Exercise: 4.2

Type: I $ax^{2n}+bx^{n}+c=0$

In Case of this type put

$x^{n}=y$ where $n\in\mathbb{R}$

then equation will be reduced in Standard Quadratic Equation

- Using Previous methods we can Solve this type of Equations.

Q.1 $x^4-6x^2+8=0$

$\therefore$ Put $x^2=y$

$y^2-6y+8=0$

This is a quadratic Equation.

$y^2-4y-2y+8=0$

$y(y-4)-2(y-4)=0$

$(y-4)(y-2)=0$

$y-4=0 \wedge y-2=0$

$y=4 \wedge y=2$

Putting $y=x^2$ back, we have

$x^2=4 \wedge x^2=2$

$x=\pm 2 \wedge x=\pm\sqrt{2}$

$S.S=\{\pm 2, \pm\sqrt{2}\}$.

Q.2 $x^{-2}-10-3x^{-1}=0$

$x^{-2}-3x^{-1}-10=0$

Put $x^{-1}=y$

$y^2-3y-10=0$

This is quadratic Equation

$y^2-5y+2y-10=0$

$y(y-5)+2(y-5)=0$

$(y-5)(y+2)=0$

$y-5=0 \wedge y+2=0$

$y=5 \wedge y=-2$

Putting $y=x^{-1}$ back, we have

$x^{-1}=5 \wedge x^{-1}=-2$

$\frac{1}{x}=5 \wedge \frac{1}{x}=-2$

$x=\frac{1}{5} \wedge x=\frac{-1}{2}$

$S.S=\{\frac{1}{5}, \frac{-1}{2}\}$

Q.3 $x^6-9x^3+8=0$

Put $x^3=y$

$y^2-9y+8=0$

This is quadratic Equation

$y^2-8y-y+8=0$

$y(y-8)-1(y-8)=0$

$(y-8)(y-1)=0$

$y-8=0 \wedge y-1=0$

$y=8 \wedge y=1$

Putting $y=x^3$ back

$x^3=8 \wedge x^3=1$

$x^3-8=0 \wedge x^3-1=0$

$x^3-(2)^3=0 \wedge x^3-(1)^3=0$

using $a^3-b^3=(a-b)(a^2+ab+b^2)$

For $x^3-2^3=0$

$(x-2)(x^2+2x+4)=0$

$x-2=0 \quad x^2+2x+4=0$

$x=2 \quad x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=2 \wedge x=\dfrac{-2\pm\sqrt{4-4.1.4}}{2}$

$x=2 \wedge x=\dfrac{-2\pm\sqrt{4-16}}{2}$

$x=2 \wedge x=\dfrac{-2\pm\sqrt{-12}}{2}$

$x=2 \wedge x=\dfrac{-2\pm2\sqrt{-3}}{2}$

$x=2 \wedge x=-1\pm\sqrt{-3}$

For $x^3-1^3=0$

$(x-1)(x^2+x+1)=0$

$x-1=0 \quad \wedge \quad x^2+x+1=0$

$x=1 \quad \wedge\ x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=1 \quad \wedge\ x=\dfrac{-1\pm\sqrt{1-4}}{2}$

$x=1 \wedge x=\dfrac{-1\pm\sqrt{-3}}{2}$

Thus

$S.S=\left\{1, 2, -1\pm\sqrt{-3}, \dfrac{-1\pm\sqrt{-3}}{2}\right\}$

Q.4 $8x^6-19x^3-27=0$

Put $x^3=y$

$8y^2-19y-27=0$

This is quadratic Equation

$8y^2-27y+8y-27=0$

$y[8y-27]+1[8y-27]=0$

$(y+1)(8y-27)=0$

$y+1=0 \quad \wedge \quad 8y-27=0$

By putting $y=x^3$ back, we have

$x^3+1=0 \quad \wedge \quad 8x^3-27=0$

$x^3+(1)^3=0$

$(x+1)(x^2-x+1)=0$

$x+1=0 \Rightarrow x=-1$

$x^2-x+1=0$

$x=\dfrac{-(-1)\pm\sqrt{(-1)^2-4(1)}}{2}$

$x=\dfrac{1\pm\sqrt{1-4}}{2}$

$x=\dfrac{1\pm\sqrt{-3}}{2}$

Thus

$x=-1, \dfrac{1\pm\sqrt{-3}}{2}$

$(2x)^3-(3)^3=0$

$(2x-3)(4x^2+6x+9)=0$

$2x-3=0 \Rightarrow x=\frac{3}{2}$

$4x^2+6x+9=0$

$x=\dfrac{-6\pm\sqrt{36-4(4)(9)}}{2(4)}$

$x=\dfrac{-6\pm\sqrt{36-144}}{8}$

$x=\dfrac{-6\pm\sqrt{-108}}{8}$

$x=\dfrac{-6\pm6\sqrt{-3}}{8}$

$x=\dfrac{6(-1\pm\sqrt{-3})}{8}$

$x=\dfrac{3(-1\pm\sqrt{-3})}{4}$

Thus

$x=\frac{3}{2}, \dfrac{3(-1\pm\sqrt{-3})}{4}$

Thus Solution Set is

$S.S.=\left\{-1, \frac{3}{2}, \dfrac{1\pm\sqrt{-3}}{2}, \dfrac{3(-1\pm\sqrt{-3})}{4}\right\}$.

