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Product of Vectors

- Scalar/Dot Product
- Vector/Cross Product

Scalar / Dot Product:-

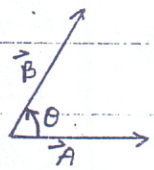
Let $\vec{A}$ and $\vec{B}$ are two vectors then dot product is denoted and defined as

$$\vec{A}\cdot\vec{B} = |A|\ |B|\ \cos\theta; \quad 0 \le \theta \le \pi$$

where θ is the angle b/w $\vec{A}$ and $\vec{B}$

For $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

$\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$



$\vec{A}\cdot\vec{B} = A_1B_1 + A_2B_2 + A_3B_3$

* Dot Product gives scalar as result.

* It is also called inner product.

Charactristics:

(i) $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

Commutative Law holds

(ii) If $\vec{A}$ and $\vec{B}$ are parallel then

$\vec{A}\cdot\vec{B} = |A|\ |B|\ \cos 0^\circ$

$\vec{A}\cdot\vec{B} = |A|\ |B| \qquad (\because \cos 0^\circ = 1)$

(iii) If $\vec{A}$ and $\vec{B}$ are anti parallel then

$\vec{A}\cdot\vec{B} = |A|\ |B|\ \cos 180^\circ$

$\vec{A}\cdot\vec{B} = -|A|\ |B| \qquad (\because \cos 180^\circ = -1)$

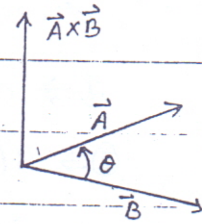
* $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}|\ |\vec{B}|}$

Vector / Cross Product:-

Let $\vec{A}$ and $\vec{B}$ are two vectors then Cross product is denoted and defined as $\vec{A}\times\vec{B} = |\vec{A}|\ |\vec{B}|\ \sin\theta\ \hat{n}$; $0 \le \theta \le \pi$

where $\hat{n}$ is unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$.

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$



* Cross product gives new vector perpendicular to $\vec{A}$ and $\vec{B}$ as result.

Charactristics:

(i) $\vec{A}\times\vec{B} \ne \vec{B}\times\vec{A}$ but $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

Commutative Law does not hold.

(ii) If $\vec{A}$ and $\vec{B}$ are parallel or Antiparallel then $\vec{A}\times\vec{B} = |A|\,|B|\ \sin\theta\ \hat{n}$ $(\theta = 0^\circ$ or $\theta = 180^\circ)$

$\vec{A}\times\vec{B} = \vec{0} \qquad (\because \sin\theta = 0$ both Cases$)$

(iii) If $\vec{A}$ and $\vec{B}$ are perpendicular then

$\vec{A}\times\vec{B} = |\vec{A}|\ |\vec{B}|\ \sin 90^\circ$

$\vec{A}\times\vec{B} = |\vec{A}|\ |\vec{B}| \qquad (\because \sin 90^\circ = 1)$

* $\sin\theta = \dfrac{|\vec{A}\times\vec{B}|}{|\vec{A}|\,|\vec{B}|}$

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(iv) If $\vec{A}$ and $\vec{B}$ are perpendicular

then $\vec{A}\cdot\vec{B} = |A|\,|B| \cos 90^\circ$

$\vec{A}\cdot\vec{B} = 0 \qquad (\because \cos 90^\circ = 0)$

(v) For unit vectors $\hat{i}, \hat{j}, \hat{k}$

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

(vi) Projections:-

Projection of $\vec{A}$ along $\vec{B}$ is defined as

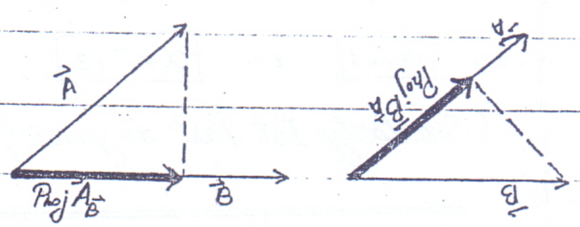
$$\text{Proj } \vec{A} \text{ along } \vec{B} = |\vec{A}|\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|B|}$$

Similarly Projection of $\vec{B}$ along $\vec{A}$

$$\text{Proj } \vec{B} \text{ along } \vec{A} = |\vec{B}|\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|A|}$$

(vii) $\vec{A}\cdot(\vec{B}\pm\vec{C}) = \vec{A}\cdot\vec{B} \pm \vec{A}\cdot\vec{C}$

