

TAHIR MEHMOOD

M.Sc Math  
0345-6510779

CH #7 (1st Year)

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Q.5 To form 6-digit numbers using 2,2,3,3,4,4 :

$$\text{Total number of 6 digit numbers} = \frac{6!}{2!2!2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2! \cdot 2! \cdot 2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2! \cdot 2! \cdot 2!} = 90 \text{ Ans.}$$

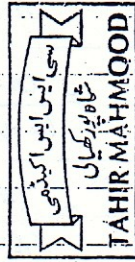
The numbers will be between 400,000 and 430,000 if 4,2 are at extreme left position so numbers.

$$\text{The numbers b/w 430,000 and 400,000} = \frac{4!}{2!1!} = \frac{4 \cdot 3 \cdot 2!}{2!} = 12 \text{ Ans.}$$

Q.6 To find number of Committees using 3,4,2,2 :

Total members = 11

$$\text{The number of Committees} = \frac{11!}{3!4!2!2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1 \cdot 4!} = 69300 \text{ Ans.}$$



Combination:-

"The arrangement of objects without considering order is called Combination."

If n are total objects and r are the taken objects then combination is defined as

$${}^n C_r = \frac{{}^n P_r}{r!} = \frac{n!}{(n-r)! r!}$$

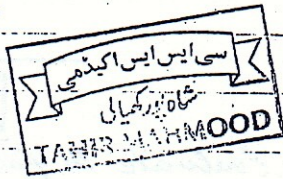
The important results:

$$* {}^n C_n = \frac{n!}{(n-n)! n!} = 1$$

$$* {}^n C_0 = \frac{n!}{0! n!} = 1$$

$$* {}^n C_r = {}^n C_{n-r}$$

(Complementary Combination)



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Permutation, Combination does not exist if r > n and if n and r are -ve integers. \* Factorial of -ve integer does not exist.

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Imp Example:- Prove that  ${}^{n-1}C_k + {}^{n-1}C_{k-1} = {}^nC_k$

$$\begin{aligned} \text{L.H.S.} &= {}^{n-1}C_k + {}^{n-1}C_{k-1} \\ &= \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-1-k+1)!} \end{aligned}$$

$$= \frac{(n-1)!}{k(k-1)!(n-k)!} + \frac{(n-1)!}{(k-1)!(n-k)!}$$

$$= \frac{(n-1)!}{(k-1)!} \left\{ \frac{1}{k(n-k)!} + \frac{1}{(n-k)(n-k)!} \right\}$$

$$= \frac{(n-1)!}{(k-1)!(n-k)!} \left\{ \frac{1}{k} + \frac{1}{n-k} \right\}$$

$$= \frac{(n-1)!}{(k-1)!(n-k)!} \left\{ \frac{n-k+k}{(n-k)k} \right\}$$

$$= \frac{(n-1)!}{(k-1)!(n-k)!} \left\{ \frac{n}{k(n-k)} \right\}$$

$$= \frac{n(n-1)!}{k(k-1)!(n-k)(n-k)!}$$

$$= \frac{n!}{k!(n-k)!} \quad \because n(n-1)! = n!$$

$$= {}^nC_k = \text{RHS}$$

Thus LHS = RHS  $\Rightarrow {}^{n-1}C_k + {}^{n-1}C_{k-1} = {}^nC_k$

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**Exercise 7.4**

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Q.1 Evaluate the followings:

$$(i) {}^{12}C_3 \Rightarrow {}^{12}C_3 = \frac{12!}{(12-3)!3!} \Rightarrow {}^{12}C_3 = \frac{12 \cdot 11 \cdot 10 \cdot 9!}{9! \cdot 3 \cdot 2 \cdot 1}$$

$${}^{12}C_3 = 220 \Rightarrow {}^{12}C_3 = 220 \text{ Ans.}$$



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Now  ${}^n C_k = 35$

Q.5 Total boys = 12

$${}^n C_3 = 35 \Rightarrow \frac{n!}{(n-3)! 3!} = 35$$

$$\frac{n(n-1)(n-2)(n-3)!}{3! (n-3)!} = 35$$

$$n(n-1)(n-2) = 35 \times 3 \times 2$$

$$n(n-1)(n-2) = 7 \times 6 \times 5$$

$$\Rightarrow \boxed{n=7}$$

Thus  $n=7$  and  $k=3$

(iii)  ${}^{n-1} C_{n-1} : {}^n C_k : {}^{n+1} C_{k+1} = 3 : 6 : 11$

Imp.