Q.5 $x^{2/5}-6x^{1/5}+8=0$

Put $x^{1/5}=y$

$y^2-6y+8=0$

This is Standard Quad. Equation.

$y^2-4y-2y+8=0$

$(y-4)(y-2)=0$

$y=4 \quad \wedge \quad y=2$

Putting $x^{1/5} = y$ back we have

$x^{1/5} = 4 \quad \wedge \quad x^{1/5} = 2$

$x = (4)^5 \quad \wedge \quad x = 2^5$

$x = 1024 \quad \wedge \quad x = 32$

Thus Solution Set is

$S.S = \{32, 1024\}$

Example(1): $x^{1/2} - x^{1/4} - 6 = 0$

$(x^{1/4})^2 - x^{1/4} - 6 = 0$

Put $y = x^{1/4}$

$y^2 - y - 6 = 0$

This is Standard Quadratic Equation.

$y^2 - 3y + 2y - 6 = 0$

$y(y-3) + 2(y-3) = 0$

$(y-3)(y+2) = 0$

$y - 3 = 0 \quad \wedge \quad y + 2 = 0$

$y = 3 \qquad y = -2$

Putting $x^{1/4} = y$ back

$x^{1/4} = 3 \quad \wedge \quad x^{1/4} = -2$

$x = (3)^4 \quad \wedge \quad x = (-2)^4$

$x = 81 \quad \wedge \quad x = 16$

Thus Solution Set is

$S.S = \{16, 81\}$

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Type-II The Equation of the type

$(x+a_1)(x+b_1)(x+a_2)(x+b_2) = K$

$\{x^2 + (a_1+b_1)x + a_1b_1\}\{x^2 + (a_2+b_2)x + a_2b_2\} = K$

Put $x^2 + (a_1+b_1)x = y$

when $x^2 + (a_1+b_1)x = x^2 + (a_2+b_2)x$

Using Substitution, we get a quadratic Equation and Solve for roots.

Q.6 $(x+1)(x+2)(x+3)(x+4) = 24$

$\{(x+1)(x+4)\}\{(x+2)(x+3)\} = 24$

$\{x^2 + x + 4x + 4\}\{x^2 + 2x + 3x + 6\} = 24$

$\{x^2 + 5x + 4\}\{x^2 + 5x + 6\} = 24$

Put $x^2 + 5x = y$

$(y+4)(y+6) = 24$

$y^2 + 4y + 6y + 24 = 24$

$y^2 + 10y = 0$

$y(y+10) = 0$

$y = 0 \quad \wedge \quad y + 10 = 0$

By Back putting $y = x^2 + 5x$

$x^2 + 5x = 0$

$x(x+5) = 0$

$x = 0 \quad \wedge \quad x = -5$

$x^2 + 5x + 10 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-5 \pm \sqrt{25 - 4 \cdot 1 \cdot 10}}{2}$

$x = \dfrac{-5 \pm \sqrt{25 - 40}}{2}$

$x = \dfrac{-5 \pm \sqrt{-15}}{2}$

Thus Solution Set is

$S.S = \left\{0, -5, \dfrac{-5 \pm \sqrt{-15}}{2}\right\}$

Q.7 $(x-1)(x+5)(x+8)(x+2)=880$

$$\{(x-1)(x+8)\}\{(x+2)(x+5)\}=880$$
$$\{x^2-x+8x-8\}\{x^2+5x+2x+10\}=880$$
$$\{x^2+7x-8\}\{x^2+7x+10\}=880$$

Put $x^2+7x=y$

$$(y-8)(y+10)=880$$
$$y^2-8y+10y-80=880$$
$$y^2+2y-80=880$$
$$y^2+2y-80-880=0$$
$$y^2+2y-960=0$$
$$y^2+32y-30y-960=0$$
$$y(y+32)-30(y+32)=0$$
$$(y+32)(y-30)=0$$
$$y+32=0 \quad \wedge \quad y-30=0$$

Putting $x^2+7x=y$ back

$$x^2+7x+32=0 \qquad\qquad x^2+7x-30=0$$

$$x=\frac{-7\pm\sqrt{49-4\times32}}{2} \qquad x=\frac{-7\pm\sqrt{49-4(1)(-30)}}{2}$$

$$x=\frac{-7\pm\sqrt{-79}}{2} \qquad x=\frac{-7\pm\sqrt{169}}{2}$$

$$x=\frac{-7\pm13}{2}$$
$$x=\frac{-7+13}{2} \wedge x=\frac{-7-13}{2}$$
$$x=\frac{6}{2} \wedge x=-\frac{20}{2}$$
$$x=3 \wedge x=-10$$