(viii) $r\vec{A}\cdot\vec{B} = \vec{A}\cdot r\vec{B} = r(\vec{A}\cdot\vec{B})$



(ix) $(r\vec{A}\cdot\lambda\vec{B}) = r\lambda(\vec{A}\cdot\vec{B}) = (\lambda\vec{A}\cdot r\vec{B})$

(x) Let $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

then

$A^2 = \vec{A}\cdot\vec{A} = A_1^2 + A_2^2 + A_3^2$

* $\vec{A}\cdot\vec{B} = 0$ iff $\vec{A} = \vec{0}$, $\vec{B} = \vec{0}$ or

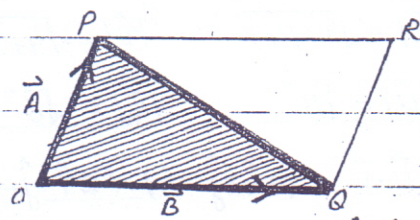
$\vec{A} \perp \vec{B}$ (i.e $\theta = 270^\circ$ or $\theta = 90^\circ$)

(iv) Geometrically

$|\vec{A}\times\vec{B}|$ = Area of parallelogram

where $\vec{A}, \vec{B}$ are adjecent sides.

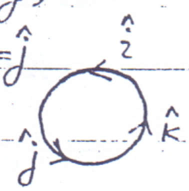
Also $\frac{1}{2}|\vec{A}\times\vec{B}|$ = Area of triangle.



(v) For unit vectors $\hat{i}, \hat{j}, \hat{k}$

$\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$

$\hat{j}\times\hat{i} = -\hat{k}$, $\hat{k}\times\hat{j} = -\hat{i}$, $\hat{i}\times\hat{k} = -\hat{j}$



$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

(vi) $\vec{A}\times r\vec{B} = r\vec{A}\times\vec{B} = r(\vec{A}\times\vec{B})$

(vii) $\vec{A}\times(\vec{B}\pm\vec{C}) = \vec{A}\times\vec{B} \pm \vec{A}\times\vec{C}$

(viii) For a vector $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

$\vec{A}\times\vec{A} = \vec{0} \qquad (\because \theta = 0^\circ$ and $\sin 0^\circ = 0)$

(ix) $r\vec{A}\times\lambda\vec{B} = r\lambda(\vec{A}\times\vec{B}) = \lambda\vec{A}\times r\vec{B}$

* Null vector is parallel also perpendicular to any vector.

* $\vec{A}\times\vec{B} = 0$ iff $\vec{A} = \vec{0}$ or $\vec{B} = \vec{0}$

or $\vec{A} \parallel \vec{B}$ (i.e. $\theta = 0^\circ$ or $\theta = 180^\circ$)

Examples of Dot and Cross Product

$W = \vec{F}\cdot\vec{d}$, Work

$\vec{\tau} = \vec{r}\times\vec{F}$, Torque/Moment of force

$\vec{L} = \vec{r}\times m\vec{v}$, Angular Momentum.

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Exercise 7.3

Q.1 Find Cosine angle θ b/w $\vec{u}$ & $\vec{v}$

(i) $\vec{u} = 3\hat{i} + \hat{j} - \hat{k}$ $\quad \vec{v} = 2\hat{i} - \hat{j} + \hat{k}$

$|\vec{u}| = \sqrt{9+1+1}$ $\quad |\vec{v}| = \sqrt{4+1+1}$

$|\vec{u}| = \sqrt{11}$ $\quad |\vec{v}| = \sqrt{6}$

$\vec{u}\cdot\vec{v} = (3\hat{i}+\hat{j}-\hat{k})\cdot(2\hat{i}-\hat{j}+\hat{k})$

$\vec{u}\cdot\vec{v} = 6-1-1 = 4$

For $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|\,|\vec{v}|}$

$\cos\theta = \dfrac{4}{\sqrt{6}\sqrt{11}} = \dfrac{4}{\sqrt{66}}$ Ans.

