TAHIR MAHMOOD

Exercise: 32

(12)

Q.14 Show that

$$\begin{bmatrix} r\cos\phi & 0 & -\sin\phi \\ 0 & r & 0 \\ r\sin\phi & 0 & \cos\phi \end{bmatrix}\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -r\sin\phi & 0 & r\cos\phi \end{bmatrix} = rI_3$$

$$LHS = \begin{bmatrix} r\cos\phi & 0 & -\sin\phi \\ 0 & r & 0 \\ r\sin\phi & 0 & \cos\phi \end{bmatrix}\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -r\sin\phi & 0 & r\cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} r\cos^2\phi+0+r\sin^2\phi & 0+0+0 & \cancel{r\sin\phi\cos\phi}+0-\cancel{r\sin\phi\cos\phi} \\ 0+0+0 & 0+r+0 & 0+0+0 \\ \cancel{r\sin\phi\cos\phi}+0-\cancel{r\sin\phi\cos\phi} & 0+0+0 & r\sin^2\phi+0+r\cos^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} r(\sin^2\phi+\cos^2\phi) & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r(\sin^2\phi+\cos^2\phi) \end{bmatrix}$$

$$= \begin{bmatrix} r(1) & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r(1) \end{bmatrix} \quad (\because \sin^2\theta+\cos^2\theta=1)$$

$$= \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}$$

Can also be written as

$$= \begin{bmatrix} 1\times r & 0\times r & 0\times r \\ 0\times r & 1\times r & 0\times r \\ 0\times r & 0\times r & 1\times r \end{bmatrix}$$

$\because$ r is Scalar So

$$r\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = r\, I_{3\times 3} \text{ (Proved).}$$

Some further Properties:-

(1) Distributive Property w.r.t addition.

$A(B+C) = AB+AC$ Left Dis. Property

$(A+B)C = AC+BC$ Right Dis. Property

(2) Scalar Multiplication Properties.

(i) Scalar Distributive Property

$c(A+B) = cA+cB$, $(c+d)A = cA+dA$

(1) If $A = [a_{ij}]_{3\times 4}$ Then Show that

(i) $I_3A = A$

$LHS = I_3A$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}+0+0 & a_{12}+0+0 & a_{13}+0+0 & a_{14}+0+0 \\ 0+a_{21}+0 & 0+a_{22}+0 & 0+a_{23}+0 & 0+0+a_{24} \\ 0+0+a_{31} & 0+0+a_{32} & 0+0+a_{33} & 0+0+a_{34} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = A$$

So $LHS = RHS \implies I_3A = A$ (Proved)

(ii) $AI_4 = A$

$LHS = AI_4$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}+0+0+0 & 0+a_{12}+0+0 & 0+0+a_{13}+0 & 0+0+0+a_{14} \\ a_{21}+0+0+0 & 0+a_{22}+0+0 & 0+0+a_{23}+0 & 0+0+0+a_{24} \\ a_{31}+0+0+0 & 0+a_{32}+0+0 & 0+0+a_{33}+0 & 0+0+0+a_{34} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$= A$

So $LHS = RHS$.

Hence $AI_4 = A$ (Proved)

Tahir Mahmood

TAHIR

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

Q.2 Find the inveres of:

(i) $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$

Let $= A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} = 3+2 = 5$

$Adj A = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$

$A^{-1} = \dfrac{Adj A}{|A|} = \dfrac{1}{5}\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1/5 & 1/5 \\ -2/5 & 3/5 \end{bmatrix}$ Ans.

(ii) $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

Let $B = \begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

$|B| = \begin{vmatrix} -2 & 3 \\ -4 & 5 \end{vmatrix} = -10+12 = 2$

$Adj B = \begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}$

$B^{-1} = \dfrac{Adj B}{|B|} = \dfrac{1}{2}\begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} 5/2 & -3/2 \\ 2 & -1 \end{bmatrix}$ Ans.

(iii) $\begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

Let $C = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$|C| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix} = -2i^2 - i^2$ $(\because i^2 = -1)$

$= -2(-1)-(-1) = 2+1 = 3$

$Adj C = \begin{bmatrix} -i & -i \\ -i & 2i \end{bmatrix}$

$C^{-1} = \dfrac{Adj C}{|C|} = \dfrac{1}{3}\begin{bmatrix} -i & -i \\ -i & 2i \end{bmatrix}$

Hence $C^{-1} = \begin{bmatrix} -i/3 & -i/3 \\ -i/3 & 2i/3 \end{bmatrix}$ Ans.

(iv) $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

Let $D = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

$|D| = \begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix} = 6-6 = 0$

$\therefore$ Matsix is Singular So no inverse canbe found.

