Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

TAHIR TAHIR

Exercise: 7.2

Q.1: Find requireds if $A(2,5)$, $B(-1,1)$, $C(2,-6)$:

(i) $\overrightarrow{AB} = ?$

$\overrightarrow{AB} = (-1-2)\hat{i} + (1-5)\hat{j}$

$\overrightarrow{AB} = -3\hat{i} - 4\hat{j}$

Thus $\overrightarrow{AB} = -3\hat{i} - 4\hat{j}$ Ans.

(ii) $2\overrightarrow{AB} - \overrightarrow{CB} = ?$

$= 2\{(-1-2)\hat{i} + (1-5)\hat{j}\} - \{(-1-2)\hat{i} + (1+6)\hat{j}\}$

$= 2[-3\hat{i} - 4\hat{j}] - [-3\hat{i} + 7\hat{j}]$

$= -6\hat{i} - 8\hat{j} + 3\hat{i} - 7\hat{j} = -3\hat{i} - 15\hat{j}$ Ans.

(iii) $2\overrightarrow{CB} - 2\overrightarrow{CA} = ?$

$= 2\{(-1-2)\hat{i} + (1+6)\hat{j}\} - 2\{(2-2)\hat{i} + (5+6)\hat{j}\}$

$= 2\{-3\hat{i} + 7\hat{j}\} - 2\{0\hat{i} + 11\hat{j}\} = -6\hat{i} + 14\hat{j} - 22\hat{j}$

$2\overrightarrow{CB} - 2\overrightarrow{CA} = -6\hat{i} - 8\hat{j}$ Ans.

Q.2: $\vec{u} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{v} = 3\hat{i} - 2\hat{j} + 2\hat{k}$, $\vec{w} = 5\hat{i} - \hat{j} + 3\hat{k}$

(i) $\vec{u} + 2\vec{v} + \vec{w} = (\hat{i} + 2\hat{j} - \hat{k}) + 2(3\hat{i} - 2\hat{j} + 2\hat{k}) + (5\hat{i} - \hat{j} + 3\hat{k})$

$\vec{u} + 2\vec{v} + \vec{w} = i + 2j - \hat{k} + 6\hat{i} - 4\hat{j} + 4\hat{k} + 5\hat{i} - \hat{j} + 3\hat{k}$

$\vec{u} + 2\vec{v} + \vec{w} = 12\hat{i} - 3\hat{j} + 6\hat{k}$. Ans.

(ii) $\vec{v} - 3\vec{w} = (3\hat{i} - 2\hat{j} + 2\hat{k}) - 3(5\hat{i} - \hat{j} + 3\hat{k})$

$\vec{v} - 3\vec{w} = 3\hat{i} - 2\hat{j} + 2\hat{k} - 15\hat{i} + 3\hat{j} - 9\hat{k}$

$\vec{v} - 3\vec{w} = -12\hat{i} + \hat{j} - 7\hat{k}$ Ans.

(iii) $|3\vec{v} + \vec{w}| = |3(3\hat{i} - 2\hat{j} + 2\hat{k}) + (5\hat{i} - \hat{j} + 3\hat{k})|$

$|3\vec{v} + \vec{w}| = |9\hat{i} - 6\hat{j} + 6\hat{k} + 5\hat{i} - \hat{j} + 3\hat{k}|$

$|3\vec{v} + \vec{w}| = |14\hat{i} - 7\hat{j} + 9\hat{k}| = \sqrt{(14)^2 + (-7)^2 + (9)^2}$

$|3\vec{v} + \vec{w}| = \sqrt{196 + 49 + 81} = \sqrt{326}$

Thus $|3\vec{v} + \vec{w}| = \sqrt{326}$ Ans.

