Adding ① and ②

$2p+8=0 \implies \boxed{p=-4}$

Subtracting ① and ②

$-2q+2=0 \implies \boxed{q=1}$

Q.16 $P(x) = x^3 - 4x^2 + ax + b$

$x = -2, 2 \qquad a = ? \quad b = ?$

$$\begin{array}{r|rrrl}
 & 1 & -4 & a & b \\
-2 & & -2 & 12 & -2a-24 \\
\hline
 & 1 & -6 & a+12 & -2a+b-24 \\
2 & & 2 & -8 & \\
\hline
 & 1 & -4 & a+4 &
\end{array}$$

Remainder

$\because$ 2, −2 are the roots of $P(x)$ so remainder must be zero.

$a+4=0 \implies \boxed{a=-4}$

$-2a+b-24=0$

$-2(-4)+b=24$

$8+b=24 \implies \boxed{b=16}$

Relation between Roots and Coefficients of a Quadratic Equation:-

Let α, β be the roots of $ax^2+bx+c=0$

$\alpha = \dfrac{-b+\sqrt{b^2-4ac}}{2a} \qquad \beta = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$

Sum of roots $= \alpha+\beta$

$\alpha+\beta = \dfrac{-b+\sqrt{b^2-4ac}}{2a} + \dfrac{-b-\sqrt{b^2-4ac}}{2a}$

$\alpha+\beta = \dfrac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-4ac}}{2a}$

$\alpha+\beta = \dfrac{-2b}{2a} \implies \alpha+\beta = -\dfrac{b}{a}$

$S = \alpha+\beta = \dfrac{-b}{a} = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of roots $= \alpha\beta$

$\alpha\beta = \left(\dfrac{-b+\sqrt{b^2-4ac}}{2a}\right)\cdot\left(\dfrac{-b-\sqrt{b^2-4ac}}{2a}\right)$

$\alpha\cdot\beta = \dfrac{(-b)^2-(\sqrt{b^2-4ac})^2}{4a^2}$

$\alpha\beta = \dfrac{\cancel{b^2}-\cancel{b^2}+4ac}{4a^2} = \dfrac{4ac}{4a^2}$

$\alpha\beta = \dfrac{c}{a} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\alpha\beta = P = \dfrac{c}{a}$

Formation of an Equation

Let $(x-\alpha)(x-\beta)=0$ has roots α, β

$\implies x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2 - Sx + P = 0$

where, S = sum of roots

and P = Product of roots.

Exercise 4.6

Q.1 If α, β are the roots of $3x^2-2x+4=0$, find the values of:

$S = \alpha+\beta = -\dfrac{b}{a} = \dfrac{+2}{3} = \dfrac{2}{3}$

$P = \alpha\beta = \dfrac{c}{a} = \dfrac{4}{3}$

(i) $\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2} = ?$

$\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2} = \dfrac{\beta^2+\alpha^2}{\alpha^2\beta^2}$

$\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2} = \dfrac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{(\alpha\beta)^2}$

$\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2} = \dfrac{(\alpha+\beta)^2-2(\alpha\beta)}{(\alpha\beta)^2}$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{(2/3)^2-2(4/3)}{(4/3)^2}=\frac{\frac{4}{9}-\frac{8}{3}}{16/9}$$

$$=\frac{\frac{4-24}{9}}{16/9}=\frac{-20}{16}=-5/4$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=-5/4 \quad \text{Ans.}$$

(ii) $\frac{\alpha}{\beta}+\beta/\alpha=?$

$$\frac{\alpha}{\beta}+\beta/\alpha=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$=\frac{(2/3)^2-2(4/3)}{(4/3)}=\frac{\frac{4}{9}-\frac{8}{3}}{4/3}$$

$$=\frac{\frac{4-24}{9}}{4/3}=\frac{-20}{4\times3}=-\frac{5}{3}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=-5/3$$

(iii) $\alpha^4+\beta^4=?$

$$\alpha^4+\beta^4=\alpha^4+\beta^4+2\alpha^2\beta^2-2\alpha^2\beta^2$$

$$=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2$$

$$=[\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta]^2-2\alpha^2\beta^2$$