$$\frac{(n-1)!}{(n-1-k+1)!(k-1)!} : \frac{n!}{(n-k)!(k-1)!} : \frac{(n+1)!}{(n+1-k-1)!(k+1)!}$$

$$= 3 : 6 : 11$$

$$\frac{(n-1)!}{(n-k)!(k-1)!} : \frac{n(n-1)!}{(n-k)!(k-1)!} : \frac{(n+1)(n)(n-1)!}{(n-k)!(k+1)k(k-1)!} = 3 : 6 : 11$$

Dividing LHS through out by  $\frac{(n-1)!}{(n-k)!(k-1)!}$ , we have

$$1 : \frac{n}{k} : \frac{(n+1)n}{(k+1)k} = 3 : 6 : 11$$

$$\Rightarrow 1 : \frac{n}{k} = 3 : 6 \quad \text{and} \quad \frac{n}{k} : \frac{n(n+1)}{k(k+1)} = 6 : 11$$

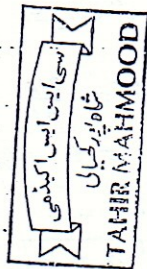
$$\Rightarrow \sqrt[n]{n/k} = \frac{3}{6} \quad \text{and} \quad 1 : \frac{n+1}{k+1} = 6 : 11$$

$$\Rightarrow \frac{k}{n} = \frac{1}{2}$$

$$\Rightarrow n = 2k$$

$$n = 2(5)$$

$$\boxed{n=10}$$



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To select at least = 3

Total Girls = 8

To select at least = 2

$$\text{Committees} = {}^{12} C_3 \cdot {}^8 C_2$$

$$= \frac{12!}{(12-3)! 3!} \times \frac{8!}{(8-2)! 2!}$$

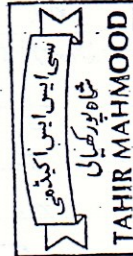
$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} \times \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1}$$

$$= 10 \cdot 11 \cdot 8 \cdot 7$$

$$= 6160 \text{ Committees.}$$

Ans.

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$$\sqrt[k+1]{k+1} = \frac{6}{11}$$

$$11(k+1) = 6(n+1)$$

$$11k+11 = 6n+6$$

$$11k+11 = 6(2k)+6$$

$$11k+11 = 12k+6$$

$$11-6 = 12k-11k$$

$$\boxed{k=5}$$

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Q.4 (a) Diagonals =  ${}^n C_2 = n$  (b) Triangles =  ${}^n C_3$

where  $k$  used sides and  $n$  are total sides

(i) 5 Sides

$$\begin{aligned} \text{Diagonals} &= {}^5 C_2 - 5 \\ &= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} - 5 \\ &= 10 - 5 = 5 \text{ Ans.} \end{aligned}$$

(ii) 8 Sides

$$\begin{aligned} \text{Diagonals} &= {}^8 C_2 - 8 \\ &= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} - 8 \\ &= 28 - 8 = 20 \text{ Ans.} \end{aligned}$$

(iii) 12 Sides

$$\begin{aligned} \text{Diagonals} &= {}^{12} C_2 - 12 \\ &= \frac{12!}{(12-2)! 2!} - 12 \\ &= \frac{6 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} - 12 \\ &= 66 - 12 = 54 \text{ Ans.} \end{aligned}$$

(i) 5 Sides

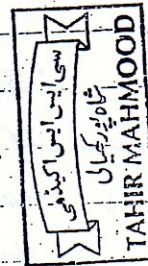
$$\begin{aligned} \text{Triangles} &= {}^5 C_3 = \frac{5 \cdot 4 \cdot 3!}{(5-3)! 3!} \\ &= 5 \cdot 2 = 10 \text{ Ans.} \end{aligned}$$

(ii) 8 Sides

$$\begin{aligned} \text{Triangles} &= {}^8 C_3 \\ &= \frac{8!}{(8-3)! 3!} \\ &= \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3! \cdot 3!} \\ &= 56 \text{ Ans.} \end{aligned}$$