(iii) $\vec{u} = -3\hat{i} + 5\hat{j}$ $\quad \vec{v} = 6\hat{i} - 2\hat{j}$

$|\vec{u}| = \sqrt{9+25}$ $\quad |\vec{v}| = \sqrt{36+4}$

$|\vec{u}| = \sqrt{34}$ $\quad |\vec{v}| = \sqrt{40}$

$\vec{u}\cdot\vec{v} = (-3\hat{i}+5\hat{j})\cdot(6\hat{i}-2\hat{j})$

$\vec{u}\cdot\vec{v} = -18-10 = -28$

$\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|\,|\vec{v}|}$

$\cos\theta = \dfrac{-28}{\sqrt{34}\sqrt{40}} = \dfrac{-28}{\sqrt{1360}}$

$\cos\theta = \dfrac{-28}{4\sqrt{85}} = \dfrac{-7}{\sqrt{85}}$ Ans.

(Similarly Remaining Parts do yourself)

Q.2 (i) $\vec{a} = \hat{i} - \hat{k}$ $\quad \vec{b} = \hat{j} + \hat{k}$

$|\vec{a}| = \sqrt{1+1} = \sqrt{2}$, $|\vec{b}| = \sqrt{1+1} = \sqrt{2}$

$\vec{a}\cdot\vec{b} = (\hat{i}-\hat{k})\cdot(\hat{j}+\hat{k}) = -1$

$\text{Proj}\,\vec{a}_{\vec{b}} = |\vec{a}|\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$\text{Proj}\,\vec{a}_{\vec{b}} = \dfrac{-1}{\sqrt{2}}$ Ans.

Now

$\text{Proj}\,\vec{b}_{\vec{a}} = |\vec{b}|\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

$\text{Proj}\,\vec{b}_{\vec{a}} = \dfrac{-1}{\sqrt{2}}$ Ans.

(Similarly (ii) Part do yourself)

Q.3 (ii) $\vec{u} = \alpha\hat{i} + 2\alpha\hat{j} - \hat{k}$, $\vec{v} = \hat{i} + \alpha\hat{j} + 3\hat{k}$

Find α if $\vec{u} \perp \vec{v}$

Soln: $\because$ $\vec{u}$ is perpendicular to $\vec{v}$ so

$\vec{u}\cdot\vec{v} = 0$

$(\alpha\hat{i}+2\alpha\hat{j}-\hat{k})\cdot(\hat{i}+\alpha\hat{j}+3\hat{k}) = 0$

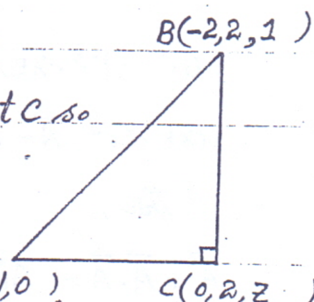
$\alpha + 2\alpha^2 - 3 = 0 \Rightarrow 2\alpha^2 + \alpha - 3 = 0$

$2\alpha^2 + 3\alpha - 2\alpha - 3 = 0 \Rightarrow (\alpha-1)(2\alpha+1) = 0$

$\Rightarrow \boxed{\alpha = 1}$ or $\boxed{\alpha = -1/2}$

(Similarly Ist Part do yourself)

Q.4 Find z where



Soln:- $\because$ Right angle is at C so

$\vec{AC} \perp \vec{BC}$

Thus $\vec{AC}\cdot\vec{BC} = 0$ — (A)

$\vec{AC} = (0-1)\hat{i} + (2+1)\hat{j} + (z-0)\hat{k}$

$\vec{AC} = -\hat{i} + 3\hat{j} + z\hat{k}$ — (1)

$\vec{BC} = (0+2)\hat{i} + (2-2)\hat{j} + (z-1)\hat{k}$

$\vec{BC} = 2\hat{i} + (z-1)\hat{k}$ — (2)

Now Consider (A) $\vec{AC}\cdot\vec{BC} = 0$

$(-\hat{i}+3\hat{j}+z\hat{k})\cdot(2\hat{i}+(z-1)\hat{k}) = 0$

$-2+0+z^2-z=0$

$z^2-z-2=0 \Rightarrow z^2-2z+z-2=0$

$(z-2)(z+1)=0$

$\Rightarrow \boxed{z=2}$ or $\boxed{z=-1}$ Ans.

Q.5- Let $V = x\hat{i}+y\hat{j}+z\hat{k}$

Now $\vec{V}\cdot\hat{i} = (x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{i} = x$

Thus $\vec{v}\cdot\hat{i}=0 \Rightarrow x=0$ ——— ①

Now $\vec{V}\cdot\hat{j} = (x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{j} = y$

Thus $\vec{V}\cdot\hat{j}=0 \Rightarrow y=0$ ——— ②

Now $\vec{V}\cdot\hat{k} = (x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{k} = z$

Thus $\vec{V}\cdot\hat{k}=0 \Rightarrow z=0$ ——— ③

Therefore $\vec{V} = 0\hat{i}+0\hat{j}+0\hat{k} = \vec{0}$ Ans.

