TAHIR MEHMOOD
M.Sc Math
0345-6510779

Properties of Union and Intersection :-

1) Commutative Property of Union:-

$$A \cup B = B \cup A$$

Proof:- Let $x \in A \cup B$

$\Rightarrow x \in A \vee x \in B$

$\Rightarrow x \in B \vee x \in A$

$\Rightarrow x \in B \cup A$

$\Rightarrow A \cup B \subseteq B \cup A$ —(i)

Conversely let $y \in B \cup A$

$\Rightarrow y \in B \vee y \in A$

$\Rightarrow y \in A \vee y \in B$

$\Rightarrow y \in A \cup B$

$\Rightarrow B \cup A \subseteq A \cup B$ —(ii)

From (i) and (ii)

$\boxed{A \cup B = B \cup A}$ (Proved)

2) Commutative Property of intersection:-

$$A \cap B = B \cap A$$

Proof:- Let $x \in A \cap B$

$\Rightarrow x \in A \wedge x \in B$

$\Rightarrow x \in B \wedge x \in A$

$\Rightarrow x \in B \cap A$

$\Rightarrow A \cap B \subseteq B \cap A$ —(i)

Conversely let $y \in B \cap A$

$\Rightarrow y \in B \wedge y \in A$

$\Rightarrow y \in A \wedge y \in B$

$\Rightarrow y \in A \cap B$

$\Rightarrow B \cap A \subseteq A \cap B$ —(ii)

From (i) and (ii)

$\boxed{A \cap B = B \cap A}$ (Proved)

3) Associative Property of Union :-

$$A \cup (B \cup C) = (A \cup B) \cup C$$

Proof:- Let $x \in A \cup (B \cup C)$

$\Rightarrow x \in A \vee x \in B \cup C$

$\Rightarrow x \in A \vee (x \in B \vee x \in C)$

$\Rightarrow (x \in A \vee x \in B) \vee x \in C$

$\Rightarrow x \in A \cup B \vee x \in C$

$\Rightarrow x \in (A \cup B) \cup C$

$\Rightarrow A \cup (B \cup C) \subseteq (A \cup B) \cup C$ —(i)

Conversely Let $y \in (A \cup B) \cup C$

$\Rightarrow y \in A \cup B \vee y \in C$

$\Rightarrow (y \in A \vee y \in B) \vee y \in C$

$\Rightarrow y \in A \vee (y \in B \vee y \in C)$

$\Rightarrow y \in A \vee y \in B \cup C$

$\Rightarrow y \in A \cup (B \cup C)$

$\Rightarrow (A \cup B) \cup C \subseteq A \cup (B \cup C)$ —(ii)

From (i) and (ii)

$\boxed{A \cup (B \cup C) = (A \cup B) \cup C}$ (Proved)

4) Associative Property of Intersection:-

$$A \cap (B \cap C) = (A \cap B) \cap C$$

Proof:- Let $x \in A \cap (B \cap C)$

$\Rightarrow x \in A \wedge x \in B \cap C$

$\Rightarrow x \in A \wedge (x \in B \wedge x \in C)$

$\Rightarrow (x \in A \wedge x \in B) \wedge x \in C$

$\Rightarrow x \in A \cap B \wedge x \in C$

$\Rightarrow x \in (A \cap B) \cap C$

$\Rightarrow A \cap (B \cap C) \subseteq (A \cap B) \cap C$ —(i)

Conversely Let $y \in (A \cap B) \cap C$

$\Rightarrow y \in A \cap B \wedge y \in C$

$\Rightarrow (y \in A \wedge y \in B) \wedge y \in C$

$\Rightarrow y \in A \wedge (y \in B \wedge y \in C)$

$\Rightarrow y \in A \wedge y \in B \cap C$

$\Rightarrow y \in A \cap (B \cap C)$

$\Rightarrow A \cap (B \cap C) \subseteq A \cap (B \cap C)$ —(ii)

From (i) and (ii)

$\boxed{A \cap (B \cap C) = (A \cap B) \cap C}$ (Proved)