Thus Solution Set is

$$S.S=\left\{3,-10,\frac{-7\pm\sqrt{-79}}{2}\right\}$$

Q.8 $(x-5)(x-7)(x+6)(x+4)=504$

$$\{(x-5)(x+4)\}\{(x-7)(x+6)\}=504$$
$$\{x^2-5x+4x-20\}\{x^2-7x+6x-42\}=504$$
$$\{x^2-x-20\}\{x^2-x-42\}=504$$

Put $y=x^2-x$

$$(y-20)(y-42)=504$$
$$y^2-20y-42y+840=504$$
$$y^2-62y+840-504=0$$
$$y^2-62y+336=0$$
$$y^2-56y-6y+336=0$$
$$y(y-56)-6(y-56)=0$$
$$(y-56)(y-6)=0$$
$$y-56=0 \quad \wedge \quad y-6=0$$

Putting $y=x^2-x$ back.

$$x^2-x-56=0 \qquad\qquad x^2-x-6=0$$
$$a=1\ \ b=-1\ \ c=-56 \qquad a=1\ \ b=-1\ \ c=-6$$
$$x=\frac{1\pm\sqrt{1-4(1)(-56)}}{2} \qquad x=\frac{1\pm\sqrt{1-4(1)(-6)}}{2}$$
$$x=\frac{1\pm\sqrt{224+1}}{2} \qquad x=\frac{1\pm\sqrt{1+24}}{2}$$
$$x=\frac{1\pm\sqrt{225}}{2} \qquad x=\frac{1\pm\sqrt{25}}{2}$$
$$x=\frac{1\pm15}{2} \qquad x=\frac{1\pm5}{2}$$
$$x=\frac{1+15}{2},\frac{1-15}{2} \qquad x=\frac{1+5}{2},\frac{1-5}{2}$$
$$x=8,-7 \qquad x=3,-2$$

Thus Solution Set is

$$S.S=\{-2,-7,3,8\}$$

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Q.9 $(x-1)(x-2)(x-8)(x+5)+360=0$

$\{(x-1)(x-2)\}\{(x-8)(x+5)\}+360=0$

$\{x^2-x-2x+2\}\{x^2-8x+5x-40\}+360=0$

$(x^2-3x+2)(x^2-3x-40)+360=0$

Put $x^2-3x=y$

$(y+2)(y-40)+360=0$

$y^2+2y-40y-80+360=0$

$y^2-38y+280=0$

$y^2-28y-10y+280=0$

$y[y-28]-10(y-28)=0$

$(y-10)(y-28)=0$

$y-10=0 \wedge y-28=0$

Putting $y=x^2-3x$

$x^2-3x-10=0$	$x^2-3x-28=0$
$x^2-5x+2x-10=0$	$x^2-7x+4x-28=0$
$x(x-5)+2(x-5)=0$	$x(x-7)+4(x-7)=0$
$(x-5)(x+2)=0$	$(x-7)(x+4)=0$
$x-5=0 \wedge x+2=0$	$x-7=0 \wedge x+4=0$
$x=5 \wedge x=-2$	$x=7 \wedge x=-4$

Thus Solution Set is

$S.S=\{-2,-4,5,7\}$

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Q.10 $(x+1)(2x+1)(2x+5)(x+3)=675$

$(x+1)(2x+5)(2x+1)(x+3)=675$

$\{2x^2+2x+5x+5\}\{2x^2+x+6x+3\}=675$

$\{2x^2+7x+5\}\{2x^2+7x+3\}=675$

Let $y=2x^2+7x$

$(y+5)(y+3)=675$

$y^2+5y+3y+15-675=0$

$y^2+8y-660=0$

$y^2+30y-22y-660=0$

$y[y+30]-22[y+30]=0$

$(y+30)(y-22)=0$

$y+30=0 \qquad y-22=0$

Putting $y=2x^2+7x$ back

$2x^2+7x+30=0$	$2x^2+7x-22=0$
$x=\dfrac{-7\pm\sqrt{49-4(2)(30)}}{2(2)}$	$x=\dfrac{-7\pm\sqrt{49-4(2)(-22)}}{2(2)}$
$x=\dfrac{-7\pm\sqrt{49-240}}{4}$	$x=\dfrac{-7\pm\sqrt{49+176}}{4}$
$x=\dfrac{-7\pm\sqrt{-191}}{4}$	$x=\dfrac{-7\pm\sqrt{225}}{4}$
$x=\dfrac{-7\pm\sqrt{191}\,i}{4}$	$x=\dfrac{-7\pm 15}{4}$
where $\sqrt{-1}=i$	$x=\dfrac{-7+15}{4}, \dfrac{-7-15}{4}$
	$x=\dfrac{8}{4}, \dfrac{-22}{4}$
	$x=2, \dfrac{-11}{2}$