Q.6 (a) Let $\vec{a},\vec{b},\vec{c}$ represents the sides of triangle such as

$\vec{a} = 3\hat{i}-2\hat{j}+\hat{k}$, $\vec{b} = \hat{i}-3\hat{j}+5\hat{k}$, $\vec{c} = 2\hat{i}+\hat{j}-4\hat{k}$

Let $\vec{a}\cdot\vec{c} = (3\hat{i}-2\hat{j}+\hat{k})\cdot(2\hat{i}+\hat{j}-4\hat{k})$

$\vec{a}\cdot\vec{c} = 6-2-4 = 6-6$

$\vec{a}\cdot\vec{c} = 0 \Rightarrow \vec{a}\perp\vec{c}$

Thus there exist $90°$ between $\vec{a}$ and $\vec{c}$ so given vectors represents sides of right $\triangle$.

(b) $P(1,3,2)$, $Q(4,1,4)$, $R(6,5,5)$

Firstly, Let us evaluate sides.

$\vec{PQ} = (4-1)\hat{i}+(1-3)\hat{j}+(4-2)\hat{k}$

$\vec{PQ} = 3\hat{i}-2\hat{j}+2\hat{k}$ ——— (i)

$\vec{PR} = (6-1)\hat{i}+(5-3)\hat{j}+(5-2)\hat{k}$

$\vec{PR} = 5\hat{i}+2\hat{j}+3\hat{k}$ ——— (ii)

$\vec{QR} = (6-4)\hat{i}+(5-1)\hat{j}+(5-4)\hat{k}$

$\vec{QR} = 2\hat{i}+4\hat{j}+\hat{k}$ ——— (iii)

Consider $\vec{PQ}\cdot\vec{QR} = (3\hat{i}-2\hat{j}+2\hat{k})\cdot(2\hat{i}+4\hat{j}+\hat{k})$

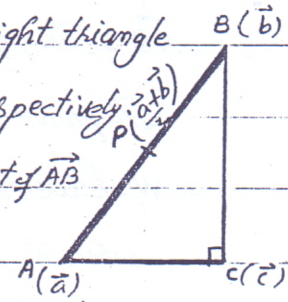
$\vec{PQ}\cdot\vec{QR} = 6-8+2 = 0$

Thus $\vec{PQ}\perp\vec{QR}$

Thus set of Points form right $\triangle$.

Q.7 Let ABC be a right triangle with p.v's $\vec{a},\vec{b},\vec{c}$ respectively

let P be the mid point of $\vec{AB}$



P.V. of $P = \frac{\vec{a}+\vec{b}}{2}$

We have to show $|\vec{AP}| = |\vec{PB}|$

Firstly $\vec{AP} = (\text{P.V. of } P) - (\text{P.V. of } A)$

$\vec{AP} = \frac{\vec{a}+\vec{b}}{2} - \vec{a} = \frac{\vec{a}+\vec{b}-2\vec{a}}{2} = \frac{\vec{b}-\vec{a}}{2}$ ——— ①

$\vec{PB} = (\text{P.V. of } B) - (\text{P.V. of } A)$

$\vec{PB} = \vec{b} - \frac{\vec{a}+\vec{b}}{2} = \frac{2\vec{b}-\vec{a}-\vec{b}}{2} = \frac{\vec{b}-\vec{a}}{2}$ ——— ②

From ① and ② $\vec{AP} = \vec{PB}$

$\Rightarrow |\vec{AP}| = |\vec{PB}|$

Thus P is equidistant from A and B.