5) Distributive Property of Union over intersection:-

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof:- Let $x \in A \cup (B \cap C)$

$\Rightarrow x \in A \vee x \in B \cap C$

$\Rightarrow x \in A \vee (x \in B \wedge x \in C)$

$\Rightarrow (x \in A \vee x \in B) \wedge (x \in A \vee x \in C)$

$\Rightarrow x \in A \cup B \wedge x \in A \cup C$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ —(i)

Conversely Let $y \in (A \cup B) \cap (A \cup C)$

$\Rightarrow y \in A \cup B \wedge y \in A \cup C$

$\Rightarrow (y \in A \vee y \in B) \wedge (y \in A \vee y \in C)$

$\Rightarrow y \in A \vee (y \in B \wedge y \in C)$

$\Rightarrow y \in A \vee y \in B \cap C$

$\Rightarrow y \in A \cup (B \cap C)$

$\Rightarrow (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ —(ii)

From (i) and (ii)

$\boxed{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$ (Proved)

6) Distributive Property of intersection over union:-

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$

Proof:- Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A \wedge x \in B \cup C$

$\Rightarrow x \in A \wedge (x \in B \vee x \in C)$

$\Rightarrow (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$

$\Rightarrow x \in A \cap B \vee x \in A \cap C$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ —(i)

Conversely Let $y \in (A \cap B) \cup (A \cap C)$

$\Rightarrow y \in A \cap B \vee y \in A \cap C$

$\Rightarrow (y \in A \wedge y \in B) \vee (y \in A \wedge y \in C)$

$\Rightarrow y \in A \wedge (y \in B \vee y \in C)$

$\Rightarrow y \in A \wedge y \in B \cup C$

$\Rightarrow y \in A \cap (B \cup C)$

$\Rightarrow (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ —(ii)

From (i) and (ii)

$\boxed{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$ (Proved)

7) De Morgan's 1st Law:-

$(A \cup B)^c = A^c \cap B^c$

Proof:- Let $x \in (A \cup B)^c$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \notin A \vee x \notin B$

$\Rightarrow x \in A^c \wedge x \in B^c$

$\Rightarrow x \in A^c \cap B^c$

$\Rightarrow (A \cup B)^c \subseteq A^c \cap B^c$ —(i)

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Conversely Let $y \in A^c \cap B^c$

$\Rightarrow y \in A^c \wedge y \in B^c$

$\Rightarrow y \notin A \vee y \notin B$

$\Rightarrow y \notin A \cup B$

$\Rightarrow y \in (A \cup B)^c$

$\Rightarrow A^c \cap B^c \subseteq (A \cap B)^c$ ___ (ii)

From (i) and (ii)

$\boxed{(A \cup B)^c = A^c \cap B^c}$ (Proved)

8) De Morgan's 2nd Law:-

$(A \cap B)^c = A^c \cup B^c$

Proof:- Let $x \in (A \cap B)^c$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A \wedge x \notin B$

$\Rightarrow x \in A^c \vee x \in B^c$

$\Rightarrow x \in A^c \cup B^c$

$\Rightarrow (A \cap B)^c \subseteq A^c \cup B^c$ ___ (i)

Conversely Let $y \in A^c \cup B^c$

$\Rightarrow y \in A^c \vee y \in B^c$

$\Rightarrow y \notin A \wedge y \notin B$

$\Rightarrow y \notin A \cap B$

$\Rightarrow y \in (A \cap B)^c$

$\Rightarrow A^c \cup B^c \subseteq (A \cap B)^c$ ___ (ii)

From (i) and (ii)

$\boxed{(A \cap B)^c = A^c \cup B^c}$ (Proved)

EXERCISE : 2.3

Q.1 Verify Commutative Laws for:

(i) $A = \{1,2,3,4,5\}$, $B = \{4,6,8,10\}$.

