$$x^2+1/x^2-2=0$$
$$\frac{x^4+1-2x^2}{x^2}=0$$
$$x^4-2x^2+1=0$$
Put $x^2=y$
$$y^2-2y+1=0$$
$$y^2-y-y+1=0$$
$$y(y-1)-1(y-1)=0$$
$$(y-1)(y-1)=0$$
$$y=1,1$$
Putting $y=x^2$
$$x^2=1 \quad \wedge \quad x^2=1$$
$$x=\pm1 \quad \wedge \quad x=\pm1$$
$$\Rightarrow x=\pm1$$

$$x^2+1/x^2-4=0$$
$$\frac{x^4+1-4x^2}{x^2}=0$$
$$x^4-4x^2+1=0$$
Put $x^2=y$
$$y^2-4y+1=0$$
$$y=\frac{4\pm\sqrt{16-4(1)(1)}}{2}$$
$$y=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm2\sqrt3}{2}$$
$$y=2\pm\sqrt3$$
$$x^2=2\pm\sqrt3$$
$$x^2=2+\sqrt3 \quad \wedge \quad x^2=2-\sqrt3$$
$$x=\pm\sqrt{2+\sqrt3} \quad \wedge \quad x=\pm\sqrt{2-\sqrt3}$$
$$x=\pm\sqrt{2+3^{1/2}},\ \pm\sqrt{2-3^{1/2}}$$

Thus the Solution Set is
$$S.S=\{\pm1, \pm\sqrt{2+3^{1/2}}, \pm\sqrt{2-3^{1/2}}\}$$

TAHIR $\boxed{\text{Exercise: 4.3}}$ **TAHIR**

Radical Equation:-

"The Equation which contains a radical sign "$\sqrt{\ }$" is called radical Equation".

Such as $\sqrt{ax^2+bx+c}=K$

$\sqrt{(ax-b)(cx+d)(ex+f)}=S$

Type-V This type of Equations is of the form $(ax^2+bx+c)+\sqrt{ax^2+bx+d}=n$ having Common ax^2+bx terms.

In this case Put $y=\sqrt{ax^2+bx+d}$ and adjust according to Quadratic Equation and Solve.

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Q.1 $3x^2+2x-\sqrt{3x^2+2x-1}=7$

(20)

Put $\sqrt{3x^2+2x-1}=y$

$y^2=3x^2+2x-1$

$3x^2+2x=y^2+1$

$(y^2+1)-y=7$

$y^2-y+1-7=0$

$y^2-y-6=0$

$y^2-3y+2y-6=0$

$y(y-3)+2(y-3)=0$

$(y-3)(y+2)=0$

$y-3=0 \quad \wedge \quad y+2=0$

Putting $y=\sqrt{3x^2+2x-1}$ back.

$\sqrt{3x^2+2x-1}-3=0$	$\sqrt{3x^2+2x-1}+2=0$
$\sqrt{3x^2+2x-1}=3$	$\sqrt{3x^2+2x-1}=-2$
Squaring	Here does not exist any root of this Eq due to -ve Sign Equal to radical expression.
$3x^2+2x-1=9$	
$3x^2+2x-9-1=0$	
$3x^2+2x-10=0$	
$x=\dfrac{-2\pm\sqrt{4-4(3)(-10)}}{2(3)}$	
$x=\dfrac{-2\pm\sqrt{4+120}}{6}$	