Thus Solution Set is

$S.S=\left\{2, \dfrac{-11}{2}, \dfrac{-7\pm\sqrt{191}\,i}{4}\right\}$

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Q.11 $(2x-7)(x^2-9)(2x+5)-91=0$

$(2x-7)(x-3)(x+3)(2x+5)=91$

$(2x-7)(x+3)(x-3)(2x+5)=91$

$\{2x^2+6x-7x-21\}\{2x^2-6x+5x-15\}=91$

$(2x^2-x-21)(2x^2-x-15)=91$

Put $2x^2-x=y$

$(y-21)(y-15)=91$

$y^2-21y-15y+315-91=0$

$y^2-36y+224=0$

$y^2-28y-8y+224=0$

$y[y-28]-8[y-28]=0$

$(y-8)(y-28)=0$

$y-8=0 \wedge y-28=0$

Putting $y=2x^2-x$ back

$2x^2-x-8=0$

$x=\frac{-(-1)\pm\sqrt{1-4(2)(-8)}}{2(2)}$

$x=\frac{1\pm\sqrt{1+64}}{4}$

$x=\frac{1\pm\sqrt{65}}{4}$

$2x^2-x-28=0$

$x=\frac{-(-1)\pm\sqrt{1-4(2)(-28)}}{2(2)}$

$x=\frac{1\pm\sqrt{1+224}}{4}$

$x=\frac{1\pm\sqrt{225}}{4}$

$x=\frac{1\pm 15}{4}$

$x=\frac{1+15}{4}, \frac{1-15}{4}$

$x=\frac{16}{4}, \frac{-14}{4}$

$x=4, \frac{-7}{2}$

Thus Solution Set is

$S.S=\left\{4, \frac{-7}{2}, \frac{1\pm\sqrt{65}}{4}\right\}$

Q.12 $(x^2+6x+8)(x^2+14x+48)=105$

$\{x^2+4x+2x+8\}\{x^2+8x+6x+48\}=105$

$(x+4)(x+2)(x+8)(x+6)=105$

$(x+4)(x+6)(x+2)(x+8)=105$

$\{x^2+4x+6x+24\}\{x^2+2x+8x+16\}=105$

$(x^2+10x+24)(x^2+10x+16)=105$

Let $y=x^2+10x$

$(y+24)(y+16)=105$

$y^2+24y+16y+384-105=0$

$y^2+40y+279=0$

$y^2+31y+9y+279=0$

$y[y+31]+9[y+31]=0$

$(y+31)(y+9)=0$

Putting $y=x^2+10x$ back

$x^2+10x+31=0$

$x=\frac{-10\pm\sqrt{100-4(1)(31)}}{2}$

$x=\frac{-10\pm\sqrt{100-124}}{2}$

$x=\frac{-10\pm\sqrt{-24}}{2}$

$x=\frac{-10\pm 2\sqrt{-6}}{2}$

$x=-5\pm\sqrt{-6}$

$x^2+10x+9=0$

$x^2+9x+x+9=0$

$x(x+9)+1(x+9)=0$

$(x+9)(x+1)=0$

$x+9=0 \wedge x+1=0$

$x=-9 \wedge x=-1$

$x=-9,-1$

Thus Solution Set is

$S.S=\{-1,-9,-5\pm\sqrt{6}\,i\}$

Q.13 $(x^2+6x-27)(x^2-2x-35)=385$

$\{x^2+9x-3x-27\}\{x^2-7x+5x-35\}=385$

$(x+9)(x-3)(x-7)(x+5)=385$

$(x+9)(x-7)(x-3)(x+5)=385$

$(x^2+9x-7x-63)(x^2-3x+5x-15)=385$

$(x^2+2x-63)(x^2+2x-15)=385$

Put $y=x^2+2x$

$(y-63)(y-15)=385$

$y^2-63y-15y+945-385=0$

$y^2-78y+560=0$

$y^2-70y-8y+560=0$

$y(y-70)-8(y-70)=0$

$(y-70)(y-8)=0$

$y-70=0 \wedge y-8=0$

Putting $y=x^2+2x$ back.