Istly $(A \cup B) = (B \cup A)$

LHS $= A \cup B = \{1,2,3,4,5\} \cup \{4,6,8,10\}$

$= \{1,2,3,4,5,6,8,10\}$

RHS $= B \cup A = \{4,6,8,10\} \cup \{1,2,3,4,5\}$

$= \{4,6,8,10,1,2,3,5\}$

$= \{1,2,3,4,5,6,8,10\}$

$\because$ LHS = RHS

$\Rightarrow \boxed{A \cup B = B \cup A}$ (Proved)

2ndly $A \cap B = B \cap A$

LHS $= A \cap B = \{1,2,3,4,5\} \cap \{4,6,8,10\}$

$= \{4\}$

RHS $= \{4,6,8,10\} \cap \{1,2,3,4,5\}$

$= \{4\}$

$\because$ LHS = RHS

$\Rightarrow \boxed{A \cap B = B \cap A}$ (Proved)

(ii) N, Z

Let $A = N$ and $B = Z$

Istly $A \cup B = B \cup A$

LHS $= A \cup B = N \cup Z$ $\quad \because N \subseteq Z$

$= Z$

RHS $= B \cup A = Z \cup N$ $\quad \because N \subseteq Z$

$= Z$

$\because$ LHS = RHS

$\Rightarrow \boxed{A \cup B = B \cup A}$ (Proved)

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2ndly $A \cap B = B \cap A$

$LHS = A \cap B = N \cap Z$

$= N \quad \because N \subseteq Z$

$RHS = B \cap A = Z \cap N$

$= N \quad \because N \subseteq Z$

$\because LHS = RHS$

$\Rightarrow \boxed{A \cap B = B \cap A}$ (Proved)

(iii) $A = \{x \mid x \in \mathbb{R} \wedge x \geq 0\}$, $B = \mathbb{R}$

Istly $A \cup B = B \cup A$

$LHS = A \cup B = \{x \mid x \in \mathbb{R} \wedge x \geq 0\} \cup \mathbb{R}$

$= \mathbb{R} \quad \because A \subseteq B$

$RHS = B \cup A = \mathbb{R} \cup \{x \mid x \in \mathbb{R} \wedge x \geq 0\}$

$= \mathbb{R} \quad \because A \subseteq B$

$\because LHS = RHS$

$\Rightarrow \boxed{A \cup B = B \cup A}$ (Proved)

2ndly $A \cap B = B \cap A$

$LHS = A \cap B = \{x \mid x \in \mathbb{R} \wedge x \geq 0\} \cap \mathbb{R}$

$= \{x \mid x \in \mathbb{R} \wedge x \geq 0\} \quad \because A \subseteq B$

$RHS = B \cap A = \mathbb{R} \cap \{x \mid x \in \mathbb{R} \wedge x \geq 0\}$

$= \{x \mid x \in \mathbb{R} \wedge x \geq 0\} \quad \because A \subseteq B$

$\because LHS = RHS$

$\Rightarrow \boxed{A \cap B = B \cap A}$ (Proved)

Q.2 Verify the following Properties for i) $A = \{1,2,3,4\}$, $B = \{3,4,5,6,7,8\}$

$C = \{5,6,7,9,10\}$

Associativity of Union:-

$A \cup (B \cup C) = (A \cup B) \cup C$

$LHS = A \cup (B \cup C)$

$= \{1,2,3,4\} \cup (\{3,4,5,6,7,8\} \cup \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cup \{3,4,5,6,7,8,9,10\}$

$= \{1,2,3,4,5,6,7,8,9,10\}$

$RHS = (A \cup B) \cup C$

$= (\{1,2,3,4\} \cup \{3,4,5,6,7,8\}) \cup \{5,6,7,9,10\}$

$= \{1,2,3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$

$= \{1,2,3,4,5,6,7,8,9,10\}$

$\because LHS = RHS$

$\Rightarrow \boxed{A \cup (B \cup C) = (A \cup B) \cup C}$ (Proved)

Associativity of Intersection:-

$A \cap (B \cap C) = (A \cap B) \cap C$

$LHS = A \cap (B \cap C)$

$= \{1,2,3,4\} \cap (\{3,4,5,6,7,8\} \cap \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cap \{5,6,7\}$