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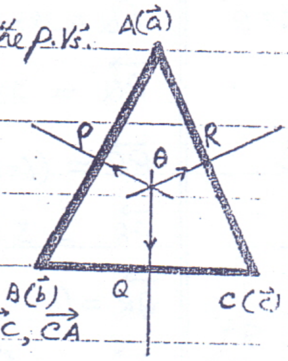
Q.8 Let $\vec{a}, \vec{b}, \vec{c}$ be the P.V's. of the vertices of $\triangle ABC$ respectively. Let P, Q, R be the mid points of $\vec{AB}, \vec{BC}, \vec{CA}$



P.V. of $P = \dfrac{\vec{a}+\vec{b}}{2}$ Also $\vec{AB} = \vec{b} - \vec{a}$

P.V. of $Q = \dfrac{\vec{b}+\vec{c}}{2}$ $\vec{BC} = \vec{c} - \vec{b}$

P.V. of $R = \dfrac{\vec{a}+\vec{c}}{2}$ $\vec{CA} = \vec{a} - \vec{c}$

$\Rightarrow \vec{AC} = \vec{c} - \vec{a}$

Let "O" be point of intersection of $\vec{OP}$ and $\vec{OQ}$ also "O" is origion with

$\vec{OP} \perp \vec{AB} \Rightarrow \left(\dfrac{\vec{a}+\vec{b}}{2}\right)\cdot(\vec{b}-\vec{a}) = 0$ — ①

$\vec{OQ} \perp \vec{BC} \Rightarrow \left(\dfrac{\vec{b}+\vec{c}}{2}\right)\cdot(\vec{c}-\vec{b}) = 0$ — ②

To prove $\vec{OP}, \vec{OQ}, \vec{OR}$ are Concurrent we have to show $\vec{OR} \perp \vec{CA}$

where $\vec{OP}, \vec{OQ}, \vec{OR}$ are $\perp$ bisectors.

Adding ① and ②

$$\frac{(\vec{a}+\vec{b})\cdot(\vec{b}-\vec{a})}{2} + \frac{(\vec{b}+\vec{c})\cdot(\vec{c}-\vec{b})}{2} = 0$$

$$\frac{\cancel{b^2} - a^2 + c^2 - \cancel{b^2}}{2} = 0$$

$$\frac{c^2 - a^2}{2} = 0 \Rightarrow \frac{(\vec{c}-\vec{a})\cdot(\vec{c}+\vec{a})}{2} = 0$$

$$\left(\frac{\vec{a}+\vec{c}}{2}\right)\cdot(\vec{c}-\vec{a}) = 0$$

$\Rightarrow \vec{OR} \perp \vec{AC}$

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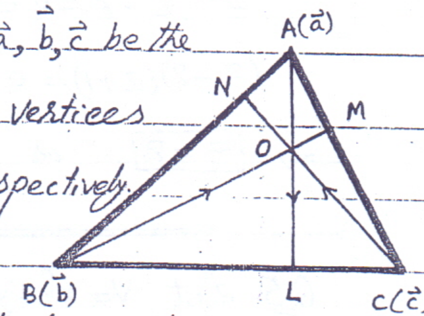
Thus perpendicular bisectors are Concurrent.

Q.9 Let $\vec{a}, \vec{b}, \vec{c}$ be the P.V's of the vertices of $\triangle ABC$ respectively. Let $\vec{AL}, \vec{BM}, \vec{CN}$ are altitudes and let 'O' be the origion and point of intersection of AL and BM such that



$\vec{AL} \perp \vec{BC} \Rightarrow \vec{AO} \perp \vec{BC}$ also $\vec{AB} = \vec{b} - \vec{a}$

and $\vec{BM} \perp \vec{CA} \Rightarrow \vec{BO} \perp \vec{CA}$ $\vec{CA} = \vec{a} - \vec{c}$

$\vec{BC} = \vec{c} - \vec{b}$

where $\vec{OA} = \vec{a}$ and $\vec{OB} = \vec{b}$, $\vec{OC} = \vec{c}$

To prove $\vec{AL}, \vec{BM}, \vec{CN}$ are Concurrent we have to show $\vec{CN} \perp \vec{AB}$

$\vec{OA} \perp \vec{BC} \Rightarrow \vec{a}\cdot(\vec{c}-\vec{b}) = 0$ — ①

$\vec{OB} \perp \vec{CA} \Rightarrow \vec{b}\cdot(\vec{a}-\vec{c}) = 0$ — ②

Adding ① and ②

$$\vec{a}\cdot(\vec{c}-\vec{b}) + \vec{b}\cdot(\vec{a}-\vec{c}) = 0$$

$$\vec{a}\cdot\vec{c} - \cancel{\vec{a}\cdot\vec{b}} - \vec{b}\cdot\vec{c} + \cancel{\vec{a}\cdot\vec{b}} = 0$$

$$\vec{a}\cdot\vec{c} - \vec{b}\cdot\vec{c} = 0 \Rightarrow (\vec{a}-\vec{b})\cdot\vec{c} = 0$$

where $\vec{c} = \vec{OC}$ $\vec{a} - \vec{b} = -\vec{AB} = \vec{BA}$

$\Rightarrow -\vec{AB} \perp \vec{OC} \Rightarrow \vec{AB} \perp \vec{OC}$

So $\vec{CN} \perp \vec{AB}$

Thef altitudes of $\triangle$ are Concurrent.

