TAHIR MEHMOOD
M.Sc Math
0345-6510779

TAHIR

Q.5 To form 6-digit numbers using 2,2,3,3,4,4 :

Total number of 6 digit numbers $= \dfrac{6!}{2!\,2!\,2!} = \dfrac{6\cdot5\cdot4\cdot3\cdot2!}{2\cdot1\cdot2\cdot1\cdot2!}$

$= 90$ Ans.

The numbers will be between 400,000 and 430,000 if 4, 2 are at extreme left position so numbers.

The numbers b/w 430,000 and 400,000 $= \dfrac{4!}{2!\,1!} = \dfrac{4\cdot3\cdot2!}{2!}$

$= 12$ Ans.

Q.6 To find number of commitees using 3,4,2,2 :

Total members = 11

The number of Commitees $= \dfrac{11!}{3!\,4!\,2!\,2!}$

$= \dfrac{11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4!}{3\cdot2\cdot1\cdot2\cdot1\cdot2\cdot1\cdot4!} = 69300$ Ans.

Combination :-

"The arrangement of objects without considering order is called Combination."

If n are total objects and r are the taken objects then Combination is defined as

$$^nC_r = \frac{^nP_r}{r!} = \frac{n!}{(n-r)!\,r!}$$

The important results:

* $^nC_n = \dfrac{n!}{(n-n)!\,n!} = 1$
* $^nC_0 = \dfrac{n!}{0!\,n!} = 1$
* $^nC_r = {}^nC_{n-r}$ (Complementry Combination)

TAHIR

* Permutation, Combination does not exist if $r > n$ and if "n and r" are $-$ve integers.
* Factorial of $-$ve integer does not exist.

TAHIR MEHMOOD
M.Sc Math
0345-6510779

TAHIR

Imp Example:- Prove that ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^{n}C_r$

$$L.H.S. = {}^{n-1}C_r + {}^{n-1}C_{r-1}$$

$$= \frac{(n-1)!}{r!\,(n-1-r)!} + \frac{(n-1)!}{(r-1)!\,(n-1-r+1)!}$$

$$= \frac{(n-1)!}{r(r-1)!\,(n-r-1)!} + \frac{(n-1)!}{(r-1)!\,(n-r)!}$$

$$= \frac{(n-1)!}{(r-1)!}\left\{\frac{1}{r(n-r-1)!} + \frac{1}{(n-r)(n-r-1)!}\right\}$$

$$= \frac{(n-1)!}{(r-1)!\,(n-r-1)!}\left\{\frac{1}{r} + \frac{1}{n-r}\right\}$$

$$= \frac{(n-1)!}{(r-1)!\,(n-r-1)!}\left\{\frac{n-r+r}{(n-r)r}\right\}$$

$$= \frac{(n-1)!}{(r-1)!\,(n-r-1)!}\left\{\frac{n}{r(n-r)}\right\}$$

$$= \frac{n\,(n-1)!}{r(r-1)!\,(n-r)(n-r-1)!}$$

$$= \frac{n!}{r!\,(n-r)!} \qquad \because n(n-1)! = n!$$

$$= {}^{n}C_r = RHS$$

Thus $LHS = RHS \Rightarrow {}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^{n}C_r$

Exercise 7.4

TAHIR

Q.1 Evaluate the followings:

(i) ${}^{12}C_3 \Rightarrow {}^{12}C_3 = \frac{12!}{(12-3)!\,3!} \Rightarrow {}^{12}C_3 = \frac{\cancel{12}^{2}\cdot 11\cdot 10\cdot \cancel{9!}}{\cancel{9!}\,\cancel{3\cdot 2}\cdot 1}$

${}^{12}C_3 = 220 \Rightarrow {}^{12}C_3 = 220$ Ans.

TAHIR MEHMOOD
M.Sc Math
0345-6510779

(ii) $^{20}C_{17} = ?$

$$^{20}C_{17} = \frac{20!}{(20-17)!\ 17!}$$

$$^{20}C_{17} = \frac{20\cdot19\cdot18\cdot\cancel{17!}}{3!\ \cancel{17!}}$$

$$^{20}C_{17} = \frac{20\cdot19\cdot\overset{3}{\cancel{18}}}{\cancel{3\cdot2}\cdot1}$$

$$^{20}C_{17} = 1140 \text{ Ans.}$$

(iii) $^{n}C_4 = ?$

$$^{n}C_4 = \frac{n!}{(n-4)!\ 4!}$$

$$^{n}C_4 = \frac{n(n-1)(n-2)(n-3)\cancel{(n-4)!}}{4!\ \cancel{(n-4)!}}$$

$$^{n}C_4 = \frac{n(n-1)(n-2)(n-3)}{4\cdot3\cdot2\cdot1}$$

$$^{n}C_4 = \frac{n(n-1)(n-2)(n-3)}{24} \text{ Ans.}$$

Q.2 Find n, when

(i) $^{n}C_5 = {}^{n}C_4$

$$\frac{n!}{(n-5)!\ 5!} = \frac{n!}{(n-4)!\ 4!}$$

$$\frac{\cancel{n!}}{\cancel{(n-5)!}\ 5\cdot\cancel{4!}} = \frac{\cancel{n!}}{\cancel{(n-5)!}\ (n-4)\cdot\cancel{4!}}$$