$x^2+2x-70=0$

$x=\frac{-2\pm\sqrt{4-4(1)(-70)}}{2}$

$x=\frac{-2\pm\sqrt{4+280}}{2}$

$x=\frac{-2\pm2\sqrt{71}}{2}$

$x=-1\pm\sqrt{71}$

$x^2+2x-8=0$

$x^2+4x-2x-8=0$

$x(x+4)-2(x+4)=0$

$(x-2)(x+4)=0$

$x-2=0 \wedge x+4=0$

$x=2 \wedge x=-4$

$x=2,-4$

Thus Solution Set is

$S.S=\{2,-4,-1\pm\sqrt{71}\}$

Example (2):

$(x-7)(x-3)(x+1)(x+5)-1680=0$

$(x-7)(x+5)(x+1)(x-3)=1680$

$[x^2-7x+5x-35][x^2+x-3x-3]=1680$

$(x^2-2x-35)(x^2-2x-3)=1680$

Put $y=x^2-2x$

$(y-35)(y-3)=1680$

$y^2-3y-35y+105-1680=0$

$y^2-38y-1575=0$

$y^2-63y+25y-1575=0$

$y(y-63)+25(y-63)=0$

$(y-63)(y+25)=0$

$y-63=0 \wedge y+25=0$

Putting $y=x^2-2x$ again

$x^2-2x-63=0$

$x^2-9x+7x-63=0$

$x(x-9)+7(x-9)=0$

$(x-9)(x+7)=0$

$x-9=0 \wedge x+7=0$

$x=9 \wedge x=-7$

$x=9,-7$

$x^2-2x+25=0$

$x=\frac{-(-2)\pm\sqrt{4-4(1)(25)}}{2}$

$x=\frac{2\pm\sqrt{4-100}}{2}$

$x=\frac{2\pm\sqrt{96}\,i}{2}$

$x=\frac{2\pm4\sqrt{6}\,i}{2}$

$x=1\pm2\sqrt{6}\,i$

$x=1\pm2\sqrt{6}\,i$

Thus Solution Set is

$S.S=\{9,-7,1\pm2\sqrt{6}\,i\}$

Type-III In this Case, the exponential Eq is in the form

$$a^{2x}+a^{x}+C=0$$

and we put $y=a^{x}$ and get the Standard Quadratic Equation.

Q.14 $4\,2^{2x+1}-9.2^{x}+1=0$

$$4\cdot 2^{2x}\cdot 2-9\cdot 2^{x}+1=0$$

$$8\cdot 2^{2x}-9\cdot 2^{x}+1=0$$

Put $2^{x}=y$

$$8y^{2}-9y+1=0$$

$$8y^{2}-8y-y+1=0$$

$$8y(y-1)-1(y-1)=0$$

$$(y-1)(8y-1)=0$$

$y-1=0$ $8y-1=0$

Putting $2^{x}=y$ back

$2^{x}-1=0$	$8\cdot 2^{x}-1=0$
$2^{x}=1$	$8\cdot 2^{x}=1$
$2^{x}=2^{0}$	$2^{x}=\frac{1}{8}=\frac{1}{2^{3}}$
$x=0$	$2^{x}=2^{-3}$
$x=0$	$x=-3$

Thus Solution set is

$S.S=\{0,-3\}$

Q.15 $2^{x}+2^{-x+6}-20=0$ **(4)**

$$2^{x}+2^{-x}\cdot 2^{6}-20=0$$

$$2^{x}+\frac{64}{2^{x}}-20=0$$

$$\frac{2^{2x}+64-20\cdot 2^{x}}{2^{x}}=0$$

$$2^{2x}+64-20\cdot 2^{x}=0$$

$$2^{2x}-20\cdot 2^{x}+64=0$$

Put $y=2^{x}$

$$y^{2}-20y+64=0$$

$$y^{2}-16y-4y+64=0$$

$$y(y-16)-4(y-16)=0$$

$$(y-4)(y-16)=0$$

$y-4=0$ $\wedge$ $y-16=0$

Putting $y=2^{x}$ again

$2^{x}-4=0$	$2^{x}-16=0$
$2^{x}=4$	$2^{x}=16$
$2^{x}=2^{2}$	$2^{x}=2^{4}$
Equating Powers	$x=4$
$x=2$	

Thus Solution Sets is

$S.S=\{2,4\}$

Q.16 $4^{x}-3\cdot 2^{x+3}+128=0$

$$(2^{2})^{x}-3\cdot 2^{x}\cdot 2^{3}+128=0$$

$$2^{2x}-3\cdot 8\cdot 2^{x}+128=0$$

$$2^{2x}-24\cdot 2^{x}+128=0$$

Put $2^x = y$

$y^2 - 24y + 128 = 0$

$y^2 - 16y - 8y + 128 = 0$

$y(y-16) - 8(y-16) = 0$

$(y-16)(y-8) = 0$

$y - 16 = 0 \quad \wedge \quad y - 8 = 0$

Putting $y = 2^x$ again

$2^x - 16 = 0$	$2^x - 8 = 0$
$2^x = 16$	$2^x = 8$
$2^x = 2^4$	$2^x = 2^3$
$x = 4$	$x = 3$

Thus, Solution Set is

$S.S = \{3, 4\}$

Q.17 $3^{2x-1} - 12 \cdot 3^x + 81 = 0$

$3^{2x} \cdot 3^{-1} - 12 \cdot 3^x + 81 = 0$

Put $3^x = y$

$3^{-1} y^2 - 12y + 81 = 0$

$\frac{y^2}{3} - 12y + 81 = 0$ — Multiplying by 3 on both sides.

