

$$x^2 + \frac{1}{x^2} - 2 = 0$$

$$\frac{x^4 + 1 - 2x^2}{x^2} = 0$$

$$x^4 - 2x^2 + 1 = 0$$

Put  $x^2 = y$

$$y^2 - 2y + 1 = 0$$

$$y^2 - y - y + 1 = 0$$

$$y(y-1) - 1(y-1) = 0$$

$$(y-1)(y-1) = 0$$

$$y = 1, 1$$

Putting  $y = x^2$

$$x^2 = 1 \quad \wedge \quad x^2 = 1$$

$$x = \pm 1 \quad \wedge \quad x = \pm 1$$

$$\Rightarrow x = \pm 1$$

Thus the solution set is

$$S.S = \left\{ \pm 1, \pm \sqrt{2+3^{1/2}}, \pm \sqrt{2-3^{1/2}} \right\}$$

TAHIR

Exercise: 4.3

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Radical Equation:-

"The Equation which is of the form

contains a radical sign " $\sqrt{\quad}$ "  $(ax^2 + bx + c) + \sqrt{ax^2 + bx + d} = n$

is called radical Equation." having common  $ax^2 + bx$  terms.

Such as  $\sqrt{ax^2 + bx + c} = k$

$$\sqrt{(ax-b)(cx+d)(ex+f)} = s$$

Type-V This type of Equations

In this case Put  $y = \sqrt{ax^2 + bx + d}$  and adjust according to Quadratic Equation and Solve.

$$x^2 + \frac{1}{x^2} - 4 = 0$$

$$\frac{x^4 + 1 - 4x^2}{x^2} = 0$$

$$x^4 - 4x^2 + 1 = 0$$

Put  $x^2 = y$

$$y^2 - 4y + 1 = 0$$

$$y = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2}$$

$$y = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$$

$$y = 2 \pm \sqrt{3}$$

$$x^2 = 2 \pm \sqrt{3}$$

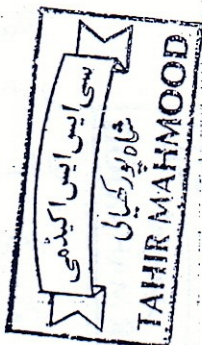
$$x^2 = 2 + \sqrt{3} \quad \wedge \quad x^2 = 2 - \sqrt{3}$$

$$x = \pm \sqrt{2 + \sqrt{3}} \quad \wedge \quad x = \pm \sqrt{2 - \sqrt{3}}$$

$$x = \pm \sqrt{2 + 3^{1/2}}, \pm \sqrt{2 - 3^{1/2}}$$

(19)

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Q.1  $3x^2 + 2x - \sqrt{3x^2 + 2x - 1} = 7$

Q.2  $x^2 - \frac{x}{2} - 7 = x - 3\sqrt{2x^2 - 3x + 2}$

Put  $\sqrt{3x^2 + 2x - 1} = y$  (20)

$y^2 = 3x^2 + 2x - 1$

$3x^2 + 2x = y^2 + 1$

$(y^2 + 1) - y = 7$

$y^2 - y + 1 - 7 = 0$

$y^2 - y - 6 = 0$

$y^2 - 3y + 2y - 6 = 0$

$y(y - 3) + 2(y - 3) = 0$

$(y - 3)(y + 2) = 0$

$y - 3 = 0 \wedge y + 2 = 0$

Putting  $y = \sqrt{3x^2 + 2x - 1}$  back.