$= \phi$

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$RHS = (A \cap B) \cap C$

$= (\{1,2,3,4\} \cap \{3,4,5,6,7,8\}) \cap \{5,6,7,9,10\}$

$= \{3,4\} \cap \{5,6,7,9,10\}$

$= \phi$

$\because LHS = RHS$

$\Rightarrow \boxed{A \cap (B \cap C) = (A \cap B) \cap C}$ (Proved)

Distributivity of union over intersection:-

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$LHS = A \cup (B \cap C)$

$= \{1,2,3,4\} \cup (\{3,4,5,6,7,8\} \cap \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cup \{5,6,7\}$

$= \{1,2,3,4,5,6,7\}$

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RHS $= (A \cup B) \cap (A \cup C)$

$= (\{1,2,3,4\} \cup \{3,4,5,6,7,8\}) \cap (\{1,2,3,4\} \cup \{5,6,7,9,10\})$

$= \{1,2,3,4,5,6,7,8\} \cap \{1,2,3,4,5,6,7,9,10\}$

$= \{1,2,3,4,5,6,7\}$

$\because$ LHS = RHS

$\Rightarrow \boxed{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$ (Proved)

Distributivity of intersection over union:-

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

LHS $= A \cap (B \cup C)$

$= \{1,2,3,4\} \cap (\{3,4,5,6,7,8\} \cup \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cap \{3,5,4,6,7,8,9,10\}$

$= \{3,4\}$

RHS $= (A \cap B) \cup (A \cap C)$

$= (\{1,2,3,4\} \cap \{3,4,5,6,7,8\}) \cup (\{1,2,3,4\} \cap \{5,6,7,9,10\})$

$= \{3,4\} \cup \{\ \}$

$= \{3,4\}$

$\because$ LHS = RHS

$\Rightarrow \boxed{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$ (Proved)

ii) $A = \phi$, $B = \{0\}$, $C = \{0,1,2\}$

Associativity of union:-

$A \cup (B \cup C) = (A \cup B) \cup C$

LHS $= A \cup (B \cup C)$

$= \phi \cup (\{0\} \cup \{0,1,2\})$

$= \phi \cup \{0,1,2\}$

$= \{0,1,2\}$

RHS $= (A \cup B) \cup C$

$= (\phi \cup \{0\}) \cup \{0,1,2\}$

$= \{0\} \cup \{0,1,2\}$

$= \{0,1,2\}$

$\because$ LHS = RHS

$\Rightarrow \boxed{A \cup (B \cup C) = (A \cup B) \cup C}$ (Proved)

Associativity of intersection:-

$A \cap (B \cap C) = (A \cap B) \cap C$

LHS $= A \cap (B \cap C)$

$= \phi \cap (\{0\} \cap \{0,1,2\})$

$= \phi \cap \{0\}$

$= \phi$

RHS $= (A \cap B) \cap C$

$= (\{\ \} \cap \{0\}) \cap \{0,1,2\}$

$= \phi \cap \{0,1,2\}$

$= \phi$

$\because$ LHS = RHS

$\Rightarrow \boxed{A \cap (B \cap C) = (A \cap B) \cap C}$ (Proved)

Distributivity of union over intersection:-

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

LHS $= A \cup (B \cap C)$

$= \phi \cup (\{0\} \cap \{0,1,2\})$

$= \phi \cup \{0\}$

$= \{0\}$

RHS $= (A \cup B) \cap (A \cup C)$

$= (\phi \cup \{0\}) \cap (\phi \cup \{0,1,2\})$

$= \{0\} \cap \{0,1,2\}$

$= \{0\}$

$\because$ LHS = RHS

$\Rightarrow \boxed{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$ (Proved)