(iii) 12 Sides

$$\begin{aligned} \text{Triangles} &= {}^{12} C_3 \\ &= \frac{12 \cdot 11 \cdot 10 \cdot 9!}{(12-3)! 3!} \\ &= \frac{24 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3! \cdot 3!} \\ &= 220 \text{ Ans.} \end{aligned}$$



Q.6 Total person = 8

Particular Person = 2

Now Committee is to select among persons = 6

Members of Committee should be = 5

Total Committees =  ${}^6 C_3$  ( $\because$  2 are all time includes)

$$\begin{aligned} \text{Total number of Committees} &= \frac{6!}{(6-3)! 3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 3!} \\ &= 20 \text{ Ans.} \end{aligned}$$

Thus total committees = 20 Ans.



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Q.7 Total members to be Selected = 11 = n

Total players including remainders = 15 = r

$$\begin{aligned} \text{Total teams} &= {}^r C_n = {}^{15} C_{11} \\ &= \frac{15!}{(15-11)! 11!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot \cancel{10!}}{4! \cdot \cancel{11!}} \\ &= \frac{15 \cdot 14 \cdot 13 \cdot 12}{24} = 1365 \end{aligned}$$

Total teams = 1365 Ans.

If one player is particular selection will be 10 out of 14

$$\begin{aligned} \text{Now total teams} &= {}^{14} C_{10} = \frac{14!}{(14-10)! 10!} \\ &= \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10!}{4! \cdot 10!} = \frac{14 \cdot 13 \cdot 12 \cdot 11}{24} \\ &= 1001 \end{aligned}$$

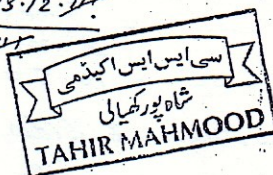
Total teams including a particular player = 1001 Ans.

Q.8 Show that  ${}^{16} C_{11} + {}^{16} C_{10} = {}^{17} C_{11}$ 

$$\begin{aligned} \text{LHS} &= {}^{16} C_{11} + {}^{16} C_{10} \Rightarrow \frac{16!}{(16-11)! 11!} + \frac{16!}{(16-10)! 10!} \\ &= \frac{16!}{5! 11 \cdot 10!} + \frac{16!}{6! 10!} \Rightarrow \frac{16!}{10!} \left\{ \frac{1}{11 \cdot 5!} + \frac{1}{6 \cdot 5!} \right\} \\ &= \frac{16!}{5! 10!} \left\{ \frac{1}{11} + \frac{1}{6} \right\} \Rightarrow \frac{16!}{5! 10!} \left\{ \frac{6+11}{11 \cdot 6} \right\} \\ &= \frac{16! (17)}{6 \cdot 5! 11 \cdot 10!} \Rightarrow \frac{17!}{6! 11!} \Rightarrow \frac{17!}{(17-11)! 11!} \\ &= {}^{17} C_{11} = \text{RHS} \end{aligned}$$

Proved that  ${}^{16} C_{11} + {}^{16} C_{10} = {}^{17} C_{11}$  (Proved)

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V-IMP

Q.10 Prove that  ${}^n C_k + {}^n C_{k-1} = {}^{n+1} C_k$

LHS =  ${}^n C_k + {}^n C_{k-1}$

=  $\frac{n!}{(n-k)! k!} + \frac{n!}{(n-k+1)! (k-1)!}$

=  $\frac{n!}{(n-k)! k \cdot (k-1)!} + \frac{n!}{(n-k+1)(n-k)! (k-1)!}$

=  $\frac{n!}{(n-k)! (k-1)!} \left\{ \frac{1}{k} + \frac{1}{n-k+1} \right\}$

=  $\frac{n!}{(n-k)! (k-1)!} \left\{ \frac{n-k+1 + k}{k(n-k+1)} \right\}$

=  $\frac{n!}{(n-k)! (k-1)!} \left\{ \frac{n+1}{k(n-k+1)} \right\}$  ( $\because n(n-1)! = n!$ )

=  $\frac{(n+1) n!}{(n-k)! (n-k+1) (k-1)! k}$

=  $\frac{(n+1)!}{(n-k+1)! k!} \Rightarrow \frac{(n+1)!}{(n+1-k)! k!}$

=  ${}^{n+1} C_k = RHS$

Thus

LHS = RHS

${}^n C_k + {}^n C_{k-1} = {}^{n+1} C_k$  (Proved)



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