$x=\dfrac{-2\pm\sqrt{124}}{6} \Rightarrow x=\dfrac{-2\pm 2\sqrt{31}}{6}$

$x=\dfrac{-1\pm\sqrt{31}}{3}$

Thus Solution Set is

$S.S=\left\{\dfrac{-1\pm\sqrt{31}}{3}\right\}$

Q.2 $x^2-\dfrac{x}{2}-7=x-3\sqrt{2x^2-3x+2}$

$\dfrac{2x^2-x-14}{2}=x-3\sqrt{2x^2-3x+2}$

Multiplying both Sides by 2

$2x^2-x-14=2x-6\sqrt{2x^2-3x+2}$

$2x^2-x-2x-14+6\sqrt{2x^2-3x+2}=0$

$2x^2-3x+6\sqrt{2x^2-3x+2}=14$

Put $\sqrt{2x^2-3x+2}=y$

$y^2=2x^2-3x+2$

$y^2-2=2x^2-3x$

$(y^2-2)+6y-14=0$

$y^2+6y-16=0$

$y^2+8y-2y-16=0$

$y(y+8)-2(y+8)=0$

$(y-2)(y+8)=0$

$y-2=0 \quad \wedge \quad y+8=0$

Putting $y=\sqrt{2x^2-3x+2}$

$\sqrt{2x^2-3x+2}+8=0$	$\sqrt{2x^2-3x+2}-2=0$
$\sqrt{2x^2-3x+2}=-8$	$\sqrt{2x^2-3x+2}=2$
There is no root due to -ve Sign Equal to radical expression.	Squaring
	$2x^2-3x+2=4$
	$2x^2-3x-4+2=0$
	$2x^2-3x-2=0$
	$2x^2-4x+x-2=0$
	$2x(x-2)+1(x-2)=0$
	$(2x+1)(x-2)=0$
	$x=2, -1/2$

Thus Solution Set is

$S.S=\{2, -1/2\}$

Type-VII This type of Equation is in the form of

$\sqrt{x\pm a} \pm \sqrt{x\pm b} = \sqrt{x\pm c}$

Squaring twice, we get quadratic Equation.

Q.3 $\sqrt{2x+8}+\sqrt{x+5} = 7$

Squaring on both sides

$(2x+8)+(x+5)+2\sqrt{(2x+8)(x+5)} = 49$

$3x+13+2\sqrt{(2x+8)(x+5)} = 49$

$2\sqrt{(2x+8)(x+5)} = 49-3x-13$

$2\sqrt{2x^2+8x+10x+40} = 36-3x$

$2\sqrt{2x^2+18x+40} = 36-3x$

Squaring again

$4(2x^2+18x+40) = 1296+9x^2-216x$

$8x^2+72x+160 = 9x^2-216x+1296$

$9x^2-8x^2-216x-72x+1296-160 = 0$

$x^2-288x+1136 = 0$

$$x = \frac{-(-288)\pm\sqrt{(-288)^2-4(1)(1136)}}{2}$$

$$x = \frac{288\pm\sqrt{82944-4544}}{2}$$

$$x = \frac{288\pm\sqrt{78400}}{2}$$

$$x = \frac{288\pm 280}{2}$$

$$x = \frac{288+280}{2}, \frac{288-280}{2}$$

$$x = \frac{568}{2}, \frac{8}{2} \Rightarrow x = 284, 4$$

Thus Solution Set is $x=284$ is extraneous root.

$S\cdot S = \{4\}$

Q.4 $\sqrt{3x+4} = 2+\sqrt{2x-4}$

$\sqrt{3x+4}-\sqrt{2x-4} = 2$

Squaring on both sides

$(3x+4)+(2x-4)-2\sqrt{(3x+4)(2x-4)} = 4$

$5x-2\sqrt{6x^2-12x+8x-16} = 4$

$5x-4 = 2\sqrt{6x^2-4x-16}$

Squaring again, we have

$25x^2+16-40x = 4(6x^2-4x-16)$

$25x^2+16-40x = 24x^2-16x-64$

$25x^2-24x^2-40x+16x+16+64 = 0$

$x^2-24x+80 = 0$

$x^2-20x-4x+80 = 0$

$x(x-20)-4(x-20) = 0$

$(x-4)(x-20) = 0$

$x = 4, 20$

Thus Solution Set is

$S\cdot S = \{4, 20\}$

Extraneous Root:-

"The root of an Equation which is obtained from Equation but does not satisfy it is called Extraneous root."

Such as $\sqrt{x} = -3$

$x = 9$ does not satisfy so $\{9\}$ is not solution set

9 is Extraneous root.

Q.5 $\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

Squaring on both sides

$(x+7)+(x+2)+2\sqrt{(x+7)(x+2)}=6x+13$

$2x+9+2\sqrt{x^2+7x+2x+14}=6x+13$

$2\sqrt{x^2+9x+14}=6x+13-2x-9$

$2\sqrt{x^2+9x+14}=4x+4$

$\sqrt{x^2+9x+14}=2x+2$

Squaring again

$x^2+9x+14=4x^2+4+8x$

$4x^2-x^2+8x-9x+4-14=0$

$3x^2-x-10=0$

$3x^2-6x+5x-10=0$

$3x(x-2)+5(x-2)=0$

$(x-2)(3x+5)=0$

$x=2,\ -5/3$

Thus Solution Set is

$S\cdot S=\{2,\ -5/3\}$.

Type·VI Q.

Q.6 $\sqrt{x^2+x+1}+\sqrt{x^2+x-1}=1$

(22)

Squaring on both sides

$(x^2+x+1)+(x^2+x-1)+2\sqrt{(x^2+x+1)(x^2+x-1)}=1$

$2x^2+2x+2\sqrt{(x^2+x)^2-(1)^2}=1$

$2\sqrt{x^4+x^2+2x^3-1}=1-2x^2-2x$

Squaring again, we have.