$$=[(\alpha+\beta)^2-2\alpha\beta]^2-2\alpha^2\beta^2$$

$$=[(2/3)^2-2(4/3)]^2-2(4/3)^2$$

$$=\left[\frac{4}{9}-\frac{8}{3}\right]^2-2\left[\frac{16}{9}\right]$$

$$=\left(\frac{4-24}{9}\right)^2-\frac{32}{9}$$

$$=\left(\frac{-20}{9}\right)^2-\frac{32}{9}=\frac{400}{81}-\frac{32}{9}$$

$$=\frac{400-288}{81}=\frac{112}{81}$$

$$\alpha^4+\beta^4=\frac{112}{81} \quad \text{Ans.}$$

(iv) $\alpha^3+\beta^3=?$

$$\alpha^3+\beta^3=\alpha^3+\beta^3+3\alpha\beta(\alpha+\beta)-3\alpha\beta(\alpha+\beta)$$

$$=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$$

$$=(2/3)^3-\cancel{3}(4/\cancel{3})(2/3)$$

$$=\frac{8}{27}-\frac{8}{3}=\frac{8-72}{27}$$

$$=\frac{-64}{27}$$

$$\alpha^3+\beta^3=-\frac{64}{27} \quad \text{Ans.}$$

(v) $\frac{1}{\alpha^3}+\frac{1}{\beta^3}=?$

$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{\alpha^3\beta^3}$$

$$=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3}$$

$$=\frac{(2/3)^3-3(4/3)(2/3)}{(4/3)^3}=\frac{-64/27}{64/27}$$

$$=-1$$

$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=-1 \quad \text{Ans.}$$

(vi) $\alpha^2-\beta^2=?$

$$\alpha^2-\beta^2=(\alpha+\beta)(\alpha-\beta)$$

$$=(\alpha+\beta)\sqrt{(\alpha-\beta)^2}$$

$$=(\alpha+\beta)\sqrt{\alpha^2+\beta^2-2\alpha\beta\pm2\alpha\beta-2\alpha\beta}$$

$$=(\alpha+\beta)\sqrt{(\alpha+\beta)^2-4\alpha\beta}$$

$$=\left(\frac{2}{3}\right)\sqrt{(2/3)^2-4(4/3)}$$

$$=\frac{2}{3}\sqrt{\frac{4}{9}-\frac{16}{3}}=\frac{2}{3}\sqrt{\frac{4-48}{9}}$$

$$\alpha^2-\beta^2=\frac{2\sqrt{-44}}{9}=\frac{4\sqrt{11}\,i}{9} \quad \text{Ans.}$$

Q.2 If α, β are the roots of $x^2 - px - p - c = 0$. prove that $(1+\alpha)(1+\beta) = 1-c$

Sol:- Sum $(S) = (\alpha+\beta) = \frac{p}{1} = p$

Product $(P) = \alpha\beta = \frac{-(p+c)}{1} = -(p+c)$

$\because (1+\alpha)(1+\beta) = 1+\alpha+\beta+\alpha\beta$

$(1+\alpha)(1+\beta) = 1+(\alpha+\beta)+\alpha\beta$

$(1+\alpha)(1+\beta) = 1+p+(-p-c)$

$(1+\alpha)(1+\beta) = 1+p-p-c$

$(1+\alpha)(1+\beta) = 1-c$ (Proved).

Q.3 Find the condition that one root of $x^2+px+q=0$ is

(i) Double the other:

Let α and 2α are the roots.

$S = \alpha+2\alpha = 3\alpha = -p$

$P = \alpha \cdot 2\alpha = 2\alpha^2 = q$

$3\alpha = -p \Rightarrow \alpha = \frac{-p}{3}$

$2\left(\frac{-p}{3}\right)^2 = q$

$2\left(\frac{p^2}{9}\right) = -q$

$2p^2 = 9q$ (req. Condition).

(ii) Square of other:

Let α and α^2 be the roots

$S = \alpha+\alpha^2 = -p$ — ①

$P = \alpha \cdot \alpha^2 = \alpha^3 = q$ — ②

$[\alpha(1+\alpha)]^3 = (-p)^3$

$\alpha^3(1+\alpha^3+3\alpha+3\alpha^2) = -p^3$

$\alpha^3+\alpha^6+3\alpha^4+3\alpha^5 = -p^3$

$\alpha^3+(\alpha^3)^2+3\alpha^3(\alpha+\alpha^2) = -p^3$

$q+q^2+3q(-p) = -p^3 \quad (\because \alpha^3 = q)$

$p^3+q+q^2 = 3pq$ (req. Cond.)

