# Bright Career Science Academy, Narowal

Trigonometric Formulas (Edition-4) Compiled By: Muzzammil Subhan

## Different Relations

- $l = r\theta$ & $A = \frac{1}{2}r^2\theta$

where θ is measure in radian

- $180^\circ = \pi$ radian
- $1^\circ = \frac{\pi}{180}$ radian & 1 radian $= \frac{180^\circ}{\pi}$
- 1 radian $\approx 57.296^\circ$
- $1^\circ \approx 0.0175$ radian
- $\pi \approx 3.1416$ & $e \approx 2.718$

## Trigonometric Ratios

- $\sin\theta = \frac{p}{h}$ • $\text{cosec}\theta = \frac{h}{p}$
- $\cos\theta = \frac{b}{h}$ • $\sec\theta = \frac{h}{b}$
- $\tan\theta = \frac{p}{b}$ • $\cot\theta = \frac{b}{p}$
- $\sin\theta = \frac{1}{\text{cosec}\theta}$ • $\text{cosec}\theta = \frac{1}{\sin\theta}$
- $\cos\theta = \frac{1}{\sec\theta}$ • $\sec\theta = \frac{1}{\cos\theta}$
- $\tan\theta = \frac{1}{\cot\theta}$ • $\cot\theta = \frac{1}{\tan\theta}$
- $\tan\theta = \frac{\sin\theta}{\cos\theta}$ • $\cot\theta = \frac{\cos\theta}{\sin\theta}$

## Trigonometric Co-ratios

- $\sin\theta \rightleftarrows \cos\theta$
- $\sec\theta \rightleftarrows \text{cosec}\theta$
- $\tan\theta \rightleftarrows \cot\theta$

## Law of Sine

- $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

## Law of Cosines

- $a^2 = b^2 + c^2 - 2bc\cos\alpha$
- $b^2 = c^2 + a^2 - 2ca\cos\beta$
- $c^2 = a^2 + b^2 - 2ab\cos\gamma$

## Law of Tangents

- $\frac{a-b}{a+b} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$
- $\frac{b-c}{b+c} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)}$
- $\frac{c-a}{c+a} = \frac{\tan\left(\frac{\gamma-\alpha}{2}\right)}{\tan\left(\frac{\gamma+\alpha}{2}\right)}$

## Types of Angles

- Acute Angle: $0<\theta<90^\circ$
- Right Angle: $\theta=90^\circ$
- Obtuse Angle: $90^\circ<\theta<180^\circ$
- Straight Angle: $\theta=180^\circ$
- Reflex Angle: $180^\circ<\theta<360^\circ$
- 1 Rotation: $\theta=360^\circ$

## Signs of Trigonometric Functions

- $\cos(-\theta) = \cos\theta$ • $\sec(-\theta) = \sec\theta$
- $\sin(-\theta) = -\sin\theta$ • $\text{cosec}(-\theta) = -\text{cosec}\theta$
- $\tan(-\theta) = -\tan\theta$ • $\cot(-\theta) = -\cot\theta$

## Fundamental Law Of Trigonometry

- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
- $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
- $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
- $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
- $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
- $\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

## Express Products as Sums or Differences

- $2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$
- $2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$
- $2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$
- $-2\sin\alpha\sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

## Express Sums or Differences as Products

- $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
- $\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$
- $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
- $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

## Area of Triangle

- $\Delta = \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha} = \frac{b^2\sin\gamma\sin\alpha}{2\sin\beta} = \frac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$
- $\Delta = \frac{1}{2}bc\sin\alpha = \frac{1}{2}ca\sin\beta = \frac{1}{2}ab\sin\gamma$
- $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ (Hero's Formula)

*where* • $s = \frac{a+b+c}{2}$

## Inverse Trigonometric Formulas

- $\sin^{-1}A + \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$
- $\sin^{-1}A - \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} - B\sqrt{1-A^2}\right)$
- $\cos^{-1}A + \cos^{-1}B = \cos^{-1}\left(AB - \sqrt{(1-A^2)(1-B^2)}\right)$
- $\cos^{-1}A - \cos^{-1}B = \cos^{-1}\left(AB + \sqrt{(1-A^2)(1-B^2)}\right)$
- $\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$
- $\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$
- $2\tan^{-1}A = \tan^{-1}\left(\frac{2A}{1-A^2}\right)$

## Half Angle Formulas

- $\sin\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$
- $\sin\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$
- $\sin\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$
- $\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$
- $\cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ca}}$
- $\cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$
- $\tan\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
- $\tan\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$
- $\tan\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

## Different Radii

- $R = \frac{a}{2\sin\alpha} = \frac{b}{2\sin\beta} = \frac{c}{2\sin\gamma}$
- $R = \frac{abc}{4\Delta}$ • $r = \frac{\Delta}{s}$
- $r_1 = \frac{\Delta}{s-a}$ • $r_2 = \frac{\Delta}{s-b}$ • $r_3 = \frac{\Delta}{s-c}$

Where R=Circum Radius

r=In Radius

$r_1$, $r_2$, $r_3$=Escribed Radius

Δ=Area of Triangle

## Pythagorean Identities

- $h^2 = p^2 + b^2$
- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \text{cosec}^2\theta$

## Half Angle Identities

- $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$
- $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$
- $2\sin^2\frac{\theta}{2} = 1 - \cos\theta$
- $2\cos^2\frac{\theta}{2} = 1 + \cos\theta$

## Double Angle Identities

- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta$
- $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
- $2\sin^2\theta = 1 - \cos 2\theta$
- $2\cos^2\theta = 1 + \cos 2\theta$

## Triple Angle Identities

- $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
- $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
- $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

## Domains , Ranges & Periods of Trigonometric Functions

| Function | Domian | Range | Period |
|---|---|---|---|
| $y = \sin x$ | $-\infty < x < +\infty$ | $-1 \le y \le 1$ | $2\pi$ |
| $y = \cos x$ | $-\infty < x < +\infty$ | $-1 \le y \le 1$ | $2\pi$ |
| $y = \tan x$ | $-\infty < x < +\infty$ , $x \ne (2n+1)\frac{\pi}{2}$ | $-\infty < y < +\infty$ | $\pi$ |
| $y = \sec x$ | $-\infty < x < +\infty$ , $x \ne (2n+1)\frac{\pi}{2}$ | $y \ge 1$ or $y \le -1$ | $2\pi$ |
| $y = \text{cosec}x$ | $-\infty < x < +\infty$, $x \ne n\pi$ | $y \ge 1$ or $y \le -1$ | $2\pi$ |
| $y = \cot x$ | $-\infty < x < +\infty$, $x \ne n\pi$ | $-\infty < y < +\infty$ | $\pi$ |

## Ranges of Inverse Trigonometric Functions

| Function | Range | Function | Range |
|---|---|---|---|
| $y = \sin^{-1}x$ | $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ | $y = \cos^{-1}x$ | $0 \le y \le \pi$ |
| $y = \tan^{-1}x$ | $-\frac{\pi}{2} < y < \frac{\pi}{2}$ | $y = \cot^{-1}x$ | $0 < y < \pi$ |
| $y = \text{cosec}^{-1}x$ | $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$; $y \ne 0$ | $y = \sec^{-1}x$ | $0 \le y \le \pi$; $y \ne \frac{\pi}{2}$ |


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