Chapter **< 1 >** **Rational Number**: A number which can be written in the form $\frac{p}{q}$ where $p, q \in Z$ and $q \neq 0$ is called rational number. Example: $5, \frac{7}{3}$ & $-\frac{17}{4}$.

Irrational Number: A number which cannot be written in the form $\frac{p}{q}$ where $p, q \in Z$ and $q \neq 0$ is called irrational number. **Example:** $\sqrt{5}, \sqrt{7}, \sqrt{12}$ & $\sqrt{\frac{5}{6}}$.

Real Number: A number that is either rational or irrational is called a real number. i.e. $R = Q \cup Q'$

Terminating Decimal: A decimal that contain finite number of digits in its decimal part is called terminating decimal.

Example: 0.5 , -0.7 & 5.373

Recurring Decimal: A decimal having an infinitely repeating digit or group of digits is called recurring decimal.

Example: $0.\overline{3} = 0.33333....$ & $0.\overline{67} = 0.676767.....$

Complex Number: A Complex number is defined as $C = \{z = a + bi \wedge a, b \in R\}$ where $i = \sqrt{-1}$.

Chapter **< 2 >** **SET**: Any collection of well define different objects is called set.

Empty or Null Set: A set contains no element is called null or empty set. It is denoted by $\{\ \} = \phi$.

Order of a set: The number of elements in a set is called Order of a set.

Singleton set: A set having only one element is called singleton set.

Induction: A result on the basis of personal experience is called induction.

Deduction: A result on the basis of well-known facts is called deduction.

Proposition: Any statement which is either true or false but not both is called proposition.

Tautology: A statement which is necessarily true for all the cases is called a tautology.

Absurdity or Contradiction: A statement which is necessarily false for all cases is called contradiction or absurdity.

Relation: Let A and B be two non-empty sets then any subset of Cartesian product $A \times B$ is called relation.

Function: A function is rule relating two sets in such a way that each element in the first set corresponds to one and only one element in the second set.

One-One or Injective function: A function in which the second elements of the order pair are different is called the One-one or injective function.

Into function: If a function $f : A \to B$ such that $Ran(f) \neq B$ then f is called into function.

Onto or Surjective function: If a function $f : A \to B$ such that $Ran(f) = B$ then f is called onto or surjective function.

Bijective Function: The function which is One-one and onto is called bijective function.

Groupoid: A non-empty set G in which closure law holds under the binary operation $*$ is called groupiod.

Semi group: A non-empty set G in which closure and associative laws holds under the binary operation $*$ is called semi group.

Monoid: A non-empty set G in which closure, associative and identity properties holds under the binary operation $*$ is called monoid.

Group: A non-empty set G in which closure, associative, identity and inverse properties holds under the binary operation $*$ is called group.

Abelian group: A non-empty set G in which closure, associative, identity, inverse and commutative properties holds under the binary operation $*$ is called Abelian group.

Finite group: A group having finite number of elements is called finite group.

Infinite group: A group having infinite number of elements is called infinite group.

Residue classes modulo n: When the natural numbers greater or equal to n are divided by n, then the remainders are called the Residue classes modulo n.

Chapter **< 3 >** **Matrix:** A rectangular array of number enclosed by a pair of bracket is called matrix.

Row Matrix / vector: A matrix which has only one row is called row matrix.

Column Matrix / Vector: A matrix which has only one column is called column matrix.

Rectangular Matrix: A matrix in which the number of row is not equal to the number of columns is called rectangular matrix.

Square Matrix: A matrix in which the number of rows is equal to number of columns is called square matrix.

Diagonal Matrix: A matrix in which each non-diagonal element is zero is called diagonal matrix.

Scalar Matrix: A matrix in which each diagonal element is same constant while the remaining all of its elements are zero is called the scalar matrix.

Identity or Unit Matrix: A square matrix in which each diagonal element is 1 (one) while the remaining all of its elements are zero is called unit or identity matrix. It is denoted by **I** .

Null or Zero Matrix: A matrix in which every element is zero is called zero or null matrix.

Order of Matrix: If a matrix has m rows and n columns then m x n is said to be it's order.

Transpose of a Matrix: The transpose of a matrix A is another matrix obtained by interchanging the rows and columns of A. it is denoted by A^t.

Rank of a Matrix: The rank of a matrix is equal to number of non-zero rows in it's echelon form.

Upper Triangular Matrix: A square matrix A is said to be an upper triangular matrix if each of it's element below the diagonal is zero.

Lower Triangular Matrix: A square matrix A is said to be an lower triangular matrix if each of its element above the diagonal is zero.

Consistent: A system of linear equation is said to be consistent if the system has a unique solution or it has infinitely many solutions.

Inconsistent: A system of liner equation is said to be inconsistent if the system has no solution.

Trivial Solution: The solution $(0, 0, 0) = (x_1, x_2, x_3)$ is called trivial solution.

Non Trivial Solution: Any solution other then trivial solution is called non-trivial solution.

Symmetric Matrix: A square matrix A is called symmetric if $A^t = A$.

Skew Symmetric Matrix: A square matrix A is called skew symmetric if $A^t = -A$.

Hermitian Matrix: A square matrix A is called hermitian if $\left(\overline{A}\right)^t = A$.

Skew Hermitian Matrix: A Square Matrix A is called skew hermitian if $\left(\overline{A}\right)^t = -A$.

Singular Matrix: A Square Matrix A is called singular if $|A| = 0$.

Non Singular Matrix: A Square Matrix A is called non-singular if $|A| \neq 0$.

Chapter **< 4 >** **Factor theorem (Statement):** The polynomial $x-a$ is factor of the polynomial $f(\mathrm{x})$ if and only if $f(a)=0$.