$4(x^4+x^2+2x^3-1)=1+4x^4+4x^2-4x^2+8x^3-4x$

$\cancel{4x^4}+\cancel{8x^3}+4x^2-4-1-\cancel{4x^4}-\cancel{4x^2}+\cancel{4x^2}-\cancel{8x^3}-4x=0$

$4x^2-4x-5=0$

$x=\dfrac{-(-4)\pm\sqrt{(-4)^2-(4)(4)(-5)}}{2(4)}$

$x=\dfrac{4\pm\sqrt{16+80}}{8}$

$x=\dfrac{4\pm\sqrt{96}}{8}\Rightarrow x=\dfrac{4\pm4\sqrt6}{8}$

$x=\dfrac{1\pm\sqrt6}{2}$

Thus Solution Set is

$S\cdot S=\left\{\dfrac{1\pm\sqrt6}{2}\right\}$

Type-VII This type of Equations is in the form of

$\sqrt{a_1x^2+b_1x+c_1}+\sqrt{a_2x^2+b_2x+c_2}=\sqrt{a_3x^2+b_3x+c_3}$

containing a Common factor. remaining Part is of the form of type VI.

$\Rightarrow$ **Q.7** $\sqrt{x^2+2x-3}+\sqrt{x^2+7x-8}=\sqrt{5(x^2+3x-4)}$

$\sqrt{x^2+3x-x-3}+\sqrt{x^2+8x-x-8}=\sqrt{5(x^2+4x-x-4)}$

$\sqrt{(x+3)(x-1)}+\sqrt{(x+8)(x-1)}=\sqrt{5(x+4)(x-1)}$

$\sqrt{(x+3)(x-1)}+\sqrt{(x+8)(x-1)}-\sqrt{5(x+4)(x-1)}=0$

$\sqrt{(x-1)}\left[\sqrt{x+3}+\sqrt{x+8}-\sqrt{5(x+4)}\right]=0$

$\sqrt{x-1}=0\ \wedge\ \sqrt{x+3}+\sqrt{x+8}-\sqrt{5(x+4)}=0$

$\sqrt{x-1}=0\Rightarrow x-1=0$

$x=1$ —— ①

$\sqrt{2x+1}=0 \wedge \sqrt{x-3}+3-\sqrt{x+12}=0$

$\sqrt{x+3}+\sqrt{x+8}=\sqrt{5(x+4)}$

Squaring on both sides.

$(x+3)+(x+8)+2\sqrt{(x+3)(x+8)}=5(x+4)$

$2x+11+2\sqrt{x^2+8x+3x+24}=5x+20$

$2\sqrt{x^2+11x+24}=5x-2x+20-11$

$2\sqrt{x^2+11x+24}=3x+9$

Squaring again

$4(x^2+11x+24)=9x^2+81+54x$

$9x^2-4x^2+54x-44x+81-96=0$

$5x^2+10x-15=0$

$$x=\frac{-(10)\pm\sqrt{(10)^2-4(5)(-15)}}{2(5)}$$

$$x=\frac{-10\pm\sqrt{100+300}}{10}$$

$$x=\frac{-10\pm\sqrt{400}}{10}$$

$$x=\frac{-10\pm 20}{10} \Rightarrow x=\frac{-10+20}{10},\frac{-10-20}{3}$$

$x=1,-3$

Thus Solution set is

$S.S=\{1,-3\}$

Q.8 $\sqrt{(2x^2-5x-3)}+3\sqrt{2x+1}=\sqrt{2x^2+25x+12}$

$\sqrt{(2x^2-6x+x-3)}+3\sqrt{2x+1}=\sqrt{2x^2+24x+x+12}$

$\sqrt{(2x+1)(x-3)}+3\sqrt{2x+1}=\sqrt{(2x+1)(x+12)}$

$\sqrt{(2x+1)(x-3)}+3\sqrt{2x+1}-\sqrt{(2x+1)(x+12)}=0$

$\sqrt{2x+1}\left[\sqrt{x-3}+3-\sqrt{x+12}\right]=0$

$\sqrt{2x+1}=0 \Rightarrow 2x+1=0$

$x=-\frac{1}{2}$ —— ①

Now $\sqrt{x-3}+3-\sqrt{x+12}=0$

$\sqrt{x+12}-\sqrt{x-3}=3$

Squaring on both sides.