$\sqrt{3x^2 + 2x - 1} - 3 = 0 \quad \sqrt{3x^2 + 2x - 1} + 2 = 0$

$\sqrt{3x^2 + 2x - 1} = 3 \quad \sqrt{3x^2 + 2x - 1} = -2$

Squaring

$3x^2 + 2x - 1 = 9$

$3x^2 + 2x - 9 - 1 = 0$

$3x^2 + 2x - 10 = 0$

$x = \frac{-2 \pm \sqrt{4 - 4(3)(-10)}}{2(3)}$

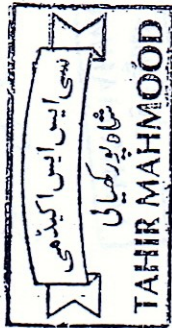
$x = \frac{-2 \pm \sqrt{4 + 120}}{6}$

$x = \frac{-2 \pm \sqrt{124}}{6} \Rightarrow x = \frac{-2 \pm 2\sqrt{31}}{6}$

$x = \frac{-1 \pm \sqrt{31}}{3}$

Thus Solution Set is

S.S =  $\left\{ \frac{-1 \pm \sqrt{31}}{3} \right\}$



$\frac{2x^2 - x - 14}{2} = x - 3\sqrt{2x^2 - 3x + 2}$

Multiplying both sides by 2

$2x^2 - x - 14 = 2x - 6\sqrt{2x^2 - 3x + 2}$

$2x^2 - x - 2x - 14 + 6\sqrt{2x^2 - 3x + 2} = 0$

$2x^2 - 3x + 6\sqrt{2x^2 - 3x + 2} = 14$

Put  $\sqrt{2x^2 - 3x + 2} = y$

$y^2 = 2x^2 - 3x + 2$

$y^2 - 2 = 2x^2 - 3x$

$(y^2 - 2) + 6y - 14 = 0$

$y^2 + 6y - 16 = 0$

$y^2 + 8y - 2y - 16 = 0$

$y(y + 8) - 2(y + 8) = 0$

$(y - 2)(y + 8) = 0$

$y - 2 = 0 \wedge y + 8 = 0$

Putting  $y = \sqrt{2x^2 - 3x + 2}$

$\sqrt{2x^2 - 3x + 2} + 8 = 0 \quad \sqrt{2x^2 - 3x + 2} - 2 = 0$

$\sqrt{2x^2 - 3x + 2} = -8 \quad \sqrt{2x^2 - 3x + 2} = 2$

There is no root

due to -ve sign

Equal to radical

expression.

Squaring

$2x^2 - 3x + 2 = 4$

$2x^2 - 3x - 4 + 2 = 0$

$2x^2 - 3x - 2 = 0$

$2x^2 - 4x + x - 2 = 0$

$2x(x - 2) + 1(x - 2) = 0$

$(2x + 1)(x - 2) = 0$

$x = 2, -1/2$

Thus Solution Set is

S.S =  $\left\{ 2, -1/2 \right\}$

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# Type-VI This type of Equation Q.4 $\sqrt{3x+4} = 2 + \sqrt{2x-4}$

is in the form of

$$\sqrt{x+a} \pm \sqrt{x+b} = \sqrt{x+c}$$

Squaring twice, we get quadratic Equation.

Q.3  $\sqrt{2x+8} + \sqrt{x+5} = 7$

Squaring on both sides

$$(2x+8) + (x+5) + 2\sqrt{(2x+8)(x+5)} = 49$$

$$3x+13 + 2\sqrt{(2x+8)(x+5)} = 49$$

$$2\sqrt{(2x+8)(x+5)} = 49 - 3x - 13$$

$$2\sqrt{2x^2+8x+10x+40} = 36 - 3x$$

$$2\sqrt{2x^2+18x+40} = 36 - 3x$$

Squaring again

$$4(2x^2+18x+40) = 1296 + 9x^2 - 216x$$

$$8x^2+72x+160 = 9x^2-216x+1296$$

$$9x^2 - 8x^2 - 216x - 72x + 1296 - 160 = 0$$

$$x^2 - 288x + 1136 = 0$$

$$x = \frac{-(-288) \pm \sqrt{(-288)^2 - 4(1)(1136)}}{2}$$

$$x = \frac{288 \pm \sqrt{82944 - 4544}}{2}$$

$$x = \frac{288 \pm \sqrt{78400}}{2}$$

$$x = \frac{288 \pm 280}{2}$$

$$x = \frac{288+280}{2}, \frac{288-280}{2}$$

$$x = \frac{568}{2}, \frac{8}{2} \Rightarrow x = 284, 4$$

Thus Solution set is  $x=284$  is extraneous root.