Remainder theorem (Statement): If a polynomial $f(\mathrm{x})$ of degree n is divided by $x-a$ till no x term exists in the remainder, then $f(\mathrm{a})$ is the remainder.

Chapter **< 5 >** **Equation:** A relation in which the equality holds for particular values of variable is called equation. **Examples:** $x^2+7x+12=0$, $x^2-9=0$.

Identity: A relation in which the equality holds for all values of variable is called identity.

Examples: $(\mathrm{x}+3)(\mathrm{x}+4)=x^2+7x+12=0$, $x^2-9=(\mathrm{x}+3)(\mathrm{x}-3)$.

Rational Fraction: The quotient of two polynomials $\frac{P(\mathrm{x})}{Q(\mathrm{x})}$ where $Q(\mathrm{x})\neq 0$ with no common factor is called a rational fraction. **Examples:** $\frac{x^4}{1-x^4}$, $\frac{1}{1-x^2}$, $\frac{3x^2}{x-2}$.

Proper Rational Fraction: The rational fraction $\frac{P(\mathrm{x})}{Q(\mathrm{x})}$ is called proper rational fraction if the degree of $\mathrm{P(x)}$ is less than the degree of $Q(\mathrm{x})$. Examples: $\frac{3}{1-x}$, $\frac{2x-5}{x^2+4}$, $\frac{9x^2}{x^3-1}$.

Improper Rational Fraction: The rational fraction $\frac{P(\mathrm{x})}{Q(\mathrm{x})}$ is called improper rational fraction if the degree of $\mathrm{P(x)}$ is equal to or greater than the degree of $Q(\mathrm{x})$. Examples: $\frac{x^4}{1-x^4}$, $\frac{x^2-3}{3x+1}$, $\frac{x^3-x^2+x+1}{x^2+5}$.

Partial Fractions: A rational fraction can be written as a sum of two or more single rational fraction is called partial fractions. Example: $\frac{7x+25}{(x+3)(\mathrm{x}+4)}=\frac{4}{x+3}+\frac{3}{x+4}$.

Chapter **< 6 >** **Sequence:** A sequence is a function whose domain is a subset of natural number.

Arithmetic Progression or Sequence (A.P.): A sequence $\{a_n\}$ is an arithmetic sequence if a_n-a_{n-1} is same for all terms. Where $n\in N$ and $n>1$.

Series: The sum of the terms of a sequence is called series.

Geometric Progression or Sequence (G.P): A sequence $\{a_n\}$ is a geometric sequence if $\frac{a_n}{a_{n-1}}$ is same for all terms. Where $n\in N$ and $n>1$.

Harmonic Progression or Sequence (H.P): A sequence of numbers whose reciprocals form an A.P. is called Harmonic progression or sequence.

Chapter **< 7 >** **Circular Permutation:** The permutation of things which can be represented by the points on a circle are called circular permutation.

Combinations: An arrangement of n different objects taken r at a time without any order is called a combination of " n " things taken " r " at a time. It is denoted by ${}^nC_r$ or $\binom{n}{r}$.

Sample Space: The collection of all possible outcomes of an experiment is called a sample space. It is denoted by **S**.

Event: A particular outcome of an experiment is called an event. It is denoted by **E**.

Probability: Probability is the numerical evaluation of a chance that a particular event occur. It is denoted by $P(\mathrm{E})$ and its value is $0 \le P(\mathrm{E}) \le 1.$

Chapter **< 8 >** **State "Principle of mathematical induction":**

If a statement $S(\mathrm{n})$ satisfies the following conditions:

(1) $S(\mathrm{n})$ is true for $n = 1.$

(2) $S(\mathrm{n})$ is true for $n = k$ implies that $S(\mathrm{n})$ is true for $n = k+1.$ Then $S(\mathrm{n})$ is true for all positive integral value of $n.$

Difference between Binomial Theorem and Binomial Series: The Binomial theorem has finite terms and exponent n is non-negative integer while the Binomial series has infinite terms and the exponent n is a negative or fraction.

Chapters **< 9 to 14 >** **Radian:** Radian is the measure of angle subtended at the center of circle by an arc, whose length is equal to the radius of the circle.

Fundamental law of the trigonometry: Let α and β any two angles then

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ is called Fundamental law of the trigonometry.

Allied Angles: The basic angle θ associated with right angle or its multiple is called an allied angle.

Examples: $90^\circ \pm \theta, 180^\circ \pm \theta, 270^\circ \pm \theta, 360^\circ \pm \theta$ are allied angles.

Period: Period of a trigonometric function is smallest positive number such that the value of the trigonometric function remains unchanged when we add the period to original angle.

Angle of Elevation: When an object is at higher level form the observer's eye then the angle made by the observer's eye is called an angle of elevation.

Angle of Depression: When an object is at lower level form the observer's eye then the angle made by the observer's eye is called an angle of depression.

In-Circle: The circle inside a triangle touching its three sides is called inscribed circle or in-circle. Its center is called in-center and its radius is called in-radius.

Escribed Circle: A circle which touches one side of the triangle externally and the outer two produced sides is called an escribed or ex-circle or e-circle.

Trigonometric Equation: The equations containing at least one trigonometric function are called trigonometric equations. Example $\cos x - 1 = 0.$

Compiled By: MuzzammilSubhan

M.Phil. Math (Minhaj University Lahore), M.Sc. Math (Quaid-i-Azam University, Islamabad)
M.Ed. (University of Sargodha), B.Sc., B.C.S. & PGD-IT

Email: muzzammilsubhan@gmail.com **Contact No: 0300-7779500**