{-2+\sqrt{89}}{5},\frac{-4+2\sqrt{89}}{5}\right),\left(\frac{-2-\sqrt{89}}{5},\frac{-4-2\sqrt{89}}{5}\right)\right\}$$

Q.10 $x^2+y^2+6x=1$ —— (i)

$x^2+y^2+2(x+y)=3$ —— (ii)

$x^2+y^2+2x+2y=3$ —— ⑤

By Eq (i) − Eq ⑤

$x^2+y^2+6x=1$

$-\ x^2+y^2+2x+2y=3$

$4x-2y=-2$

$4x+2=2y$

$y=2x+1$

Putting in (i), we have

$x^2+(2x+1)^2+6x=1$

$x^2+4x^2+4x+1+6x-1=0$

$5x^2+10x=0$

$5x(x+10)=0$ $\quad 5\neq 0$

$x=0$ | $x+10=0$

$x=0$ | $x=-10$

$y=2x+1$ | $y=2(x)+1$

$y=2(0)+1$ | $y=2(-10)+1$

$y=1$ | $y=-20+1$

$y=-19$

Thus solution set is

$\{(0,1),(-10,-19)\}$