$$S.S = \{4\}$$

Q.4  $\sqrt{3x+4} = 2 + \sqrt{2x-4}$

$$\sqrt{3x+4} - \sqrt{2x-4} = 2 \quad (21)$$

Squaring on both sides

$$(3x+4) + (2x-4) - 2\sqrt{(3x+4)(2x-4)} = 4$$

$$5x - 2\sqrt{6x^2 - 12x + 8x - 16} = 4$$

$$5x - 4 = 2\sqrt{6x^2 - 4x - 16}$$

Squaring again, we have

$$25x^2 + 16 - 40x = 4(6x^2 - 4x - 16)$$

$$25x^2 + 16 - 40x = 24x^2 - 16x - 64$$

$$25x^2 - 24x^2 - 40x + 16x + 16 + 64 = 0$$

$$x^2 - 24x + 80 = 0$$

$$x^2 - 20x - 4x + 80 = 0$$

$$x(x-20) - 4(x-20) = 0$$

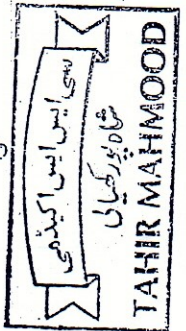
$$(x-4)(x-20) = 0$$

$$x = 4, 20$$

Thus Solution set is

$$S.S = \{4, 20\}$$

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## Extraneous Root:-

"The root of an Equation which is obtained from Equation but does not satisfy it is called Extraneous root."

Such as  $\sqrt{x} = -3$

$x = 9$  does not satisfy

so  $\{9\}$  is not solution set

9 is Extraneous root.

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Q.5  $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

Squaring on both sides

$(x+7) + (x+2) + 2\sqrt{(x+7)(x+2)} = 6x+13$

$2x+9 + 2\sqrt{x^2+9x+14} = 6x+13$

$2\sqrt{x^2+9x+14} = 6x+13-2x-9$

$2\sqrt{x^2+9x+14} = 4x+4$

$\sqrt{x^2+9x+14} = 2x+2$

Squaring again

$x^2+9x+14 = 4x^2+4+8x$

$4x^2-x^2+8x-9x+4-14=0$

$3x^2-x-10=0$

$3x^2-6x+5x-10=0$

$3x(x-2)+5(x-2)=0$

$(x-2)(3x+5)=0$

$x=2, -5/3$

Thus Solution Set is

S.S =  $\{2, -5/3\}$

Type-VI Q.

Q.6  $\sqrt{x^2+x+1} + \sqrt{x^2+x-1} = 1$

Squaring on both sides

$(x^2+x+1) + (x^2+x-1) + 2\sqrt{(x^2+x+1)(x^2+x-1)} = 1$

$2x^2+2x+2\sqrt{(x^2+x)^2-(1)^2} = 1$

$2\sqrt{x^4+x^2+2x^3-1} = 1-2x^2-2x$

Squaring again, we have

$4(x^4+x^2+2x^3-1) = 1+4x^4+4x^2-4x^2+8x^3-4x$

$4x^4+8x^3+4x^2-4-1-4x^4-4x^2+4x^2-8x^3-4x=0$

$4x^2-4x-5=0$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - (4)(-5)}}{2(4)}$

$x = \frac{4 \pm \sqrt{16+80}}{8}$

$x = \frac{4 \pm \sqrt{96}}{8} \Rightarrow x = \frac{4 \pm 4\sqrt{6}}{8}$

$x = \frac{1 \pm \sqrt{6}}{2}$

Thus Solution Set is

S.S =  $\{ \frac{1 \pm \sqrt{6}}{2} \}$

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Type-VII This type of Equations  $\Rightarrow$

is in the form of

$\sqrt{a_1x^2+b_1x+c_1} + \sqrt{a_2x^2+b_2x+c_2} = \sqrt{a_3x^2+b_3x+c_3}$

containing a Common factor.

remaining Parts of the form

of type VI.

