Ordered Pair:-

A pair of numbers of the form (a,b) is called an ordered pair. "a" is called Abscissa and "b" is called ordinate of (a,b)

Note that $(a,b) \neq (b,a)$ in general but $(a,b) = (b,a)$ iff $a = b$

Cartisian Product:-

let "A" and "B" be the two sets then $A \times B$ is called Cartisian Product of A and B and contains all the ordered pairs (a,b) such that $a \in A, b \in B$

$$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

Now $A \times B \neq B \times A$ in general but $A \times B = B \times A$ iff $A = B$.

Binary Relation:-

The subset of $A \times B$ is called a binary relation from A to B.

Now if $n(A) = p$ and $n(B) = q$ then total number of binary relations are 2^{pq}.

Domain of a Binary Relation:-

let $R = \{(a,b) \mid a \in A \wedge b \in B\} \subseteq A \times B$ be a binary relation from A to B then the set of all Ist elements of ordered pairs of R is called domain of R.

$Dom(R) = \{a \mid a \in (a,b)\}$.

Range of a Binary Relation:-

Let R be a binary relation from A to B then set of all 2nd elements of the ordered pairs of R is called range of R.

$Ran(R) = \{b \mid b \in (a,b)\}$

Function:-

Let A, B are two sets and "f" be a relation from A to B. "f" is called function from A to B if (i) $Dom(f) = A$

(ii) There is no repitation of first element of ordered pair in any two distinct pairs!

It is denoted as $f : A \to B$.

Types of functions:-

1) Into Function:-

Let $f: A \to B$ be a function then f is called into function if $Ran(f) \subset B$ but $Ran(f) \neq B$

2) Onto Function:-

Let $f: A \to B$ be a function then f is called onto function if $Ran(f) = B$. or Surjective function

3) One-One function:-

A function $f: A \to B$ is called one-one if there is no repitation in 2nd elements of the ordered pairs of "f"

4) Injective Function:-

A function $f: A \to B$ is called injective function if f is into and one-one

5) Bijective Function:-

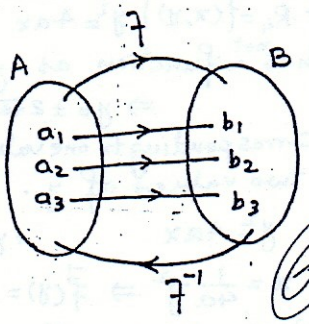
A function $f: A \to B$ is called bijective function if f is one-one and onto.

* A function is symbolically written as $y = f(x)$ or $b = f(a)$
* A function $y = mx + c$ is called Linear function having graph a Straight Line.
* A function $y = ax^2 + bx + c$ is called Quadratic function having graph a parabola.

6) Inverse Function:-

Let $f: A \to B$ be a bijective function then $f^{-1}: B \to A$ is called inverse function of "f"

If $b = f(a) \iff a = f^{-1}(b)$ where f is bijective



EXERCISE: 2.6

Q.1 $A = \{1,2,3,4\}$

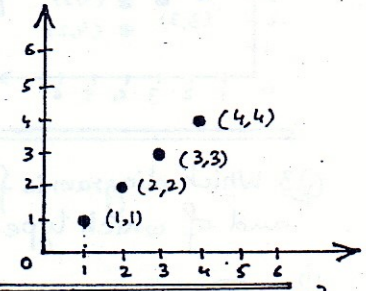
$A \times A = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}$