Distributivity of intersection over union:-

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

LHS $= A \cap (B \cup C)$

$= \phi \cap (\{0\} \cup \{0,1,2\})$

$= \phi \cap \{0\}$

$= \phi$

$RHS = (A\cap B)\cup(A\cap C)$
$= (\phi\cap\{0\})\cup(\phi\cap\{0,1,2\})$
$= \phi\cup\phi$
$= \phi$
$\because LHS = RHS$
$\Rightarrow \boxed{A\cap(B\cup C) = (A\cap B)\cup(A\cap C)}$ (Proved)

iii) $A = N,\ B = Z,\ C = Q.$

Associativity of union:-

$A\cup(B\cup C) = (A\cup B)\cup C$

$LHS = A\cup(B\cup C)$
$= N\cup(Q\cup Z)$
$= N\cup Q \quad \because N\subseteq Z\subseteq Q$
$= Q$

$RHS = (A\cup B)\cup C$
$= (N\cup Z)\cup Q$
$= Z\cup Q$
$= Q$
$\because LHS = RHS$
$\Rightarrow \boxed{A\cup(B\cup C) = (A\cup B)\cup C}$ (Proved)

Associativity of intersection:-

$A\cap(B\cap C) = (A\cap B)\cap C$

$LHS = A\cap(B\cap C)$
$= N\cap(Z\cap Q)\ (\because N\subseteq Z\subseteq Q)$
$= N\cap Z$
$= N$

$RHS = (A\cap B)\cap C$
$= (N\cap Z)\cap Q$
$= N\cap Q$
$= N$
$\because LHS = RHS$
$\Rightarrow \boxed{A\cap(B\cap C) = (A\cap B)\cap C}$ (Proved)

Distributivity of union over intersection:-

$A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

$LHS = A\cup(B\cap C)$
$= N\cup(Z\cap Q) \quad \because N\subseteq Z\subseteq Q$
$= N\cup Z$
$= Z$

$RHS = (A\cup B)\cap(A\cup C)$
$= (N\cup Z)\cap(N\cup Q)$
$= Z\cap Q$
$= Z$
$\because LHS = RHS$
$\Rightarrow \boxed{A\cup(B\cap C) = (A\cup B)\cap(A\cup C)}$ (Proved)

Distributivity of intersection over union:-

$A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$

$LHS = A\cap(B\cup C)$
$= N\cap(Z\cup Q)$
$= N\cap Q$
$= N$

$RHS = (A\cap B)\cup(A\cap C)$
$= (N\cap Z)\cup(N\cap Q)$
$= N\cup N$
$= N$
$\because LHS = RHS$
$\Rightarrow \boxed{A\cap(B\cup C) = (A\cap B)\cup(A\cap C)}$ (Proved)

Q.3 Verify De Morgan's Laws for sets:

$U = \{1,2,3,\ldots,20\},\quad A = \{2,4,6,\ldots,20\}$
$B = \{1,3,5,\ldots,19\}$

$A^c = U - A = \{1,2,3,\ldots,20\} - \{2,4,6,\ldots,20\}$
$A^c = \{1,3,5,\ldots,19\}$

$B^c = U - B = \{1,2,3,\ldots,20\} - \{1,3,5,\ldots,19\}$
$B^c = \{2,4,6,\ldots,20\}$

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$A\cup B=\{1,3,5,\ldots,19\}\cup\{2,4,6,\ldots,20\}$
$=\{1,2,3,\ldots,20\}$

$(A\cup B)^c=U-(A\cup B)$
$=\{1,2,3,\ldots,20\}-\{1,2,3,\ldots,20\}$
$=\phi$

$A\cap B=\{2,4,6,\ldots,20\}\cap\{1,3,5,\ldots,19\}$
$=\phi$

$(A\cap B)^c=U-(A\cap B)$
$=\{1,2,3,\ldots,20\}-\phi$
$=\{1,2,3,\ldots,20\}=U$

Now
$A^c\cup B^c=\{1,3,5,\ldots,19\}\cup\{2,4,6,\ldots,20\}$
$=\{1,2,3,\ldots,20\}=U$

Thus $\boxed{(A\cap B)^c=A^c\cup B^c}$ (Proved)