$y^2 - 36y + 243 = 0$

$y^2 - 27y - 9y + 243 = 0$

$y(y-27) - 9(y-27) = 0$

$(y-27)(y-9) = 0$

$y - 27 = 0 \quad \wedge \quad y - 9 = 0$

Putting $y = 3^x$ back

$3^x - 27 = 0$	$3^x - 9 = 0$
$3^x = 27$	$3^x = 9$
$3^x = 3^3$	$3^x = 3^2$
$x = 3$	$x = 2$

Thus the Solution Set is

$S.S = \{2, 3\}$

Example (4): $4^{1+x} + 4^{1-x} = 10$

$4 \cdot 4^x + 4 \cdot 4^{-x} = 10$

$4 \cdot 4^x + \frac{4}{4^x} = 10$

$4 \cdot 4^{2x} + 4 = 10 \cdot 4^x$

$4 4^{2x} - 10 \cdot 4^x + 4 = 0$

Put $4^x = y$

$4y^2 - 10y + 4 = 0$

$4y^2 - 8y - 2y + 4 = 0$

$4y(y-2) - 2(y-2) = 0$

$(y-2)(4y-2) = 0$

$y - 2 = 0 \quad \wedge \quad 4y - 2 = 0$

Putting $y = 4^x$

$4^x - 2 = 0$	$4 \cdot 4^x - 2 = 0$
$4^x = 2$	$4^x = \frac{2}{4}$
$4^x = 4^{1/2}$	$4^x = \frac{1}{2} = \frac{1}{4^{1/2}}$
$4^x = 4^{1/2}$	$4^x = 4^{-1/2}$
$x = 1/2$	$x = -1/2$

Thus the Solution Set is

$S.S = \{-\frac{1}{2}, \frac{1}{2}\}$

Type-IV This type is in the form of Reciprocal Equations

$\left(x \pm \frac{1}{x}\right)^2 + a\left(x \pm \frac{1}{x}\right) + c = 0$

we put $x \pm \frac{1}{x} = y$ and get a standard Quadratic Eq.

Q.18 $\left(x+\frac{1}{x}\right)^2 - 3\left(x+\frac{1}{x}\right) - 4 = 0$

Put $x + \frac{1}{x} = y$

$y^2 - 3y - 4 = 0$

$y^2 - 4y + y - 4 = 0$

$y(y-4) + 1(y-4) = 0$

$(y-4)(y+1) = 0$

$y - 4 = 0 \quad \wedge \quad y + 1 = 0$

Putting $y = x + \frac{1}{x}$ back

$x+\frac{1}{x}-4=0$	$x+\frac{1}{x}+1=0$
$\frac{x^2+1-4x}{x}=0$	$\frac{x^2+1+x}{x}=0$
$x^2-4x+1=0$	$x^2+x+1=0$
$x=\frac{4\pm\sqrt{16-4}}{2}$	$x=\frac{-1\pm\sqrt{1-4}}{2}$
$x=\frac{4\pm\sqrt{12}}{2}$	$x=\frac{-1\pm\sqrt{-3}}{2}$
$x=\frac{4\pm 2\sqrt{3}}{2}$	$x=\frac{-1\pm\sqrt{3}i}{2}$
$x=2\pm\sqrt{3}$	

Thus Solution Set is

$S.S = \left\{2 \pm \sqrt{3}, \frac{-1\pm\sqrt{3}i}{2}\right\}$

Q.19 $x^2 + x - 4 + \frac{1}{x} + \frac{1}{x^2}$

$\left(x^2+\frac{1}{x^2}\right) + \left(x+\frac{1}{x}\right) - 4$

Put $x + \frac{1}{x} = y$

$\left(x+\frac{1}{x}\right)^2 = y^2 \Rightarrow x^2 + \frac{1}{x^2} + 2 = y^2$

$x^2 + \frac{1}{x^2} = y^2 - 2$

$(y^2-2) + y - 4 = 0$

$y^2 + y - 6 = 0$

$y^2 + 3y - 2y - 6 = 0$

$y(y+3) - 2(y+3) = 0$

$(y+3)(y-2) = 0$

$y + 3 = 0 \quad \wedge \quad y - 2 = 0$

Putting $y = x + \frac{1}{x}$ back

$x+\frac{1}{x}+3=0$	$x+\frac{1}{x}-2=0$
$\frac{x^2+1+3x}{x}=0$	$\frac{x^2+1-2x}{x}=0$
$x^2+3x+1=0$	$x^2-2x+1=0$
$x=\frac{-3\pm\sqrt{9-4}}{2}$	$(x-1)^2=0$
$x=\frac{-3\pm\sqrt{5}}{2}$	$(x-1)(x-1)=0$
	$x=1,1$