(i) let $R_1 = \{(x,y) \mid y = x\}$

$R_1 = \{(1,1),(2,2),(3,3),(4,4)\}$

$Dom(R_1) = \{1,2,3,4\}$

$Ran(R_1) = \{1,2,3,4\}$

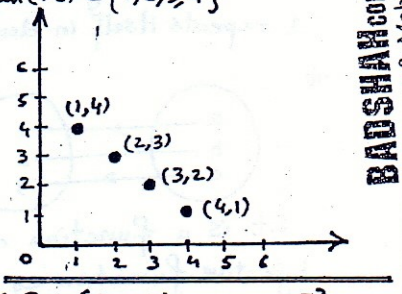


(ii) $R_2 = \{(x,y) \mid y + x = 5\}$

$R_2 = \{(1,4),(2,3),(3,2),(4,1)\}$

$Dom(R_2) = \{1,2,3,4\}$

$Ran(R_2) = \{1,2,3,4\}$

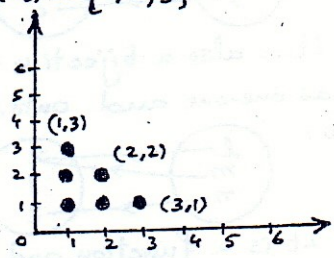


(iii) Let $R_3 = \{(x,y) \mid y + x < 5\}$

$R_3 = \{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)\}$

$Dom(R_3) = \{1,2,3\}$

$Ran(R_3) = \{1,2,3\}$

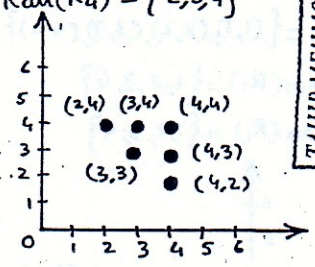


(iv) Let $R_4 = \{(x,y) \mid x+y > 5\}$

$R_4 = \{(2,4),(3,3),(3,4),(4,2),(4,3),(4,4)\}$

$Dom(R_4) = \{2,3,4\}$

$Ran(R_4) = \{2,3,4\}$

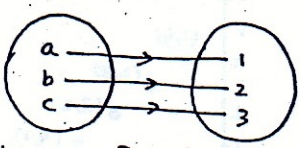


Q.3 Which diagram is function and of which type?

(i)

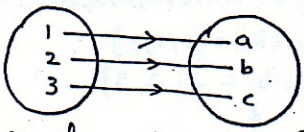
It is not a function because 1 has two images which means 1 repeats itself in domain.

(ii)



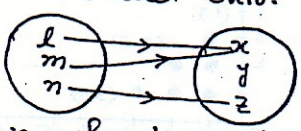
It is a function and is bijective function as it is one-one and onto.

(iii)



It is also a bijective function as one-one and onto.

(iv)



It is a function and is into function as y is unmapped in the second set.

Q.4 Find the inverse relation and tell which is a function and which is not?

(i) Let $R_1 = \{(2,1),(3,2),(4,3),(5,4),(6,5)\}$

Clearly R_1 is a function with

$Dom(R_1) = \{2,3,4,5,6\}$

Now $R_1^{-1} = \{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

which is also a function whose domain is $Dom(R_1^{-1}) = \{1,2,3,4,5\}$

(ii) Let $R_2 = \{(1,3),(2,5),(3,7),(4,9),(5,11)\}$

Clearly R_2 is a function with

$Dom(R_2) = \{1,2,3,4,5\}$

Now $R_2^{-1} = \{(3,1),(5,2),(7,3),(9,4),(11,5)\}$

which is also a function whose domain is $Dom(R_2^{-1}) = \{3,5,7,9,11\}$

(iii) Let $R_3 = \{(x,y) \mid y = 2x+3,\ x \in \mathbb{R}\}$

R_3 is a function

Now let $y = f(x) \Rightarrow y = f(x) = 2x+3$

$x = \frac{y-3}{2} \Rightarrow f^{-1}(y) = \frac{y-3}{2} \quad \because x = f^{-1}(y)$

Replacing y by x $\quad f^{-1}(x) = \frac{x-3}{2}$

Thus $R_3^{-1} = \{(x,y) \mid y = \frac{x-3}{2},\ x \in \mathbb{R}\}$

which is also a function.

(iv) let $R_4 = \{(x,y) \mid y^2 = 4ax \quad x \geq 0\}$

which is not a function as $y^2 = 4ax$

$\Rightarrow y = \pm 2\sqrt{a}\sqrt{x}$

i.e. Corresponding to one value of x there are two values of y.

Now $y^2 = 4ax \quad \because y = f(x)$

$x = \frac{1}{4a}y^2 \Rightarrow f^{-1}(y) = \frac{1}{4a}y^2$

Replacing y by x $\quad f^{-1}(x) = \frac{1}{4a}x^2$

$R_4^{-1} = \{(x,y) \mid y = \frac{1}{4a}x^2 \quad x \geq 0\}$

which is a function.