$$\frac{1}{5} = \frac{1}{n-4}$$

$$5 = n-4$$

$$n = 5+4 = 9$$

$$\boxed{n=9} \text{ Ans.}$$

(ii) $^{n}C_{10} = \frac{12\times11}{2!}$

$$\frac{n!}{(n-10)!\ 10!} = \frac{12\times11\times10!}{2!\times10!}$$

$$\frac{n!}{(n-10)!\ \cancel{10!}} = \frac{12!}{2!\ \cancel{10!}}$$

$$\frac{n!}{(n-10)!} = \frac{12!}{(12-10)!}$$

$$^{n}P_{10} = {}^{12}P_{10}$$

$$\boxed{n=12}$$

(iii) $^{n}C_{12} = {}^{n}C_6$

$\because\ ^{n}C_r = {}^{n}C_{n-r}$

$$\Rightarrow {}^{n}C_{12} = {}^{n}C_{n-12}$$

$$^{n}C_{n-12} = {}^{n}C_6$$

$$n-12 = 6$$

$$n = 6+12$$

$$\boxed{n=18}$$

Q.3 Find n and r if

(i) $^{n}C_r = 35$ and $^{n}P_r = 210$

$$\because\ ^{n}C_r = \frac{^{n}P_r}{r!}$$

$$35 = \frac{210}{r!} \Rightarrow r! = \frac{\overset{6}{\cancel{210}}}{\cancel{35}}$$

$$r! = 6 \Rightarrow r! = 3\cdot2\cdot1$$

$$r! = 3!$$

$$\boxed{r=3}$$

TAHIR MEHMOOD
M.Sc Math
0345-6510779

Now $^nC_r = 35$

$^nC_3 = 35 \Rightarrow \dfrac{n!}{(n-3)!\,3!} = 35$

$\dfrac{n(n-1)(n-2)\cancel{(n-3)!}}{3!\,\cancel{(n-3)!}} = 35$

$n(n-1)(n-2) = 35 \times 3 \times 2$

$n(n-1)(n-2) = 7 \times 6 \times 5$

$\Rightarrow \boxed{n = 7}$

Thus $n = 7$ and $r = 3$

Q.5 Total boys $= 12$

To select at least $= 3$

Total Girls $= 8$

To select at least $= 2$

Committees $= {}^{12}C_3 \cdot {}^{8}C_2$

$= \dfrac{12!}{(12-3)!\,3!} \times \dfrac{8!}{(8-2)!\,2!}$

$= \dfrac{\cancel{12}^{2}\cdot 10 \cdot 11 \cdot \cancel{9!}}{\cancel{9!}\cdot \cancel{3}\cdot \cancel{2} \cdot 1} \times \dfrac{8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!}\cdot \cancel{2} \cdot 1}$

$= 10 \cdot 11 \cdot 8 \cdot 7$

$= 6160$ Committees.

Ans.

(ii) Imp. $^{n-1}C_{r-1} : {}^nC_r : {}^{n+1}C_{r+1} = 3 : 6 : 11$

$\dfrac{(n-1)!}{(n-1-r+1)!\,(r-1)!} : \dfrac{n!}{(n-r)!\,r!} : \dfrac{(n+1)!}{(n+1-r-1)!\,(r+1)!}$

$= 3 : 6 : 11$

$\dfrac{(n-1)!}{(n-r)!\,(r-1)!} : \dfrac{n(n-1)!}{(n-r)!\,r(r-1)!} : \dfrac{(n+1)(n)(n-1)!}{(n-r)!\,(r+1)r(r-1)!} = 3 : 6 : 11$

Dividing LHS through out by $\dfrac{(n-1)!}{(n-r)!\,(r-1)!}$, we have

$1 : \dfrac{n}{r} : \dfrac{(n+1)n}{(r+1)r} = 3 : 6 : 11$

$\Rightarrow 1 : \dfrac{n}{r} = 3 : 6$ and $\dfrac{n}{r} : \dfrac{n(n+1)}{r(r+1)} = 6 : 11$

$\Rightarrow \dfrac{1}{n/r} = \dfrac{3}{6}$ and $1 : \dfrac{n+1}{r+1} = 6 : 11$

$\Rightarrow \dfrac{r}{n} = \dfrac{1}{2}$

$\Rightarrow n = 2r$

$\dfrac{1}{\left(\frac{n+1}{r+1}\right)} = \dfrac{6}{11}$

$11(r+1) = 6(n+1)$

$11r + 11 = 6n + 6$

$11r + 11 = 6(2r) + 6$

$11r + 11 = 12r + 6$

$11 - 6 = 12r - 11r$

$\boxed{r = 5}$

$n = 2(5)$

$\boxed{n = 10}$

TAHIR MEHMOOD
M.Sc Math
0345-6510779

Q.4 (a) Diagonals $= {}^nC_r - n$ (b) Triangles $= {}^nC_r$

where r used sides and n are total sides...