(iii) Additive inverse of other:

Let α and $-\alpha$ are the roots

$S = \alpha+(-\alpha) = -p \Rightarrow -p = 0$

$P = -\alpha \cdot \alpha = q \Rightarrow q = -\alpha^2$

(req. Condition) $p = 0$

(iv) Multiplicative inverse of other:

Let α and $1/\alpha$ are the roots

$S = \alpha + \frac{1}{\alpha} = -p$

$P = \alpha \cdot \frac{1}{\alpha} = q$

$q = 1$ (req. Condition)

Q.4 If the roots of equation $x^2-px+q=0$ differ by unity prove that $p^2 = 4q+1$

Let α and $\alpha-1$ be the roots

$(S) = \alpha+(\alpha-1) = p \Rightarrow 2\alpha = p+1$

$(P) = \alpha(\alpha-1) = q$

$\Rightarrow \left(\frac{p+1}{2}\right)\left(\frac{p+1}{2}-1\right) = q$

$\Rightarrow \left(\frac{p+1}{2}\right)\left(\frac{p+1-2}{2}\right) = q$

$$\left(\frac{p+1}{2}\right)\left(\frac{p-1}{2}\right) = q$$

$$\frac{p^2-1}{4} = q$$

$$p^2-1 = 4q$$

$$\Rightarrow p^2 = 4q+1 \quad \text{(Proved)}$$

Q.5 Find the Condition that $\frac{a}{x-a} + \frac{b}{x-b} = 5$ may have roots equal but opposite in sign:

Sol: $\because \frac{a}{x-a} + \frac{b}{x-b} = 5$

$$\frac{a(x-b)+b(x-a)}{(x-a)(x-b)} = 5$$

$$ax-ab+bx-ab = 5(x^2-ax-bx+ab)$$

$$(a+b)x-2ab = 5x^2-5(a+b)x+5ab$$

$$0 = 5x^2-5(a+b)x-(a+b)x+5ab+2ab$$

$$5x^2-6(a+b)x+7ab = 0$$

Let the roots are α and $-\alpha$

$$S = \alpha+(-\alpha) = \frac{6(a+b)}{5}$$

$$\frac{6(a+b)}{5} = 0 \qquad \frac{6}{5} \neq 0$$

$$a+b = 0 \quad \text{(req. Condition)}.$$

$$P = \alpha \cdot -\alpha = -\alpha^2 = \frac{7ab}{5}$$

Thus

req. Condition is $a+b=0$

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Q.6 If the roots of the Eq $px^2+qx+q=0$ are α and β then prove that

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\beta/\alpha} + \sqrt{q/p} = 0$$

Sol:-

Sum of roots $(S) = \alpha+\beta = \frac{-q}{p}$ — ①

Product of roots $(P) = \alpha\beta = \frac{q}{p}$ — ②

② $\Rightarrow \sqrt{\alpha\beta} = \sqrt{\frac{q}{p}}$ — ③

$$\frac{\alpha+\beta}{\sqrt{\alpha\beta}} = \frac{-q/p}{\sqrt{q/p}}$$

$$\frac{\alpha}{\sqrt{\alpha\beta}} + \frac{\beta}{\sqrt{\alpha\beta}} = -\frac{\sqrt{q^2/p^2}}{\sqrt{q/p}}$$

$$\sqrt{\frac{\alpha^2}{\alpha\beta}} + \sqrt{\frac{\beta^2}{\alpha\beta}} = -\sqrt{\frac{q^2 p}{p^2 \cdot q}}$$

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = -\sqrt{\frac{q}{p}}$$

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{q/p} = 0 \quad \text{(Proved)}$$

Q.7 If α, β are the roots of $ax^2+bx+c=0$, form eq having roots

(i) α^2, β^2

$$\alpha+\beta = \frac{-b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

(S) Sum of given roots $= \alpha^2+\beta^2$

$$(S) = (\alpha+\beta)^2 - 2\alpha\beta$$

$$= \left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$$

$$(S) = \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$$

Product $(P) = \alpha^2\beta^2 = (\alpha\beta)^2$

$$(P) = \left(\frac{c}{a}\right)^2 = \frac{c^2}{a^2}$$

Thus Eq is

$$y^2 - Sy + P = 0$$

$$y^2 - \left(\frac{b^2 - 2ac}{a^2}\right)y + \frac{c^2}{a^2} = 0$$

$$a^2y^2 - (b^2 - 2ac)y + c^2 = 0$$

(Req. Eq).