$(x+12)+(x-3)-2\sqrt{(x+12)(x-3)}=9$

$2x+9-2\sqrt{x^2+12x-3x-36}=9$

$2x+\cancel{9}-\cancel{9}=2\sqrt{x^2+9x-36}$

$2x=2\sqrt{x^2+9x-36}$

$x=\sqrt{x^2+9x-36}$

Squaring on both sides, again

$x^2=x^2+9x-36$

$x^2-x^2+9x-36=0$

$9x-36=0 \Rightarrow 9x=36$

$x=\frac{36}{9} \Rightarrow x=4$ —— ②

From ① and (2) Solution set is

$S.S=\{4,-\frac{1}{2}\}$

Q.9 $\sqrt{3x^2-5x+2}+\sqrt{6x^2-11x+5}=\sqrt{5x^2-9x+}$

$\sqrt{3x^2-3x-2x+2}+\sqrt{6x^2-6x-5x+5}=\sqrt{5x^2-5x-4x+}$

$\sqrt{(3x-2)(x-1)}+\sqrt{(6x-5)(x-1)}=\sqrt{(5x-4)(x-1)}$

$\sqrt{(3x-2)(x-1)}+\sqrt{(6x-5)(x-1)}-\sqrt{(5x-4)(x-1)}=$

$\sqrt{(x-1)}\left(\sqrt{3x-2}+\sqrt{6x-5}-\sqrt{5x-4}\right)=0$

$\sqrt{(x-1)}=0 \wedge \sqrt{3x-2}+\sqrt{6x-5}-\sqrt{5x-4}=0$

$\sqrt{x-1}=0 \Rightarrow x-1=0$

$x=1$ —— ①

$\sqrt{3x-2}+\sqrt{6x-5}-\sqrt{5x-4}=0$

$\sqrt{3x-2}+\sqrt{6x-5}=\sqrt{5x-4}$

Squaring on both sides

$(3x-2)+(6x-5)+2\sqrt{(3x-2)(6x-5)}=5x-4$

$9x-7+2\sqrt{18x^2-15x-12x+10}=5x-4$

$2\sqrt{18x^2-27x+10}=5x-4-9x+7$

$2\sqrt{18x^2-27x+10}=3-4x$

Squaring again, we have

$4(18x^2-27x+10)=9+16x^2-24x$

$72x^2-108x+40-9-16x^2+24x=0$

$56x^2-84x+31=0$

$x=\dfrac{-(-84)\pm\sqrt{(-84)^2-4(56)(31)}}{2(56)}$

$x=\dfrac{84\pm\sqrt{7056-6944}}{112}$

$x=\dfrac{84\pm\sqrt{112}}{112}$

$x=\dfrac{84\pm4\sqrt{7}}{112}$

$x=\dfrac{24\pm\sqrt{7}}{28}$ —— ②

Thus from ① and ② Solution set is

$S\cdot S=\left\{1,\ \dfrac{24\pm\sqrt{7}}{28}\right\}$

Q.10 $(x+4)(x+1)=\sqrt{x^2+2x-15}+3x+31$

$(x^2+4x+x+4)=\sqrt{x^2+2x-15}+3x+31$

$x^2+5x+4-3x-31=\sqrt{x^2+2x-15}$

$x^2+2x-27-\sqrt{x^2+2x-15}=0$

This is the Eq of type V so

Put $y=\sqrt{x^2+2x-15}$

$y^2=x^2+2x-15$

$y^2+15=x^2+2x$

$y^2+15-27-y=0$

$y^2-y-12=0$

$y^2-4y+3y-12=0$

$(y-4)(y+3)=0$

$y-4=0 \quad \wedge \quad y+3=0$

Putting the Value of $y=\sqrt{x^2+2x-15}$

$\sqrt{x^2+2x-15}-4=0$

$\sqrt{x^2+2x-15}=4$

Squaring

$x^2+2x-15=16$

$x^2+2x-15-16=0$

$x^2+2x-31=0$

$x=\dfrac{-2\pm\sqrt{4-4(1)(31)}}{2}$

$x=\dfrac{-2\pm\sqrt{4+124}}{2}$

$x=\dfrac{-2\pm\sqrt{128}}{2}$

$x=\dfrac{-2\pm8\sqrt{2}}{2}$

$x=-1\pm4\sqrt{2}$

$\sqrt{x^2+2x-15}+3=0$

$\sqrt{x^2+2x-15}=-3$

There does not exist any root satisfying Eq because radical is -ve

Thus the Solution is

$S\cdot S=\{-1\pm4\sqrt{2}\}$