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Aristotlian and Non-Aristotlian Logic:-

A Logic inwhich every statement is either true or false but nothing other possibility is called Aristotlian Logic.

A Logic inwhich statement has Third Possibility other than true and False is called non-Aristotlian logic.

Negation:-

Let "p" be a proposition then its negation is denoted by "$\sim p$" and read as "not p". It is true if p is false and False if p is true. Its Logic Table is:

p	$\sim p$
T	F
F	T

Conjunction:-

A conjuction of statements "p" and "q" is denoted by "$p \wedge q$" read as "p and q" and is true only if both "p" and "q" are true otherwise false. Its Logic table is:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

e.g. $4<5$ and $2+2=4$

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Disjunction:-

A disjunction of statements "p" and "q" is denoted by "$p \vee q$" read as "p or q" and false if both are false otherwise true. Its Logic Table is:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

e.g $2+2=4$ or It is morning.

Implication / Conditional:-

Let p and q be the two propositions then implication of p and q is denoted by

$$p \longrightarrow q$$

and read as: "p implies q"

"p" is called antecedent or hypothesis and "q" is called Consequent or Conclusion.

It is false only if p is true and q is false. Its Logic table is:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional:-

The proposition $p \longleftrightarrow q$ is called biconditional or equivalence. It is read as "p" iff "q"

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It is infact
$p \to q$ and $q \to p$
It is true if both "p" and "q" are true or both false. Its table is:

p	q	$p\to q$	$q\to p$	$p\leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Let $p \to q$ be a given proposition then

(i) $q \to p$ is called Converse of $p \to q$
(ii) $\sim p \to \sim q$ is called inverse of $p \to q$
(iii) $\sim q \to \sim p$ is called Contrapositive of $p \to q$.

Tautology:-

A statement which is true for all possible values of variable involved in it is called Tautology.
e.g $(p\wedge q) \to p$

Absurdity:-

A Statement which is already false is called an absurdity or a Contradiction.
e.g $p \to \sim q$

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Contingency:-

A statement which can be true or false depending upon the truth values of the variables involved in it is called Contingency. e.g
$(p \to q) \wedge (p \vee q)$.

EXERCISE: 2.4

Q.1 Write Converse, inverse and Contrapositive of the following:-

(i) $\sim p \to q$

Converse: $q \to \sim p$

Inverse: $\sim(\sim p) \to \sim q$
$\Rightarrow p \to \sim q$

Contrapositive: $\sim q \to \sim(\sim p)$
$\Rightarrow \sim q \to p$

(ii) $q \to p$

Converse: $p \to q$

Inverse: $\sim q \to \sim p$

Contrapositive: $\sim p \to \sim q$.

(iii) $\sim p \to \sim q$

Converse: $\sim q \to \sim p$

Inverse: $\sim(\sim p) \to \sim(\sim q)$
$\Rightarrow p \to q$

Contrapositive: $\sim(\sim q) \to \sim(\sim p)$
$\Rightarrow q \to p$.

(iv) $\sim q \to \sim p$

Converse: $\sim p \to \sim q$

Inverse: $\sim(\sim q) \to \sim(\sim p)$
$\Rightarrow q \to p$

ContraPositive: $\sim(\sim p) \to \sim(\sim q)$
$\Rightarrow p \to q$

$\forall$ is called universal quantifier
$\exists$ is called existential quantifier.
$\sim(\sim p) = p$ $\quad \sim(\sim q) = q$

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Q.2 Construct truth table:-

(i) $(p \to \sim p) \vee (p \to q)$

p	q	$\sim p$	$p \to q$	$p \to \sim p$	$(p \to \sim p) \vee (p \to q)$
T	T	F	T	F	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T

(ii) $(p \wedge \sim p) \to q$

p	q	$\sim p$	$p \wedge \sim p$	$(p \wedge \sim p) \to q$
T	T	F	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(iii) $\sim(p \to q) \leftrightarrow (p \wedge \sim q)$

p	q	$\sim q$	$p \to q$	$\sim(p \to q)$	$(p \wedge \sim q)$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

Clearly $\sim(p \to q)$ and $(p \wedge \sim q)$ have same truth values so $\sim(p \to q) \leftrightarrow (p \wedge \sim q)$.

Q.3 Show that following statements are tautologies:

(i) $(p \wedge q) \to p$

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p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Statement is true for all values of variable so it is tautology.

(ii) $p \to (p \vee q)$

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Statement is true for all values of variable so it is tautology.

(iii) $\sim(p \to q) \to p$

p	q	$p \to q$	$\sim(p \to q)$	$\sim(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

Statement is true for all values of variable so it is tautology.

(iv) $\sim q \wedge (p \to q) \to \sim p$

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \wedge (p \to q)$	$\sim q \wedge (p \to q) \to \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Statement is true for all values of variable so it is tautology

Q.4 Discuss the nature of statement:

(i) $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

It is false always so it is Absurdity

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(ii) $p \to (q \to p)$

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

Statement is true for all values of variable so it is tautology.

(iii) $q \vee (\sim q \vee p)$

p	q	$\sim q$	$\sim q \vee p$	$q \vee (\sim q \vee p)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

Statement is true for all values of variable so it is Tautology.

Q.5 Prove that

$p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge \sim q$	$p \vee (\sim p \wedge \sim q)$	$p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$
T	T	F	F	T	F	T	T
T	F	F	T	F	F	T	T
F	T	T	F	F	F	F	F
F	F	T	T	F	T	T	T

which proves that $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$.

Logic Forms of Set Operations:-

If p is for A and q is for B then

A^c	$\sim p$
$A \cup B$	$p \vee q$
$A \cap B$	$p \wedge q$
$(A \cup B)^c$	$\sim (p \vee q)$
$(A \cap B)^c$	$\sim (p \wedge q)$
$A^c \cup B^c$	$\sim p \vee \sim q$
$A^c \cap B^c$	$\sim p \wedge \sim q$

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EXERCISE: 2.5

Convert in Logical form and Prove by Constructing the truth Table:-

Q.1 $(A \cap B)' = A' \cup B'$

Logic Form: $\sim (p \wedge q) = \sim p \vee \sim q$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim (p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Clearly $\sim (p \wedge q)$ and $\sim p \vee \sim q$ have same values so $\sim (p \wedge q) = \sim p \vee \sim q$

Hence $A' \cup B' = (A \cap B)'$ (Proved)

ASSIGNMENTS:-

* Show Logically that

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cup B')' = A' \cap B$

(iii) $(A' \cap B)' = A \cup B'$

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