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

Contact No: 03456510779

Q.3: Find the magnitudes and direction Cosines of $\vec{v}$:

(i) $\vec{v} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$|\vec{v}| = \sqrt{(2)^2 + (3)^2 + (4)^2}$

$|\vec{v}| = \sqrt{4+9+16} = \sqrt{29}$

Direction Cosines are

$\cos\alpha = \frac{v_1}{|\vec{v}|}$, $\cos\beta = \frac{v_2}{|\vec{v}|}$, $\cos\gamma = \frac{v_3}{|\vec{v}|}$

$\cos\alpha = \frac{2}{\sqrt{29}}$, $\cos\beta = \frac{3}{\sqrt{29}}$, $\cos\gamma = \frac{4}{\sqrt{29}}$

(ii) $\vec{v} = \hat{i} - \hat{j} - \hat{k}$

$|\vec{v}| = \sqrt{(1)^2 + (-1)^2 + (-1)^2} = \sqrt{1+1+1}$

$|\vec{v}| = \sqrt{3}$

Direction Cosines are

$\cos\alpha = \frac{v_1}{|v|}$, $\cos\beta = \frac{v_2}{|v|}$, $\cos\gamma = \frac{v_3}{|v|}$

$\cos\alpha = \frac{1}{\sqrt{3}}$, $\cos\beta = \frac{-1}{\sqrt{3}}$, $\cos\gamma = \frac{-1}{\sqrt{3}}$

(iii) $\vec{v} = 4\hat{i} - 5\hat{j} + 0\hat{k}$

$|\vec{v}| = \sqrt{(4)^2 + (-5)^2 + (0)^2} = \sqrt{16+25+0} = \sqrt{41}$

Thus direction Cosines are

$\cos\alpha = \frac{v_1}{|\vec{v}|}$, $\cos\beta = \frac{v_2}{|\vec{v}|}$, $\cos\gamma = \frac{v_3}{|\vec{v}|}$

$\cos\alpha = \frac{4}{\sqrt{41}}$, $\cos\beta = \frac{-5}{\sqrt{41}}$, $\cos\gamma = \frac{0}{\sqrt{41}} = 0$

Imp.

Q.4: Find $\alpha = ?$ where $|\alpha\hat{i} + (\alpha+1)\hat{j} + 2k| = 3$

Soln:- $\sqrt{(\alpha)^2 + (\alpha+1)^2 + (2)^2} = 3$

$\sqrt{\alpha^2 + (\alpha^2 + 2\alpha + 1) + 4} = 3 \Rightarrow \sqrt{2\alpha^2 + 2\alpha + 5} = 3$

Squaring both sides $2\alpha^2 + 2\alpha + 5 = 9$

$\Rightarrow 2\alpha^2 + 2\alpha - 9 + 5 = 0 \Rightarrow 2\alpha^2 + 2\alpha - 4 = 0$

$2 \neq 0$ $\alpha^2 + \alpha - 2 = 0 \Rightarrow \alpha^2 + 2\alpha - \alpha - 2 = 0$

$\Rightarrow \alpha(\alpha+2) - 1(\alpha+2) = 0 \Rightarrow (\alpha+2)(\alpha-1) = 0$

$\alpha + 2 = 0$ or $\alpha - 1 = 0$

$\alpha = -2$ or $\alpha = 1$ Ans.

Tahir Mahmood
M.Sc. (Math)
Mob No: 0346-6510779

Q.5 Find a unit vector

$\vec{V} = \hat{i} + 2\hat{j} - \hat{k}$

$|\vec{V}| = \sqrt{(1)^2 + (2)^2 + (-1)^2}$

$|\vec{V}| = \sqrt{1+4+1} = \sqrt{6}$

The unit vector in the direction of $\vec{V}$

$\hat{V} = \dfrac{\vec{V}}{|\vec{V}|}$

$\hat{V} = \dfrac{1}{\sqrt{6}}(\hat{i} + 2\hat{j} - \hat{k})$ Ans.