Now. $A^c\cap B^c=\{1,3,5,\ldots,19\}\cap\{2,4,\ldots,20\}$
$=\phi$

Thus $\boxed{(A\cup B)^c=A^c\cap B^c}$ (Proved)

Q.4 Verify De Morgan's Laws for
U = Set of the English Alphabets.
$A=\{x\mid x \text{ is a vowel}\}$
$B=\{y\mid y \text{ is a Consonant}\}$

Sol:-
$A^c=U-A=$ Set of all Consonants $=B$
$B^c=U-B=$ Set of all vowels $=A$
$A\cup B=$ Set of vowels $\cup$ Set of Consonants
$=$ Set of all alphabets
$A\cup B=U$

$(A\cup B)^c=U-(A\cup B)$
$=U-U$
$=\phi$

$A\cap B=$ Set of vowels $\cap$ Set of Consonants
$=\phi$ ($\because A,B$ are disjoint)

$(A\cap B)^c=U-(A\cap B)$
$=U-\phi$
$=U$

De Morgan's Ist Law:-

$(A\cup B)^c=A^c\cap B^c$

LHS $=(A\cup B)^c$
$=\phi$

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RHS $=A^c\cap B^c$
$=B\cap A$ $\because A,B$ are disjoint
$=\phi$

$\because$ LHS $=$ RHS

$\Rightarrow \boxed{(A\cup B)^c=A^c\cap B^c}$ (Proved)

De Morgan's 2nd Law:-

$(A\cap B)^c=A^c\cup B^c$

LHS $=(A\cap B)^c$
$=U$

RHS $=A^c\cup B^c$
$=B\cup A$
$=U$

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$\because$ LHS $=$ RHS

$\Rightarrow \boxed{(A\cap B)^c=A^c\cup B^c}$

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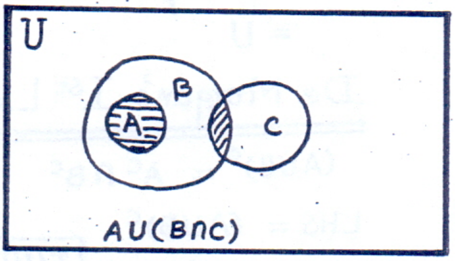
Q.5 Using Venn Diagrams, Prove Distributive Laws:

i) $A \subseteq B$, $A \cap C = \phi$, B and C overlapping.

Distributivity of Union over Intersection:-

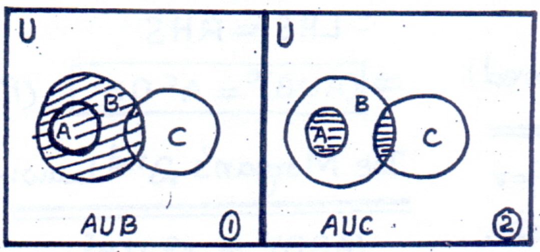
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

LHS = $A \cup (B \cap C)$



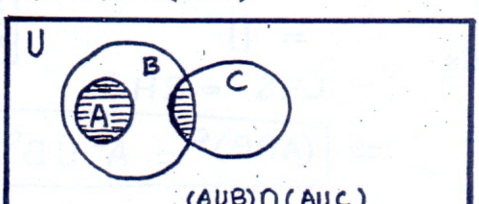
▨ Shaded region represents $B \cap C$ and whole shaded region represents $A \cup (B \cap C)$.

RHS = $(A \cup B) \cap (A \cup C)$



Shaded region ① represents $A \cup B$ and Shaded region ② reprensts $A \cup C$

Now Combining ① and ② to get $(A \cup B) \cap (A \cup C)$



Shaded region represents $(A \cup B) \cap (A \cup C)$

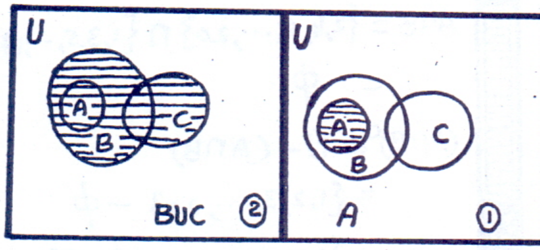
$\Rightarrow$ LHS = RHS

so $\boxed{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$ (Proved)