Q.7  $\sqrt{x^2+2x-3} + \sqrt{x^2+7x-8} = \sqrt{5(x^2+3x-4)}$

$\sqrt{x^2+3x-x-3} + \sqrt{x^2+8x-x-8} = \sqrt{5(x^2+4x-x-4)}$

$\sqrt{(x+3)(x-1)} + \sqrt{(x+8)(x-1)} = \sqrt{5(x+4)(x-1)}$

$\sqrt{(x+3)(x-1)} + \sqrt{(x+8)(x-1)} - \sqrt{5(x+4)(x-1)} = 0$

$\sqrt{x-1} \left( \sqrt{x+3} + \sqrt{x+8} - \sqrt{5(x+4)} \right) = 0$

$\sqrt{x-1} = 0 \wedge \sqrt{x+3} + \sqrt{x+8} - \sqrt{5(x+4)} = 0$

$\sqrt{x-1} = 0 \Rightarrow x-1 = 0$

$x = 1$  — ①

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$$\sqrt{2x+1} = 0 \wedge \sqrt{x-3} + 3 - \sqrt{x+12} = 0$$

$$\sqrt{x+3} + \sqrt{x+8} = \sqrt{5(x+4)} \quad (23)$$

Squaring on both sides.

$$(x+3) + (x+8) + 2\sqrt{(x+3)(x+8)} = 5(x+4)$$

$$2x+11 + 2\sqrt{x^2+8x+3x+24} = 5x+20$$

$$2\sqrt{x^2+11x+24} = 5x-2x+20-11$$

$$2\sqrt{x^2+11x+24} = 3x+9$$

Squaring again

$$4(x^2+11x+24) = 9x^2+81+54x$$

$$9x^2-4x^2+54x-44x+81-96 = 0$$

$$5x^2+10x-15 = 0$$

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(5)(-15)}}{2(5)}$$

$$x = \frac{-10 \pm \sqrt{100+300}}{10}$$

$$x = \frac{-10 \pm \sqrt{400}}{10}$$

$$x = \frac{-10 \pm 20}{10} \Rightarrow x = \frac{-10+20}{10}, \frac{-10-20}{10}$$

$$x = 1, -3$$

Thus Solution Set is

$$S.S = \{1, -3\}$$

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Q.8  $\sqrt{(2x^2-5x-3)} + 3\sqrt{2x+1} = \sqrt{2x^2+25x+12}$

$$\sqrt{(2x^2-6x+x-3)} + 3\sqrt{2x+1} = \sqrt{2x^2+24x+x+12}$$

$$\sqrt{(2x+1)(x-3)} + 3\sqrt{2x+1} = \sqrt{(2x+1)(x+12)}$$

$$\sqrt{(2x+1)(x-3)} + 3\sqrt{2x+1} - \sqrt{(2x+1)(x+12)} = 0$$

$$\sqrt{2x+1} (\sqrt{x-3} + 3 - \sqrt{x+12}) = 0$$

$$\sqrt{2x+1} = 0 \Rightarrow 2x+1 = 0$$

$$x = -\frac{1}{2} \quad \text{--- (1)}$$

Now  $\sqrt{x-3} + 3 - \sqrt{x+12} = 0$

$$\sqrt{x+12} - \sqrt{x-3} = 3$$

Squaring on both sides.