Thus Solution Set is

$S.S = \left\{1, \frac{-3\pm\sqrt{5}}{2}\right\}$

Q.20 $\left(x-\frac{1}{x}\right)^2 + 3\left(x+\frac{1}{x}\right) = 0$

$x^2 + \frac{1}{x^2} - 2 + 3\left(x+\frac{1}{x}\right) = 0$

$\left(x^2+\frac{1}{x^2}\right) + 3\left(x+\frac{1}{x}\right) - 2 = 0$

Put $y = x + \frac{1}{x} \Rightarrow y^2 = \left(x+\frac{1}{x}\right)^2$

$y^2 - 2 = x^2 + \frac{1}{x^2}$

$(y^2-2)+3y-2=0$

$y^2+3y-4=0$

$y^2+4y-y-4=0$

$y(y+4)-1(y+4)=0$

$(y+4)(y-1)=0$

$y+4=0 \wedge y-1=0$

Putting $y=x+1/x$

$x+1/x+4=0$ | $x+1/x-1=0$

$\frac{x^2+1+4x}{x}=0$ | $\frac{x^2+1-x}{x}=0$

$x^2+4x+1=0$ | $x^2-x+1=0$

$x=\frac{-4\pm\sqrt{16-4}}{2}$ | $x=\frac{1\pm\sqrt{1-4}}{2}$

$x=\frac{-4\pm\sqrt{12}}{2}$ | $x=\frac{1\pm\sqrt{3}\,i}{2}$

$x=\frac{-4\pm2\sqrt{3}}{2}$

$x=-2\pm\sqrt{3}$ $\quad x=\frac{1\pm\sqrt{3}\,i}{2}$

Thus Solution Set is

$S.S=\left\{-2\pm\sqrt{3},\ \frac{1\pm\sqrt{3}\,i}{2}\right\}$

Q.21 $2x^4-x^3-x^2-x+2=0$

Dividing both Sides by x^2

$2x^2-x-1-\frac{1}{x}+\frac{2}{x^2}=0$

$2(x^2+1/x^2)-(x+1/x)-1=0$

Put $x+1/x=y$

$(x+1/x)^2=y^2 \Rightarrow y^2-2=x^2+1/x^2$

$2y^2-y-1=0$

$2y^2-2y+y-1=0$

$2y(y-1)+1(y-1)=0$

$(y-1)(2y+1)=0$

$y-1=0 \wedge 2y+1=0$

Put $y=x+1/x$ back

$x+\frac{1}{x}-1=0$ | $2(x+1/x)+1=0$

$\frac{x^2+1-x}{x}=0$ | $2x+\frac{2}{x}+1=0$

$x^2+1-x=0$ | $\frac{2x^2+2+x}{x}=0$

$x^2-x+1=0$ | $2x^2+x+2=0$

$x=\frac{1\pm\sqrt{1-4}}{2}$ | $x=\frac{-1\pm\sqrt{1-4(2)(2)}}{2(2)}$

$x=\frac{1\pm\sqrt{-3}}{2}$ | $x=\frac{-1\pm\sqrt{-15}}{4}$

$x=\frac{1\pm\sqrt{3}\,i}{2}$ $\quad x=\frac{-1\pm\sqrt{15}\,i}{4}$

Thus the Solution Set is

$S.S=\left\{\frac{1\pm\sqrt{3}\,i}{2},\ \frac{-1\pm\sqrt{15}\,i}{4}\right\}$

Q.22 $2x^4+3x^3-4x^2-3x+2=0$

Dividing both Sides by x^2

$2x^2+3x-4-\frac{3}{x}+\frac{2}{x^2}=0$

$2(x^2+\frac{1}{x^2})+3(x-1/x)-4=0$

Put $x-1/x=y \Rightarrow (x-1/x)^2=y^2$

$x^2+1/x^2-2=y^2 \Rightarrow y^2+2=x^2+1/x^2$

$2(y^2+2)+3y-4=0$

$2y^2+3y-4+4=0$

$2y^2+3y=0$

$(2y+3)y=0$

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$y = 0 \quad \wedge \quad 2y+3=0$

Putting $y = x - 1/x$ back.