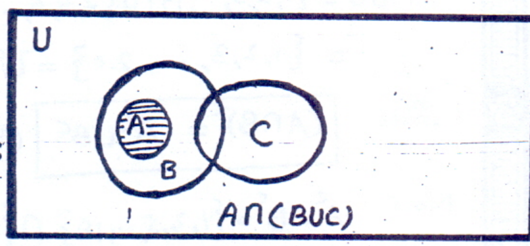
Distributivity of intersection over union:-

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

LHS = $A \cap (B \cup C)$

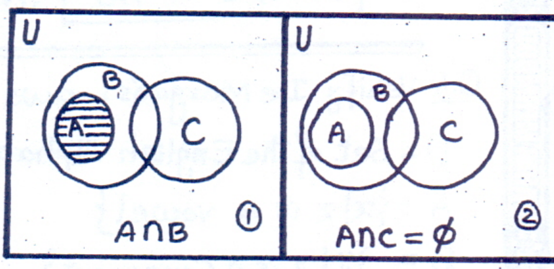


Shaded region ① represents A and Shaded region ② represents $B \cup C$



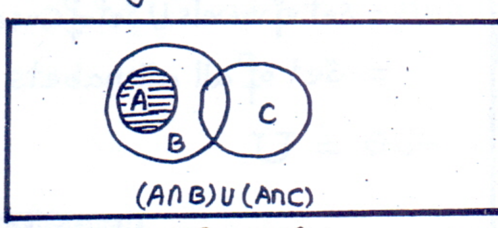
Shaded region represents $A \cap (B \cup C)$

RHS = $(A \cap B) \cup (A \cap C)$



Shaded region in ① represents $A \cap B$ and ② does not represent any region as $A \cap C = \phi$

Combining ① and ②



Thus LHS = RHS

$\boxed{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$ (Proved)

TAHIR MEHMOOD
M.Sc Math
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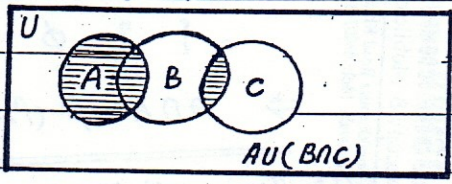
(ii) A, B are overlapping, B, C are overlapping but A and C are disjoint sets.

Distributivity of Union over Intersection:-

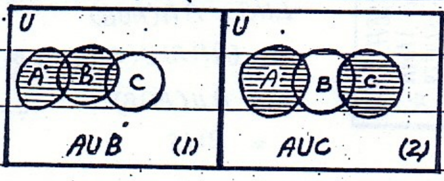
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

LHS = $A \cup (B \cap C)$

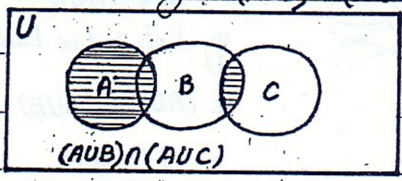


The whole shaded region represents $A \cup (B \cap C)$

RHS = $(A \cup B) \cap (A \cup C)$



Shaded region in (1) represents $A \cup B$ and in (2) represents $A \cup C$. Now combining (1) and (2) to get $(A \cup B) \cap (A \cup C)$



Shaded region represents $(A \cup B) \cap (A \cup C)$

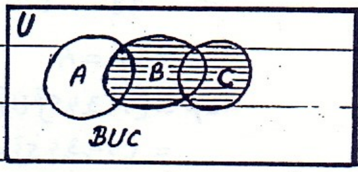
From both sides, the diagrams are similar this shows that

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Distributivity of Intersection over Union:

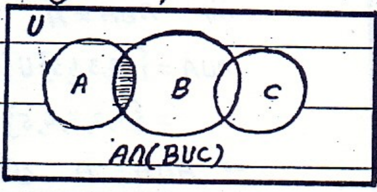
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