$$(x+12) + (x-3) - 2\sqrt{(x+12)(x-3)} = 9$$

$$2x+9 - 2\sqrt{x^2+12x-3x-36} = 9$$

$$2x+9-9 = 2\sqrt{x^2+9x-36}$$

$$2x = 2\sqrt{x^2+9x-36}$$

$$x = \sqrt{x^2+9x-36}$$

Squaring on both sides, again

$$x^2 = x^2+9x-36$$

$$x^2 - x^2 + 9x - 36 = 0$$

$$9x - 36 = 0 \Rightarrow 9x = 36$$

$$x = \frac{36}{9} \Rightarrow x = 4 \quad \text{--- (2)}$$

From (1) and (2) Solution Set is

$$S.S = \{4, -\frac{1}{2}\}$$

Q.9  $\sqrt{3x^2-5x+2} + \sqrt{6x^2-11x+5} = \sqrt{5x^2-9x+4}$

$$\sqrt{3x^2-3x-2x+2} + \sqrt{6x^2-6x-5x+5} = \sqrt{5x^2-5x-4x+4}$$

$$\sqrt{(3x-2)(x-1)} + \sqrt{(6x-5)(x-1)} = \sqrt{(5x-4)(x-1)}$$

$$\sqrt{(3x-2)(x-1)} + \sqrt{(6x-5)(x-1)} - \sqrt{(5x-4)(x-1)} = 0$$

$$\sqrt{(x-1)} (\sqrt{3x-2} + \sqrt{6x-5} - \sqrt{5x-4}) = 0$$

$$\sqrt{(x-1)} = 0 \wedge \sqrt{3x-2} + \sqrt{6x-5} - \sqrt{5x-4} = 0$$

$$\sqrt{x-1} = 0 \Rightarrow x-1 = 0$$

$$x = 1 \quad \text{--- (1)}$$

$$\sqrt{3x-2} + \sqrt{6x-5} - \sqrt{5x-4} = 0$$

$$\sqrt{3x-2} + \sqrt{6x-5} = \sqrt{5x-4}$$

- Squaring on both sides

$$(3x-2) + (6x-5) + 2\sqrt{(3x-2)(6x-5)} = 5x-4$$

$$9x-7 + 2\sqrt{18x^2-15x-12x+10} = 5x-4$$

$$2\sqrt{18x^2-27x+10} = 5x-4-9x+7$$

$$2\sqrt{18x^2-27x+10} = 3-4x$$

Squaring again, we have

$$4(18x^2-27x+10) = 9+16x^2-24x$$

$$72x^2-108x+40-9-16x^2+24x = 0$$

$$56x^2-84x+31 = 0$$

$$x = \frac{-(-84) \pm \sqrt{(-84)^2 - 4(56)(31)}}{2(56)}$$

$$x = \frac{84 \pm \sqrt{7056 - 6944}}{112}$$

$$x = \frac{84 \pm \sqrt{112}}{112}$$

$$x = \frac{84 \pm 4\sqrt{7}}{112}$$

$$x = \frac{24 \pm \sqrt{7}}{28} \quad \text{--- (2)}$$

Thus from (1) and (2) Solution Set is

$$S \cdot S = \left\{ 1, \frac{24 \pm \sqrt{7}}{28} \right\}$$

Q.10  $(x+4)(x+1) = \sqrt{x^2+2x-15} + 3x+31$

$$(x^2+4x+x+4) = \sqrt{x^2+2x-15} + 3x+31$$

$$x^2+5x+4-3x-31 = \sqrt{x^2+2x-15}$$

$$x^2+2x-27 - \sqrt{x^2+2x-15} = 0$$

This is the Eq of type V. So

(24)

Put  $y = \sqrt{x^2+2x-15}$

$$y^2 = x^2+2x-15$$

$$y^2+15 = x^2+2x$$

$$y^2+15-27-y = 0$$

$$y^2-y-12 = 0$$

$$y^2-4y+3y-12 = 0$$

$$(y-4)(y+3) = 0$$

$$y-4 = 0 \quad \wedge \quad y+3 = 0$$

Putting the value of  $y = \sqrt{x^2+2x-15}$

$$\sqrt{x^2+2x-15} - 4 = 0 \quad \sqrt{x^2+2x-15} + 3 = 0$$

$$\sqrt{x^2+2x-15} = 4$$

$$\sqrt{x^2+2x-15} = -3$$

Squaring

$$x^2+2x-15 = 16$$

$$x^2+2x-15-16 = 0$$

$$x^2+2x-31 = 0$$

$$x = \frac{-2 \pm \sqrt{4-4(1)(-31)}}{2}$$

$$x = \frac{-2 \pm \sqrt{4+124}}{2}$$

$$x = \frac{-2 \pm \sqrt{128}}{2}$$

$$x = \frac{-2 \pm 8\sqrt{2}}{2}$$

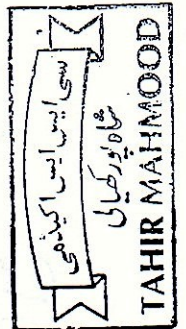
$$x = -1 \pm 4\sqrt{2}$$

Thus the Solution is

$$S \cdot S = \left\{ -1 \pm 4\sqrt{2} \right\}$$

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There does not exist any root satisfying Eq because radical is -ve



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