(ii) $\frac{1}{\alpha}, \frac{1}{\beta}$

$$S = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-b/a}{c/a} = -b/c$$

$$P = \frac{1}{\alpha}\cdot\frac{1}{\beta} = \frac{1}{\alpha\beta} = 1/\tfrac{c}{a} = a/c$$

Thus Eq is

$$y^2 - Sy + P = 0$$

$$y^2 + \frac{b}{c}y + \frac{a}{c} = 0$$

$$cy^2 + by + a = 0 \text{ (Req. Eq)}.$$

(iii) $\frac{1}{\alpha^2}, \frac{1}{\beta^2}$

$$S = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$S = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = \frac{\left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}$$

$$S = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{c^2/a^2} = \frac{\frac{b^2 - 2ac}{a^2}}{c^2/a^2}$$

$$S = \frac{b^2 - 2ac}{c^2}$$

$$P = \frac{1}{\alpha^2}\cdot\frac{1}{\beta^2} = \frac{1}{(\alpha\beta)^2}$$

$$P = \frac{1}{(c/a)^2} = \frac{a^2}{c^2}$$

Thus Eq $y^2 - Sy + P = 0$

$$y^2 - \left(\frac{b^2 - 2ac}{c^2}\right)y + \frac{a^2}{c^2} = 0$$

$$c^2y^2 - (b^2 - 2ac)y + a^2 = 0$$

(iv) α^3, β^3

$$S = \alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$$

$$S = \left(-\frac{b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right)$$

$$= \frac{-b^3}{a^3} + \frac{3bc}{a^2} = \frac{-b^3 + 3abc}{a^3}$$

$$P = \alpha^3\beta^3 = (\alpha\beta)^3 = \left(\frac{c}{a}\right)^3 = \frac{c^3}{a^3}$$

Eq $y^2 - Sy + P = 0$

$$y^2 + \left(\frac{b^3 - 3abc}{a^3}\right)y + \frac{c^3}{a^3} = 0$$

$$a^3y^2 + (b^3 - 3abc)y + c^3 = 0$$

(v) $\frac{1}{\alpha^3}, 1/\beta^3$

$$S = \frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3 + \beta^3}{(\alpha\beta)^3}$$

$$S = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3}$$

$$S = \frac{-b^3 + 3abc}{a^3} \Big/ c^3/a^3$$

$$S = -\frac{b^3 + 3abc}{c^3}$$

$$P = \frac{1}{\alpha^3}\frac{1}{\beta^3} = \frac{1}{(\alpha\beta)^3} = \frac{a^3}{c^3}$$

(vi) $\alpha+\frac{1}{\alpha}, \quad \beta+\frac{1}{\beta}$

Sum (S) $= \left(\alpha+\frac{1}{\alpha}\right)+\left(\beta+\frac{1}{\beta}\right)$

$S = (\alpha+\beta)+\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$

$= (\alpha+\beta)+\left(\frac{\alpha+\beta}{\alpha\beta}\right)$

$= \left(\frac{-b}{a}\right)+\left(\frac{-b/a}{c/a}\right)$

$= \frac{-b}{a}+\left(\frac{-b}{c}\right) = \frac{-b}{a}-\frac{b}{c}$

$S = -\left(\frac{bc+ab}{ac}\right)$

Product (P) $= \left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)$

$(P) = \alpha\beta+\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+\frac{1}{\alpha\beta}$

$P = \alpha\beta+\frac{1}{\alpha\beta}+\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)$

$= \alpha\beta+\frac{1}{\alpha\beta}+\left[\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\right]$

$= \frac{c}{a}+\frac{1}{(c/a)}+\left[\frac{(-b/a)^2-2(c/a)}{c/a}\right]$

$= \frac{c}{a}+\frac{a}{c}+\left[\frac{b^2/a^2-\frac{2c}{a}}{c/a}\right]$

$P = \frac{c}{a}+\frac{a}{c}+\frac{(b^2-2ac)a}{c.a^2}$

$= \frac{c}{a}+\frac{a}{c}+\frac{b^2-2ac}{ac}$

$= \frac{c^2+a^2+b^2-2ac}{ac}$

$P = \frac{c^2+a^2-2ca+b^2}{ac}$

$P = \frac{(c-a)^2+b^2}{ac}$

Eq is $y^2-Sy+P=0$

$y^2+\left(\frac{bc+ab}{ac}\right)y+\frac{(c-a)^2+b^2}{ac}=0$

$ac\,y^2+(bc+ab)y+(c-a)^2+b^2=0$

(vii) $(\alpha-\beta)^2, (\alpha+\beta)^2$

Sum (S) $= (\alpha-\beta)^2+(\alpha+\beta)^2$

$S = \alpha^2+\beta^2\cancel{-2\alpha\beta}+\alpha^2+\beta^2\cancel{+2\alpha\beta}$