LHS: $A \cap (B \cup C)$



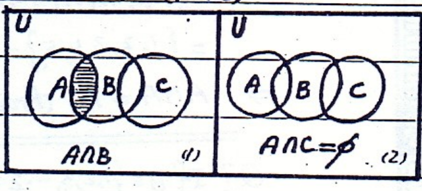
Shaded region represents $B \cup C$

Now



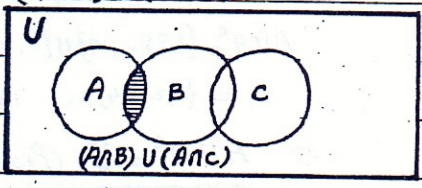
Shaded region represents $A \cap (B \cup C)$

RHS: $(A \cap B) \cup (A \cap C)$



Shaded region in (1) shows $A \cap B$ and (2) does not contain any shaded region because A and C are disjoint.

Combining (1) and (2) to get $(A \cap B) \cup (A \cap C)$



Shaded region represents $(A \cap B) \cup (A \cap C)$

From the sides

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Q.6 Verify the following if $A=\{1,2,3,4,5\}$

(i) $A\cup\phi = A$

$A\cup\phi = \{1,2,3,4,5\}\cup\{\ \}$

$= \{1,2,3,4,5\} = A$

$\Rightarrow A\cup\phi = A$ (Proved)

(ii) $A\cup A = A$

$A\cup A = \{1,2,3,4,5\}\cup\{1,2,3,4,5\}$

$= \{1,2,3,4,5\} = A$

$\Rightarrow A\cup A = A$ (Proved)

(iii) $A\cap A = A$

$A\cap A = \{1,2,3,4,5\}\cap\{1,2,3,4,5\}$

$= \{1,2,3,4,5\} = A$

$\Rightarrow A\cap A = A$ (Proved)

Q.7 If $U=\{1,2,3,\cdots 20\}$, $A=\{1,3,5,\cdots,19\}$ then verify that:

(i) $A\cup A^c = U$

$\because A = \{1,3,5,\cdots,19\}$

$A^c = U-A = \{1,2,3,\cdots,20\} - \{1,3,5,\cdots,19\}$

$= \{2,4,6,\cdots,20\}$

$A\cup A^c = \{1,3,5,\cdots,19\}\cup\{2,4,6,\cdots,20\}$

$= \{1,2,3,4,\cdots 19,20\} = U$

$\Rightarrow A\cup A^c = U$ (Proved)

(ii) $A\cap U = A$

$A\cap U = \{1,3,5,\cdots,19\}\cap\{1,2,3,4,\cdots,20\}$

$= \{1,3,5,\cdots,19\} = A$

$\Rightarrow A\cap U = A$ (Proved)

(iii) $A\cap A^c = \phi$

$A\cap A^c = \{1,3,5,\cdots,19\}\cap\{2,4,6,\cdots,20\}$

$= \{\ \} = \phi$

$\Rightarrow A\cap A^c = \phi$ (Proved)

Q.8 From suitable Properties of union and intersection show that:

(i) $A\cap(A\cup B) = A\cup(A\cap B)$

LHS $= A\cap(A\cup B)$

$= (A\cap A)\cup(A\cap B)$ (By distributive Law)

$= A\cup(A\cap B)$ $\because A\cap A = A$

$=$ RHS

$\Rightarrow A\cap(A\cup B) = A\cup(A\cap B)$ (Proved)

(ii) $A\cup(A\cap B) = A\cap(A\cup B)$

LHS $= A\cup(A\cap B)$

By distributive Law

$= (A\cup A)\cap(A\cup B)$

$= A\cap(A\cup B)$ $\because A\cup A = A$

$=$ RHS

$\Rightarrow A\cup(A\cap B) = A\cap(A\cup B)$ (Proved)

Proposition:-

A declarative statement which may be true or false but not both is called Proposition.

For example:

$2+2=4$ (True) and $2+3=10$ (False)