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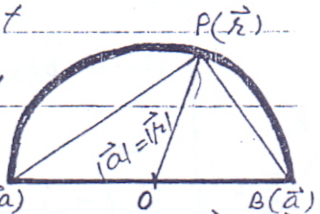
Q.10 Let "O" be origion as well as centre of semi circle.

To prove that angle inscribed in semicircle is right, we have to show $\vec{AP} \perp \vec{BP}$



Firstly $\vec{AP} = (\text{P.V. of } P) - (\text{P.V. of } A)$

$\vec{AP} = \vec{r} + \vec{a}$ ——— (1)

Now $\vec{BP} = (\text{P.V. of } P) - (\text{P.V. of } B)$

$\vec{BP} = \vec{r} - \vec{a}$ ——— (2)

Consider $\vec{AP} \cdot \vec{BP} = (\vec{r}+\vec{a}) \cdot (\vec{r}-\vec{a})$

$\vec{AP} \cdot \vec{BP} = |\vec{r}|^2 - |\vec{a}|^2$

But $|\vec{r}| = |\vec{a}|$ being radius of circle.

$\vec{AP} \cdot \vec{BP} = |\vec{a}|^2 - |\vec{a}|^2 = 0$

$\vec{AP} \cdot \vec{BP} = 0 \Rightarrow \vec{AP} \perp \vec{BP}$

Thus angle inscribed in semi-circle is right.

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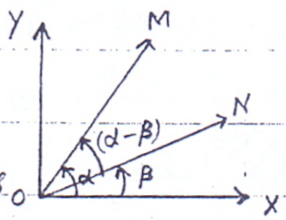
Q.11 Show $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

Proof:-



Let $\vec{OM}$ and $\vec{ON}$ be the unit vectors in XY plane with angles α, β to horizontal (X-axis) respectively

$m\angle MON = \alpha - \beta$

$\vec{OM} = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$ $\quad \vec{ON} = \cos\beta\,\hat{i} + \sin\beta\,\hat{j}$

Also $|\vec{OM}| = |\vec{ON}| = 1$

Consider $\vec{OM} \cdot \vec{ON} = |\vec{OM}|\,|\vec{ON}|\cos(\alpha-\beta)$

$(1)(1)\cos(\alpha-\beta) = (\cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}) \cdot (\cos\beta\,\hat{i} + \sin\beta\,\hat{j})$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

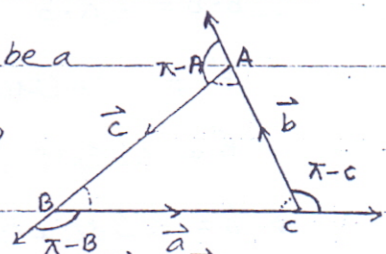
Replacing β by $-\beta$, we have

$\cos(\alpha+\beta) = \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta)$

But $\cos(-\beta) = \cos\beta$, $\sin(-\beta) = -\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ (Proved)

Q.12 (iii) Let ABC be a triangle having sides



$\vec{AB} = \vec{c}$, $\vec{BC} = \vec{a}$, $\vec{CA} = \vec{b}$

Using triangle Law of Vector addition

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\Rightarrow \vec{b} = -(\vec{a} + \vec{c})$

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Squaring both sides, we have

$(\vec{b})^2 = \{-(\vec{a}+\vec{c})\}^2 = (\vec{a}+\vec{c})^2$

$(\because A^2 = \vec{A} \cdot \vec{A})$

$b^2 = (\vec{a}+\vec{c}) \cdot (\vec{a}+\vec{c})$

$b^2 = \vec{a}\cdot\vec{a} + 2\vec{a}\cdot\vec{c} + \vec{c}\cdot\vec{c}$

$b^2 = a^2 + c^2 + 2ac\cos(\pi - B)$

$b^2 = a^2 + c^2 - 2ac\cos B$ $\quad (\because \cos(\pi-\theta) = -\cos\theta)$

$b^2 = a^2 + c^2 - 2ac\cos B$ (Proved)

(Similarly (iv) Part.)

* This Law is known as COSINE Law.