Q.3 Solve the Systems of linear Equations:

(i) $\left.\begin{array}{l} 2x_1 - 3x_2 = 5 \\ 5x_1 + x_2 = 4 \end{array}\right\}$

In Matsix form

$\begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

Let $A = \begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}$ $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

$\therefore AX = B$

$A^{-1}AX = A^{-1}B \implies IX = A^{-1}B \implies X = A^{-1}B$

$\therefore |A| = \begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} = 2+15 = 17$

$Adj A = \begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix}$

$A^{-1} = \dfrac{Adj A}{|A|} = \dfrac{1}{17}\begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix}$

$\therefore X = A^{-1}B$

$X = \dfrac{1}{17}\begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 4 \end{bmatrix}$

$X = \dfrac{1}{17}\begin{bmatrix} 5+12 \\ -25+8 \end{bmatrix} = \dfrac{1}{17}\begin{bmatrix} 17 \\ -17 \end{bmatrix}$

$X = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \implies \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$x_1 = 1$ $x_2 = -1$

Hence Solution Set $\{(x_1, x_2)\} = \{(1,-1)\}$

(ii) $\left.\begin{array}{l} 4x_1 + 3x_2 = 5 \\ 3x_1 - x_2 = 7 \end{array}\right\}$

In the Matsix form

$\begin{bmatrix} 4 & 3 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

Let $A = \begin{bmatrix} 4 & 3 \\ 3 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$\because AX = B \implies X = A^{-1}B$

$|A| = \begin{vmatrix} 4 & 3 \\ 3 & -1 \end{vmatrix} = -4-9 = -13$

$$\text{Adj}\,A = \begin{pmatrix} -1 & -3 \\ -3 & 4 \end{pmatrix}$$

$$A^{-1} = \frac{\text{Adj}A}{|A|} = \frac{-1}{13}\begin{pmatrix} -1 & -3 \\ -3 & 4 \end{pmatrix} = \frac{1}{13}\begin{pmatrix} 1 & 3 \\ 3 & -4 \end{pmatrix}$$

$$\because \quad X = A^{-1}B$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{13}\begin{pmatrix} 1 & 3 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{13}\begin{pmatrix} 5+21 \\ 15-28 \end{pmatrix} = \frac{1}{13}\begin{pmatrix} 26 \\ -13 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \implies x_1 = 2 \ \wedge \ x_2 = -1$$

Hence Solution Set $= \{(2,-1)\}$

(iii) $\left.\begin{aligned} 3x - 5y &= 1 \\ -2x + y &= -3 \end{aligned}\right\}$

In the matrix form

$$\begin{pmatrix} 3 & -5 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

Let $A = \begin{pmatrix} 3 & -5 \\ -2 & 1 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$$\because \quad AX = B \implies X = A^{-1}B$$

$$|A| = \begin{vmatrix} 3 & -5 \\ -2 & 1 \end{vmatrix} = 3 - 10 = -7$$

$$\text{Adj}A = \begin{pmatrix} 1 & 5 \\ 2 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{\text{Adj}A}{|A|} = \frac{-1}{7}\begin{pmatrix} 1 & 5 \\ 2 & 3 \end{pmatrix}$$

$$X = A^{-1}B$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{-1}{7}\begin{pmatrix} 1 & 5 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{-1}{7}\begin{pmatrix} 1-15 \\ 2-9 \end{pmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{7}\begin{pmatrix} -14 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$x = 2 \qquad y = 1$

Hence Solution Set $= \{(2,1)\}$

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

سی ایس ایس اکیڈمی
علی پور چٹھہ
TAHIR MAHMOOD

Q.4 Find the following if

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 0 \\ 3 & 4 & -1 \end{bmatrix}$$

(i) $A - B = ?$

$$A-B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 1-2 & -1-1 & 2+1 \\ 3-1 & 2-3 & 5-4 \\ -1+1 & 0-2 & 4-1 \end{bmatrix}$$

$$A-B = \begin{bmatrix} -1 & -2 & 3 \\ 2 & -1 & 1 \\ 0 & -2 & 3 \end{bmatrix} \text{ Ans.}$$

(ii) $B - A = ?$

$$B-A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix}$$

$$B-A = \begin{bmatrix} 2-1 & 1+1 & -1-2 \\ 1-3 & 3-2 & 4-5 \\ -1+1 & 2-0 & 1-4 \end{bmatrix}$$

$$B-A = \begin{bmatrix} 1 & 2 & -3 \\ -2 & 1 & -1 \\ 0 & 2 & -3 \end{bmatrix} \text{ Ans.}$$