$x - 1/x = 0$

$\frac{x^2-1}{x} = 0$

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

$2(x - 1/x) + 3 = 0$

$2x - \frac{2}{x} + 3 = 0$

$\frac{2x^2 - 2 + 3x}{x} = 0$

$2x^2 + 3x - 2 = 0$

$2x^2 + 4x - x - 2 = 0$

$2x(x+2) - 1(x+2) = 0$

$(x+2)(2x-1) = 0$

$x+2=0 \quad \wedge \quad 2x-1=0$

$x = -2 \quad \wedge \quad x = 1/2$

Thus Solution Set is

$S.S = \{1, -1, -2, \frac{1}{2}\}$

$(3y-10)(2y-5) = 0$

$3y - 10 = 0 \quad \wedge \quad 2y - 5 = 0$

Putting $y = x + 1/x$ back

$3(x + 1/x) - 10 = 0$

$3x + \frac{3}{x} - 10 = 0$

$\frac{3x^2 + 3 - 10x}{x} = 0$

$3x^2 - 10x + 3 = 0$

$3x^2 - 9x - x + 3 = 0$

$3x(x-3) - 1(x-3) = 0$

$(x-3)(3x-1) = 0$

$x = 3, \frac{1}{3}$

$2(x + 1/x) - 5 = 0$

$2x + \frac{2}{x} - 5 = 0$

$\frac{2x^2 + 2 - 5x}{x} = 0$

$2x^2 - 5x + 2 = 0$

$2x^2 - 4x - x + 2 = 0$

$2x(x-2) - 1(x-2) = 0$

$(2x-1)(x-2) = 0$

$x = 2, 1/2$

Thus Solution Set is

$S.S = \{2, 3, \frac{1}{2}, \frac{1}{3}\}$

Q.23 $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

Dividing both sides by x^2

$6x^2 - 35x + 62 - \frac{35}{x} + \frac{6}{x^2} = 0$

$6(x^2 + \frac{1}{x^2}) - 35(x + 1/x) + 62 = 0$

Put $y = x + 1/x$

$y^2 = x^2 + \frac{1}{x^2} - 2 \Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$

$6(y^2 - 2) - 35y + 62 = 0$

$6y^2 - 35y + 62 - 12 = 0$

$6y^2 - 35y + 50 = 0$

$6y^2 - 20y - 15y + 50 = 0$

$2y(3y-10) - 5(3y-10) = 0$

Q.24 $x^4 - 6x^2 + 10 - \frac{6}{x^2} + 1/x^4 = 0$

$(x^4 + 1/x^4) - 6(x^2 + 1/x^2) + 10 = 0$

Put $y = x^2 + 1/x^2$

$y^2 - 2 = x^4 + 1/x^4$

$y^2 - 2 - 6y + 10 = 0$

$y^2 - 6y + 8 = 0$

$y^2 - 4y - 2y + 8 = 0$

$y(y-4) - 2(y-4) = 0$

$y - 2 = 0 \quad \wedge \quad y - 4 = 0$

Putting $y = x^2 + 1/x^2$ back.

$$x^2+1/x^2-2=0$$

$$\frac{x^4+1-2x^2}{x^2}=0$$

$$x^4-2x^2+1=0$$

Put $x^2=y$

$$y^2-2y+1=0$$

$$y^2-y-y+1=0$$

$$y(y-1)-1(y-1)=0$$

$$(y-1)(y-1)=0$$

$$y=1,1$$

Putting $y=x^2$

$$x^2=1 \quad \wedge \quad x^2=1$$

$$x=\pm 1 \quad \wedge \quad x=\pm 1$$

$$\Rightarrow x=\pm 1$$

$$x^2+1/x^2-4=0$$

$$\frac{x^4+1-4x^2}{x^2}=0$$

$$x^4-4x^2+1=0$$

Put $x^2=y$

$$y^2-4y+1=0$$

$$y=\frac{4\pm\sqrt{16-4(1)(1)}}{2}$$

$$y=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm 2\sqrt{3}}{2}$$

$$y=2\pm\sqrt{3}$$

$$x^2=2\pm\sqrt{3}$$

$$x^2=2+\sqrt{3} \quad \wedge \quad x^2=2-\sqrt{3}$$

$$x=\pm\sqrt{2+\sqrt{3}} \quad \wedge \quad x=\pm\sqrt{2-\sqrt{3}}$$

$$x=\pm\sqrt{2+3^{1/2}},\ \pm\sqrt{2-3^{1/2}}$$

Thus the Solution Set is

$$S.S=\{\pm 1, \pm\sqrt{2+3^{1/2}}, \pm\sqrt{2-3^{1/2}}\}$$

TAHIR Exercise:4.3 TAHIR

Radical Equation:-

"The Equation which contains a radical sign "$\sqrt{\ }$" is called radical Equation".

Such as

$$\sqrt{ax^2+bx+c}=K$$

$$\sqrt{(ax-b)(cx+d)(ex+f)}=S$$

Type-V This type of Equations is of the form

$$(ax^2+bx+c)+\sqrt{ax^2+bx+d}=n$$

having Common ax^2+bx terms.

In this case Put $y=\sqrt{ax^2+bx+d}$ and adjust according to Quadratic Equation and Solve.