$= 2(\alpha^2+\beta^2)$

$= 2(\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta)$

$= 2\{(\alpha+\beta)^2-2\alpha\beta\}$

$= 2\left((-b/a)^2-2\left(\frac{c}{a}\right)\right)$

$= 2\left(\frac{b^2}{a^2}-\frac{2c}{a}\right)$

$= 2\left[\frac{b^2-2ac}{a^2}\right]$

Product (P) $= (\alpha-\beta)^2\cdot(\alpha+\beta)^2$

$P = \{(\alpha+\beta)^2-4\alpha\beta\}(\alpha+\beta)^2$

$= \left\{\left(\frac{-b}{a}\right)^2-4\left(\frac{c}{a}\right)\right\}\cdot\left(\frac{-b}{a}\right)^2$

$= \left\{\frac{b^2}{a^2}-\frac{4c}{a}\right\}\cdot\frac{b^2}{a^2}$

$= \left(\frac{b^2-4ac}{a^2}\right)\left(\frac{b^2}{a^2}\right)$

$P = \frac{b^4-4acb^2}{a^4}$

Thus Eq $y^2-Sy+P=0$

$y^2-\frac{2(b^2-2ac)}{a^2}y+\frac{b^4-4ab^2c}{a^4}=0$

$a^4y^2-2a^2(b^2-2ac)y+b^2-4ab^2c=0$

(req. Equation)

(viii) $-\frac{1}{\alpha^3}, -\frac{1}{\beta^3}$

$$\text{Sum}(S') = -\frac{1}{\alpha^3} + \left(-\frac{1}{\beta^3}\right) = -\left\{\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right\}$$

$$= -\left\{\frac{\alpha^3+\beta^3}{(\alpha\beta)^3}\right\}$$

$$= -\left\{\frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3}\right\}$$

$$= -\left\{\frac{-b^3+3abc}{c^3}\right\} \text{ from (v)}$$

$$S = \frac{b^3-3abc}{c^3}$$

$$\text{Product}(P) = \left(-\frac{1}{\alpha^3}\right)\cdot\left(-\frac{1}{\beta^3}\right)$$

$$P = \frac{1}{(\alpha\beta)^3} = \frac{1}{(c/a)^3} = \frac{a^3}{c^3}$$

Thus Eq $y^2 - Sy + P = 0$

$$y^2 - \left(\frac{b^3-3abc}{c^3}\right)y + \frac{a^3}{c^3} = 0$$

$$c^3y^2 - (b^3-3abc)y + a^3 = 0$$

required Eq.

Q.8 If α, β are the roots of $5x^2 - x - 2 = 0$, form Equation whose roots are $\frac{3}{\alpha}, \frac{3}{\beta}$.

Sol:

$$\text{Sum of roots}(S) = (\alpha+\beta) = \frac{1}{5}$$

$$\text{Product}(P) = \alpha\beta = \frac{-2}{5}$$

The roots of required Eq are $\frac{3}{\alpha}, 3/\beta$

$$\text{Sum}(S) = \frac{3}{\alpha} + \frac{3}{\beta} = \frac{3(\alpha+\beta)}{\alpha\beta}$$

$$S = \frac{3(1/5)}{-2/5} = \frac{3\cdot5}{5\cdot-2} = -3/2$$

$$\text{Product}(P) = \frac{3}{\alpha}\cdot\frac{3}{\beta} = \frac{3\cdot3}{\alpha\beta}$$

$$P = \frac{9}{(-2/5)} = \frac{-45}{2}$$

Thus required Eq

$$y^2 - Sy + P = 0$$

$$y^2 + \frac{3}{2}x - \frac{45}{2} = 0$$

$$2y^2 + 3x - 45 = 0 \text{ (req. Eq).}$$

Q.9 If α, β are the roots of $x^2 - 3x + 5 = 0$, form Eq whose roots are $\frac{1-\alpha}{1+\alpha}, \frac{1-\beta}{1+\beta}$.

Sol: Sum of roots $(S) = \alpha+\beta = 3/1 = 3$

Product $(P) = \alpha\beta = 5/1 = 5$

The roots of req uired Eq are $\frac{1-\alpha}{1+\alpha}, \frac{1-\beta}{1+\beta}$

$$\text{Sum}(S) = \frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta}$$

$$(S) = \frac{(1-\alpha)(1+\beta) + (1-\beta)(1+\alpha)}{(1+\alpha)(1+\beta)}$$

$$S = \frac{(1-\alpha+\beta-\alpha\beta) + (1-\beta+\alpha-\alpha\beta)}{1+(\alpha+\beta)+\alpha\beta}$$

$$S = \frac{1-\cancel{\alpha}+\cancel{\beta}-\alpha\beta+1-\cancel{\beta}+\cancel{\alpha}-\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

$$S = \frac{2-2\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

$$S = \frac{2-2(5)}{1+3+5} = \frac{2-10}{9} = -\frac{8}{9}$$