(iii) $(A-B) - C = ?$

$$(A-B)-C = \left(\begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix}\right) - \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 0 \\ 3 & 4 & -1 \end{bmatrix}$$

$$(A-B)-C = \begin{bmatrix} 1-2 & -1-1 & 2+1 \\ 3-1 & 2-3 & 5-4 \\ -1+1 & 0-2 & 4-1 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 0 \\ 3 & 4 & -1 \end{bmatrix}$$

$$(A-B)-C = \begin{bmatrix} -1 & -2 & 3 \\ 2 & -1 & 1 \\ 0 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 0 \\ 3 & 4 & -1 \end{bmatrix}$$

$$(A-B)-C = \begin{bmatrix} -1-1 & -2-3 & 3+2 \\ 2+1 & -1-2 & 1-0 \\ 0-3 & -2-4 & 3+1 \end{bmatrix}$$

$$(A-B)-C = \begin{bmatrix} -2 & -5 & 5 \\ 3 & -3 & 1 \\ -3 & -6 & 4 \end{bmatrix} \text{ Ans.}$$

(iv) $A - (B-C) = ?$

$$A-(B-C) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix} - \left(\begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 0 \\ 3 & 4 & -1 \end{bmatrix}\right)$$

$$A-(B-C) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 2-1 & 1-3 & -1+2 \\ 1+1 & 3-2 & 4-0 \\ -1-3 & 2-4 & 1+1 \end{bmatrix}$$

$$A-(B-C) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 4 \\ -4 & -2 & 2 \end{bmatrix}$$

$$A-(B-C) = \begin{bmatrix} 1-1 & -1+2 & 2-1 \\ 3-2 & 2-1 & 5-4 \\ -1+4 & 0+2 & 4-2 \end{bmatrix}$$

$$A-(B-C) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 2 \end{bmatrix}$$

Ans.

Q.5 Show that for

$$A=\begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix},\ B=\begin{bmatrix} -i & 1 \\ 2i & i \end{bmatrix},\ C=\begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$$

(i) $(AB)C = A(BC)$

LHS $= (AB)C$

$$= \left(\begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix}\begin{bmatrix} -i & 1 \\ 2i & i \end{bmatrix}\right)\begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} -i^2+4i^2 & i+2i^2 \\ -i-2i^2 & 1-i^2 \end{bmatrix}\begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} 3(-1) & i+2(-1) \\ -i-(-1)2 & 1-(-1) \end{bmatrix}\begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} -3 & i-2 \\ 2-i & 2 \end{bmatrix}\begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} -6i-i^2+2i & 3+i^2-2i \\ 4i-2i^2-2i & -2+i+2i \end{bmatrix}$$

$$= \begin{bmatrix} -4i-i^2 & 3+i^2-2i \\ 2i-2i^2 & -2+i3 \end{bmatrix} \quad (\because i^2=-1)$$

$$= \begin{bmatrix} -4i-(-1) & 3+(-1)-2i \\ 2i-2(-1) & -2+3i \end{bmatrix}$$

$$= \begin{bmatrix} -4i+1 & 3-1-2i \\ 2i+2 & -2+3i \end{bmatrix}$$

$$= \begin{bmatrix} 1-4i & 2-2i \\ 2+2i & -2+3i \end{bmatrix} \text{ ——— } (1)$$

RHS $= A(BC)$

$$= \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix}\left(\begin{bmatrix} -i & 1 \\ 2i & i \end{bmatrix}\begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}\right)$$

$$= \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix}\begin{bmatrix} -2i^2-i & +i+i \\ 4i^2-i^2 & -2i+i^2 \end{bmatrix}$$

$$= \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix}\begin{bmatrix} -2(-1)-i & 2i \\ 4(-1)-(-1) & -2i+(-1) \end{bmatrix}$$

$$= \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix}\begin{bmatrix} 2-i & 2i \\ -3 & -2i-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2i-i^2-6i & 2i^2-4i^2-2i \\ 2-i+3i & 2i+2i^2+i \end{bmatrix}$$

$$= \begin{bmatrix} -i^2-4i & -2i^2-2i \\ 2+2i & 2i^2+3i \end{bmatrix}$$

$$= \begin{bmatrix} -(-1)-4i & -2(-1)-2i \\ 2+2i & 2(-1)+3i \end{bmatrix}$$

$$= \begin{bmatrix} 1-4i & 2-2i \\ 2+2i & -2+3i \end{bmatrix} \text{ ——— } (2)$$

From (1) and (2)