Q.6: $\vec{a} = 3\hat{i} - \hat{j} - 4\hat{k}$, $\vec{b} = -2\hat{i} - 4\hat{j} - 3\hat{k}$

$\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

let $\vec{d} = 3\vec{a} - 2\vec{b} + 4\vec{c}$

$\vec{d} = 3(3\hat{i} - \hat{j} - 4\hat{k}) - 2(-2\hat{i} - 4\hat{j} - 3\hat{k}) + 4(\hat{i} + 2\hat{j} - \hat{k})$

$\vec{d} = 9\hat{i} - 3\hat{j} - 12\hat{k} + 4\hat{i} + 8\hat{j} + 6\hat{k} + 4\hat{i} + 8\hat{j} - 4\hat{k}$

$\vec{d} = 17\hat{i} + 13\hat{j} - 10\hat{k}$

$|\vec{d}| = \sqrt{(17)^2 + (13)^2 + (-10)^2}$

$= \sqrt{289 + 169 + 100} = \sqrt{558}$

Thus unit vector $\parallel$ to $\vec{d}$ is

$\hat{d} = \dfrac{\vec{d}}{|\vec{d}|}$

$\hat{d} = \dfrac{1}{\sqrt{558}}(17\hat{i} + 13\hat{j} - 10\hat{k})$

$= \dfrac{17}{\sqrt{558}}\hat{i} + \dfrac{13}{\sqrt{558}}\hat{j} - \dfrac{10}{\sqrt{558}}\hat{k}$ Ans.

Q.7: Find a vector if

(i) Magnitude $= 4$ parallel to $2\hat{i} - 3\hat{j} + 6\hat{k}$

Let $\vec{A}$ be the required vector

so $|\vec{A}| = 4$ let $\vec{u} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$\hat{A} = \hat{u} = \dfrac{\vec{u}}{|\vec{u}|}$

$|\vec{u}| = \sqrt{(2)^2 + (-3)^2 + (6)^2}$

$|\vec{u}| = \sqrt{4+9+36} = \sqrt{49} = 7$

$\hat{A} = \dfrac{1}{7}(2\hat{i} - 3\hat{j} + 6\hat{k})$

Thus $\vec{A} = |A|\ \hat{A}$

$\vec{A} = \dfrac{4}{7}(2\hat{i} - 3\hat{j} + 6\hat{k})$

$\vec{A} = \dfrac{8}{7}\hat{i} - \dfrac{12}{7}\hat{j} + \dfrac{24}{7}\hat{k}$

Similarly do (ii) Part yourself.

Q.8: $\vec{u} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{v} = -\hat{i} + 3\hat{j} - \hat{k}$

$\vec{w} = \hat{i} + 6\hat{j} + z\hat{k}$ $z = ?$

using Law of triangle for vectors.

$\vec{u} + \vec{v} = \vec{w}$

$(2\hat{i} + 3\hat{j} + 4\hat{k}) + (-\hat{i} + 3\hat{j} + (-\hat{k})) = \hat{i} + 6\hat{j} + z\hat{k}$

$\hat{i} + 6\hat{j} + 3\hat{k} = \hat{i} + 6\hat{j} + z\hat{k}$

Comparing the Coefficients of $\hat{k}$

$\boxed{z = 3}$

NOTE: Parallel vectors always have same unit vectors.

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

Let O be fixed point w.r.t which P.V. taken

Q.9:- $\vec{OA} = 2\hat{i} - \hat{j} + \hat{k}$ $\quad \vec{OB} = 3\hat{i} + \hat{j} + 0\hat{k}$

$\vec{OC} = 2\hat{i} + 4\hat{j} - 2\hat{k}$ $\quad \vec{OD} = -\hat{i} - 2\hat{j} + \hat{k}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$\vec{AB} = (3\hat{i} + \hat{j} + 0\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$

$\vec{AB} = (3-2)\hat{i} + (1+1)\hat{j} + (0-1)\hat{k}$

$\vec{AB} = \hat{i} + 2\hat{j} - \hat{k}$ ——— (1)

$\vec{CD} = \vec{OD} - \vec{OC}$

$\vec{CD} = (-\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + 4\hat{j} - 2\hat{k})$

$\vec{CD} = (-1-2)\hat{i} + (-2-4)\hat{j} + (1+2)\hat{k}$

$\vec{CD} = -3\hat{i} - 6\hat{j} + 3\hat{k}$

$\vec{CD} = -3(\hat{i} + 2\hat{j} - \hat{k})$

$\vec{CD} = -3\ \vec{AB}$

$\because$ $\vec{CD}$ is scalar multiple of $\vec{AB}$

so $\vec{AB}$ is parallel to $\vec{CD}$.

