(ii) $P \rightarrow (q \rightarrow p)$

p	q	$q \rightarrow p$	$P \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

Statement is true for all values of variable so it is tautology.

(iii) $q \vee (\sim q \vee p)$

p	q	$\sim q$	$\sim q \vee p$	$q \vee (\sim q \vee p)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

Statement is true for all values of variable so it is Tautology.

Q.5 Prove that

$p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

F	F	T	T	p
F	T	F	T	q
T	T	F	F	$\sim p$
T	F	T	F	$\sim q$
F	F	F	T	$p \wedge q$
T	F	F	F	$\sim p \wedge \sim q$
T	F	T	T	$p \vee (\sim p \wedge \sim q)$
T	F	T	T	$p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$

which Proves that $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$.

Logic Forms of Set Operations:-

If p is for A and q is for B then

A^c	$\sim p$
$A \cup B$	$p \vee q$
$A \cap B$	$p \wedge q$
$(A \cup B)^c$	$\sim(p \vee q)$
$(A \cap B)^c$	$\sim(p \wedge q)$
$A^c \cup B^c$	$\sim p \vee \sim q$
$A^c \cap B^c$	$\sim p \wedge \sim q$

EXERCISE: 2.5

Convert in Logical form and Prove by Constructing the truth Table:-

Q.1 $(A \cap B)' = A' \cup B'$

Logic Form: $\sim(p \wedge q) = \sim p \vee \sim q$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Clearly $\sim(p \wedge q)$ and $\sim p \vee \sim q$ have same values so $\sim(p \wedge q) = \sim p \vee \sim q$

Hence $A' \cup B' = (A \cap B)'$ (Proved)

ASSIGNMENTS:-

* Show Logically that

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cup B')' = A' \cap B$

(iii) $(A' \cap B)' = A \cup B'$ *

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Q.2 $A \cup (B \cup C) = (A \cup B) \cup C$

Logic Form: $P \vee (Q \vee R) = (P \vee Q) \vee R$

P	Q	R	$P \vee Q$	$Q \vee R$	$P \vee (Q \vee R)$	$(P \vee Q) \vee R$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

Clearly $P \vee (Q \vee R)$ and $(P \vee Q) \vee R$ have same values so $P \vee (Q \vee R) = (P \vee Q) \vee R$

$\Rightarrow$ $A \cup (B \cup C) = (A \cup B) \cup C$ (Proved)

Q.3 $A \cap (B \cap C) = (A \cap B) \cap C$

Logic Form: $P \wedge (Q \wedge R) = (P \wedge Q) \wedge R$

P	Q	R	$P \wedge Q$	$Q \wedge R$	$P \wedge (Q \wedge R)$	$(P \wedge Q) \wedge R$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Result holds as $P \wedge (Q \wedge R)$ and $(P \wedge Q) \wedge R$ have same values.

$\Rightarrow$ $A \cap (B \cap C) = (A \cap B) \cap C$ (Proved)

Q.4 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Logic Form: $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$

P	Q	R	$Q \wedge R$	$P \vee Q$	$P \vee R$	$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

Clearly result holds as $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge (P \vee R)$ have same values

so $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$