LHS = RHS $\Longrightarrow (AB)C = A(BC)$

(ii) $(A+B)C = AC+BC$ **(15)**

LHS $= (A+B)C$

$$= \left(\begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix}+\begin{bmatrix} -i & 1 \\ 2i & i \end{bmatrix}\right)\begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} i-i & 2i+1 \\ 1+2i & -i+i \end{bmatrix}\begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1+2i \\ 1+2i & 0 \end{bmatrix}\begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} 0-i-2i^2 & -0+i+2i^2 \\ 2i+4i^2-0 & -1-2i+0 \end{bmatrix}$$

$$= \begin{bmatrix} -i-2(-1) & i+2(-1) \\ 2i+4(-1) & -1-2i \end{bmatrix}$$

$$= \begin{bmatrix} -i+2 & i-2 \\ 2i-4 & -1-2i \end{bmatrix}$$

$$= \begin{bmatrix} 2-i & i-2 \\ 2i-4 & -1-2i \end{bmatrix} \text{ ——— } (1)$$

RHS $= AC+BC$

$$= \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix}\begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix} + \begin{bmatrix} -i & 1 \\ 2i & i \end{bmatrix}\begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} 2i^2-2i^2 & -i+2i^2 \\ 2i+i^2 & -1-i^2 \end{bmatrix} + \begin{bmatrix} -2i^2-i & i+i \\ 4i^2-i^2 & -2i+i^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -i+2(-1) \\ 2i+(-1) & -1-(-1) \end{bmatrix} + \begin{bmatrix} -2(-1)-i & 2i \\ 3(-1) & -2i+(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2-i \\ 2i-1 & 0 \end{bmatrix} + \begin{bmatrix} 2-i & 2i \\ -3 & -2i-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+2-i & -2-i+2i \\ 2i-1-3 & 0-2i-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-i & i-2 \\ 2i-4 & -2i-1 \end{bmatrix} \text{ ——— } (2)$$

From ① and ②

TAHIR

LHS = RHS $\Longrightarrow (A+B)C = AC+BC$

Q.6 If A & B are Same Ordered Matrices then show that and give reason?

(i) $(A+B)^2 \neq A^2+2AB+B^2$

LHS $= (A+B)^2 = (A+B)(A+B)$

$= A^2+AB+BA+B^2$

In General $AB \neq BA$ so.

$(A+B)^2 \neq A^2+2AB+B^2$ (Prove)

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

TAHIR MAHMOOD
M.Sc. (Math)
Mob No: 0345-6510779

(ii) $(A-B)^2 \neq A^2 - AB2 + B^2$

LHS $= (A-B)^2 = (A-B)(A-B)$

$= A^2 - AB - BA + B^2$

In General $AB \neq BA$ so

$(A-B)^2 \neq A^2 - 2AB + B^2$ (Prove)

(iii) $A^2 - B^2 \neq (A-B)(A+B)$

RHS $= (A-B)(A+B)$

$= A^2 + AB - BA - B^2$

$\because$ In General $AB \neq BA$ so

$(A-B)(A+B) \neq A^2 - B^2$ (Prove)

Q.7 If $A = \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix}$ find AA^t and A^tA

Soln:- $A^t = \begin{bmatrix} 2 & 1 & -3 \\ -1 & 0 & 5 \\ 3 & 4 & 2 \\ 0 & -2 & -1 \end{bmatrix}$

$$AA^t = \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -3 \\ -1 & 0 & 5 \\ 3 & 4 & 2 \\ 0 & -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+9+0 & 2-0+12-0 & -6-5+6-0 \\ 2-0+12-0 & 1+0+16+4 & -3+0+8+2 \\ -6-5+6-0 & -3+0+8+2 & 9+25+4+1 \end{bmatrix}$$

$$AA^t = \begin{bmatrix} 14 & 14 & -5 \\ 14 & 21 & 7 \\ -5 & 7 & 39 \end{bmatrix}$$ Ans.

Now

$$A^tA = \begin{bmatrix} 2 & 1 & -3 \\ -1 & 0 & 5 \\ 3 & 4 & 2 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+9 & -2+0-15 & 6+4-6 & 0-2+3 \\ -2+0-15 & 1+0+25 & -3+0+10 & 0-0-5 \\ 6+4-6 & -3+0+10 & 9+16+4 & 0-8-2 \\ 0-2+3 & 0-0-5 & 0-8-2 & 0+4+1 \end{bmatrix}$$

$$\because A^tA = \begin{bmatrix} 14 & -17 & 4 & 1 \\ -17 & 26 & 7 & -5 \\ 4 & 7 & 29 & -10 \\ 1 & -5 & -10 & 5 \end{bmatrix}$$ Ans.