TAHIR
M.Sc.(Math)

Q.10 (a) Let $\pm\vec{A}$ be the required vectors

$|A| = 2 \qquad \hat{A} = \pm\ \hat{v}$

Firstly $\vec{v} = 2\hat{i} - 4\hat{j} + 4\hat{k}$

$|\vec{v}| = \sqrt{(2)^2 + (-4)^2 + (4)^2}$

$|\vec{v}| = \sqrt{4+16+16} = \sqrt{36} = 6$

Thus $\hat{v} = \dfrac{\vec{v}}{|v|} = \dfrac{1}{6}[2\hat{i} - 4\hat{j} + 4\hat{k}]$

$\hat{v} = \frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

(The vector parallel to $\vec{v}$ in the direction of $\vec{v}$ is $\vec{A}$

$\vec{A} = |A|(\hat{v})$

$\vec{A} = 2\left[\frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}\right]$

$\vec{A} = \frac{2}{3}\hat{i} - \frac{4}{3}\hat{j} + \frac{4}{3}\hat{k}$ ——— (1)

(The vector parallel to $\vec{v}$ in the opposite direction of $\vec{v}$ is $-\vec{A}$

$-\vec{A} = |A|(-\hat{v})$

$-\vec{A} = 2\left[\frac{-1}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}\right]$

$-\vec{A} = -\frac{2}{3}\hat{i} + \frac{4}{3}\hat{j} - \frac{4}{3}\hat{k}$ ——— (2)

(1) and (2) represents required vectors.

(b) $a = ?$ $\qquad \vec{w} \parallel \vec{v}$

$\because$ $\vec{v}$ and $\vec{w}$ are parallel so $\hat{v} = \hat{w}$

$$\frac{\vec{v}}{|v|} = \frac{\vec{w}}{|w|}$$

$$\frac{\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{1+9+16}} = \frac{a\hat{i} + 9\hat{j} - 12\hat{k}}{\sqrt{a^2+81+144}}$$

$$\frac{\hat{i}}{\sqrt{26}} - \frac{3}{\sqrt{26}}\hat{j} + \frac{4}{\sqrt{26}}\hat{k} = \frac{a\hat{i} + 9\hat{j} - 12\hat{k}}{\sqrt{a^2+225}}$$

Comparing the Coefficients of $\hat{i}$ (may $\hat{j}$ or $\hat{k}$)

$$\frac{1}{\sqrt{26}} = \frac{a}{\sqrt{a^2+225}}$$

Squaring both sides

$$\frac{1}{26} = \frac{a^2}{a^2+225} \Rightarrow a^2 + 225 = 26a^2$$

$$25a^2 = 225 \Rightarrow a^2 = 9$$

$$\boxed{a = \pm 3}$$

(c) Hint:- Take $-$ve of unit vector of $\vec{v}$ and use $\vec{A} = |A|(-\hat{v})$

Do yourself.