(i) 5 Sides

Diagonals $= {}^5C_2 - 5$

$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3! \cdot 2 \cdot 1} - 5$

$= 10 - 5 = 5$ Ans.

(ii) 8 Sides

Diagonals $= {}^8C_2 - 8$

$= \frac{8 \cdot 7 \cdot 6!}{6! \, 2!} - 8$

$= 28 - 8 = 20$ Ans.

(iii) 12 Sides

Diagonals $= {}^{12}C_2 - 12$

$= \frac{12!}{(12-2)! \, 2!} - 12$

$= \frac{12 \cdot 11 \cdot 10!}{10! \, 2 \cdot 1} - 12$

$= 66 - 12 = 54$ Ans.

(i) 5 Sides

Triangles $= {}^5C_3 = \frac{5 \cdot 4 \cdot 3!}{(5-3)! \, 3!}$

$= 5 \cdot 2 = 10$ Ans.

(ii) 8 Sides

Triangles $= {}^8C_3$

$= \frac{8!}{(8-3)! \, 3!}$

$= \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3 \cdot 2 \cdot 1}$

$= 56$ Ans.

(iii) 12 Sides

Triangles $= {}^{12}C_3$

$= \frac{12 \cdot 11 \cdot 10 \cdot 9!}{(12-3)! \, 3!}$

$= \frac{12 \cdot 11 \cdot 10 \cdot 9!}{9! \, 3 \cdot 2}$

$= 220$ Ans.

Q.6 Total person = 8

Particular Person = 2

Now Committe is to select among persons = 6

Members of Committee should be = 5

Total Committees $= {}^6C_3$ ($\because$ 2 are all time includes)

Total number of Committees $= \frac{6!}{(6-3)! \, 3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3 \cdot 2 \cdot 1 \cdot 3!}$

$= 20$ Ans.

Thus total committees = 20 Ans.

TAHIR MEHMOOD
M.Sc Math
0345-6510779

TAHIR

Q.7 Total members to be Selected = 11 = r

Total players including remainders = 15 = n

Total teams = ${}^{n}C_{r} = {}^{15}C_{11}$

$= \frac{15!}{(15-11)!\,11!} = \frac{15.14.13.12.\cancel{11!}}{4!\,\cancel{11!}}$

$= \frac{15.\cancel{14}^{7}.13.\cancel{12}}{\cancel{24}_{2}} = 1365$

Total teams = 1365 Ans.

If one player is particular selection will be 10 out of 14

Now total teams = ${}^{14}C_{10} = \frac{14!}{(14-10)!\,10!}$

$= \frac{14.13.12.11.\cancel{10!}}{4!\,\cancel{10!}} = \frac{\cancel{14}^{7}.13.\cancel{12}.11}{\cancel{24}_{2}}$

$= 1001$

Total teams including a particular player = 1001 Ans.

Q.8 Show that ${}^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$

LHS = ${}^{16}C_{11} + {}^{16}C_{10} \Rightarrow \frac{16!}{(16-11)!\,11!} + \frac{16!}{(16-10)!\,10!}$

$= \frac{16!}{5!\,11.10!} + \frac{16!}{6!\,10!} \Rightarrow \frac{16!}{10!}\left\{\frac{1}{11.5!} + \frac{1}{6.5!}\right\}$

$= \frac{16!}{5!\,10!}\left\{\frac{1}{11} + \frac{1}{6}\right\} \Rightarrow \frac{16!}{5!\,10!}\left\{\frac{6+11}{11.6}\right\}$

$= \frac{16!\,(17)}{6.5!\,11.10!} \Rightarrow \frac{17!}{6!\,11!} \Rightarrow \frac{17!}{(17-11)!\,11!}$

$= {}^{17}C_{11} = RHS =$

TAHIR

Proved that ${}^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$ (Proved) :

TAHIR MEHMOOD
M.Sc Math
0345-6510779

TAHIR

V.Imp.

Q.10 Prove that ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

LHS $= {}^{n}C_{r} + {}^{n}C_{r-1}$

$$= \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\;r.(r-1)!} + \frac{n!}{(n-r+1)(n-r)!\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\;(r-1)!}\left\{\frac{1}{r} + \frac{1}{n-r+1}\right\}$$

$$= \frac{n!}{(n-r)!\;(r-1)!}\left\{\frac{n-r+1+r}{r(n-r+1)}\right\}$$

$$= \frac{n!}{(n-r)!\;(r-1)!}\left\{\frac{n+1}{r(n-r+1)}\right\} \qquad (\because n(n-1)! = n!)$$

$$= \frac{(n+1)\;n!}{(n-r)!\,(n-r+1)\,(r-1)!\,r}$$

$$= \frac{(n+1)!}{(n-r+1)!\;\;r!} \Rightarrow \frac{(n+1)!}{(n+1-r)!\;\;r!}$$

$$= {}^{n+1}C_{r} = RHS$$

Thus

LHS = RHS

TAHIR

${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$ (Proved)

C.S.S. Academy

By: Tahir Mahmood
M.Sc. Mathematics.

Tahir Mahmood.
Contact No:
0300 6419294