Q.8 Find X matrix if

$A = \begin{bmatrix} 2 & 3 & -2 \\ -1 & 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix}$

(i) $3X - 2A = B$

$= 3X - 2\begin{bmatrix} 2 & 3 & -2 \\ -1 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix}$

$3X - \begin{bmatrix} 4 & 6 & -4 \\ -2 & 2 & 10 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix}$

$3X = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix} + \begin{bmatrix} 4 & 6 & -4 \\ -2 & 2 & 10 \end{bmatrix}$

$3X = \begin{bmatrix} 2+4 & -3+6 & 1+(-4) \\ 5+(-2) & 4+2 & -1+10 \end{bmatrix}$

$3X = \begin{bmatrix} 6 & 3 & -3 \\ 3 & 6 & 9 \end{bmatrix}$

$X = \frac{1}{3}\begin{bmatrix} 6 & 3 & -3 \\ 3 & 6 & 9 \end{bmatrix}$

$X = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ Ans.

(ii) $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

$2X - 3A = B$

$2X - 3\begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

$2X - \begin{bmatrix} 3 & -3 & 6 \\ -6 & 12 & 15 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

$2X = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -3 & 6 \\ -6 & 12 & 15 \end{bmatrix}$

$2X = \begin{bmatrix} 3+3 & -1+(-3) & 0+6 \\ 4+(-6) & 2+12 & 1+15 \end{bmatrix}$

$2X = \begin{bmatrix} 6 & -4 & 6 \\ -2 & 14 & 16 \end{bmatrix}$

$X = \frac{1}{2}\begin{bmatrix} 6 & -4 & 6 \\ -2 & 14 & 16 \end{bmatrix}$

$X = \begin{bmatrix} 3 & -2 & 3 \\ -1 & 7 & 8 \end{bmatrix}$ Ans.

Tahir Mahmood.
TAHIR

Q.9 Solve the matrix Equations for A :

(i) $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}A - \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 3 & 6 \end{bmatrix}$

$= \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}A = \begin{bmatrix} -1 & -4 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$

$\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}A = \begin{bmatrix} -1+2 & -4+3 \\ 3+(-1) & 6+(-2) \end{bmatrix}$

$\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$

Let $B = \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$

$\therefore \quad BA = C \Rightarrow B^{-1}BA = B^{-1}C$

$IA = B^{-1}C \Rightarrow A = B^{-1}C$

$|B| = \begin{vmatrix} 4 & 3 \\ 2 & 2 \end{vmatrix} = 8-6 = 2$

$Adj\,B = \begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}$

$B^{-1} = \frac{Adj B}{|B|} = \frac{1}{2}\begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}$

$\because \quad A = B^{-1}C$

$A = \frac{1}{2}\begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$

$A = \frac{1}{2}\begin{bmatrix} 2-6 & -2-12 \\ -2+8 & 2+16 \end{bmatrix}$

$A = \frac{1}{2}\begin{bmatrix} -4 & -14 \\ 6 & 18 \end{bmatrix}$

$A = \begin{bmatrix} -2 & -7 \\ 3 & 9 \end{bmatrix}$ Ans.

(ii) $A\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix}$

$A\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$

$A\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2-1 & 0+2 \\ -1+3 & 5+1 \end{bmatrix}$

$A\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$

Let $B = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$

$\therefore \quad AB = C \Rightarrow ABB^{-1} = CB^{-1}$

$AI = CB^{-1} \Rightarrow A = CB^{-1}$

$|B| = \begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} = 6-4 = 2$

$Adj\,B = \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$

TAHIR

$B^{-1} = \frac{Adj B}{|B|} = \frac{1}{2}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$

$A = CB^{-1}$

$A = \frac{1}{2}\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$

$A = \frac{1}{2}\begin{bmatrix} 2-8 & -1+6 \\ 4-24 & -2+18 \end{bmatrix}$

$A = \frac{1}{2}\begin{bmatrix} -6 & 5 \\ -20 & 16 \end{bmatrix}$

$A = \begin{bmatrix} -3 & 5/2 \\ -10 & 8 \end{bmatrix}$ Ans.

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

Minor Of an Element:-

"If A is an m×n matrix then minor of an element is defined as determinant of matrix of (m-1)×(n-1) order."

This implies that a minor is a matrix having one row and one Column less.

Minor is denoted by M_{ij} and its is the determinant of the matrix M_{ij}.

Cofactor of an Element:-

The Cofactor of an element is denoted by A_{ij} and is defined as

$A_{ij} = (-1)^{i+j} M_{ij}$

where M_{ij} is the minor of row i and Column j.

Determinant of Matrix of order $n \geq 3$

Let

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{bmatrix}$