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

TAHIR

(d) $a = ?\ b = ?$

where $3\hat{i}-\hat{j}+4\hat{k} \parallel a\hat{i}+b\hat{j}-2\hat{k}$

let $\vec{u}=3\hat{i}-\hat{j}+4\hat{k}$ $\quad \vec{v}=a\hat{i}+b\hat{j}-2\hat{k}$

$\because \vec{u}\parallel\vec{v}$ so $\hat{u}=\hat{v}$

$$\frac{3\hat{i}-\hat{j}+4\hat{k}}{\sqrt{9+1+16}}=\frac{a\hat{i}+b\hat{j}-2\hat{k}}{\sqrt{a^2+b^2+4}}$$

$$\frac{3\hat{i}-\hat{j}+4\hat{k}}{\sqrt{26}}=\frac{a\hat{i}+b\hat{j}-2\hat{k}}{\sqrt{a^2+b^2+4}}$$

Comparing the Coefficients of $\hat{j}$ and $\hat{k}$

$$\frac{-1}{\sqrt{26}}=\frac{b}{\sqrt{a^2+b^2+4}} \quad \& \quad \frac{4}{\sqrt{26}}=\frac{-2}{\sqrt{a^2+b^2+4}}$$

Squaring above equations.

$$\frac{1}{26}=\frac{b^2}{a^2+b^2+4} \quad \& \quad \frac{16}{26}=\frac{4}{a^2+b^2+4}$$

$a^2+b^2+4=26b^2 \quad \& \quad 2a^2+2b^2+8=13$

$a^2-25b^2+4=0$ —① $\quad 2a^2+2b^2-5=0$ —②

① $\Rightarrow a^2=25b^2-4$

② $\Rightarrow 2(25b^2-4)+2b^2-5=0$

$50b^2-8+2b^2-5=0$

$52b^2-13=0 \Rightarrow 52b^2=13$

$4b^2=1 \Rightarrow \boxed{b=\pm\frac{1}{2}}$ Ans.

Now $a^2 = 25(1/4)-4$

$a^2 = \frac{25-16}{4} = \frac{9}{4}$

$\boxed{a=\pm\frac{3}{2}}$ Ans.

Q.11 (i), (ii) Do yourself Similar to Q.3

(iii) $P(2,1,5)$ $\quad Q(1,3,1)$

$\overrightarrow{PQ}=(1-2)\hat{i}+(3-1)\hat{j}+(1-5)\hat{k}$

let $\vec{u}=\overrightarrow{PQ}=-\hat{i}+2\hat{j}-4\hat{k}$

$|\vec{u}|=\sqrt{1+4+16}=\sqrt{21}$

The direction Cosines are

$$\cos\alpha=\frac{u_1}{|u|},\ \cos\beta=\frac{u_2}{|u|},\ \cos\gamma=\frac{u_3}{|u|}$$

$$\cos\alpha=\frac{-1}{\sqrt{21}},\ \cos\beta=\frac{2}{\sqrt{21}},\ \cos\gamma=\frac{-4}{\sqrt{21}}$$

Q.12 (i) Let $(\alpha,\beta,\gamma)=(45°,45°,60°)$

Consider $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=l^2+m^2+n^2$

where $l=\cos\alpha$ $\quad m=\cos\beta$ $\quad n=\cos\gamma$

$l^2+m^2+n^2=(\cos 45°)^2+(\cos 45°)^2+(\cos 60°)^2$

$l^2+m^2+n^2=\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{2}\right)^2$

$l^2+m^2+n^2=\frac{1}{2}+\frac{1}{2}+\frac{1}{4}=\frac{2+2+1}{4}$

$l^2+m^2+n^2=5/4\neq 1$

Thus (α,β,γ) are not direction angles.

(iii) Let $(\alpha,\beta,\gamma)=(45°,60°,60°)$

Consider $l^2+m^2+n^2=\cos^2\alpha+\cos^2\beta+\cos^2\gamma$

$l^2+m^2+n^2=(\cos 45°)^2+(\cos 60°)^2+(\cos 60°)^2$

$l^2+m^2+n^2=\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2$

$l^2+m^2+n^2=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}=\frac{2+1+1}{4}$

$l^2+m^2+n^2=4/4=1$

$l^2+m^2+n^2=1$

Thus (α,β,γ) are direction angles.

Similarly do (ii) part.

TAHIR M.Sc. (Math)

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779