Permutation:-

"An arrangement of a finite number of objects some or all at a time is called Permutation."

If n are the total objects and r objects are taken at a time, then permutation is written as

$${}^nP_r = \frac{n!}{(n-r)!} \quad \text{where } r \leq n$$

* ${}^nP_n = \frac{n!}{(n-n)!} = n!$ * ${}^nP_0 = \frac{n!}{(n-0)!} = 1$

Fundamental Principle of Counting:

Suppose A and B are two events. A occurs in p different ways and B occurs in q different ways. The number of ways of occurance of the two events are the product $p.q$

*This principle can be extended for more than two events.

Exercise: 7.2

Q.1 Evaluate the followings

(i) ${}^{20}P_3 = \frac{20!}{(20-3)!} = \frac{20!}{17!}$

${}^{20}P_3 = \frac{20.19.18.17!}{17!} = 20.19.18$

${}^{20}P_3 = 6840$ Ans.

(ii) ${}^{16}P_4 = \frac{16!}{(16-4)!}$

${}^{16}P_4 = \frac{16!}{12!} = \frac{16.15.14.13.12!}{12!}$

${}^{16}P_4 = 43680$ Ans.

(iii) ${}^{12}P_5 = \frac{12!}{(12-5)!} = \frac{12!}{7!}$

${}^{12}P_5 = \frac{12.11.10.9.8.7!}{7!}$

${}^{12}P_5 = 95040$ Ans

(iv) ${}^{10}P_7 = \frac{10!}{(10-7)!} = \frac{10!}{3!}$

${}^{10}P_7 = \frac{10.9.8.7.6.5.4.3!}{3!}$

${}^{10}P_7 = 604800$ Ans.

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(v) ${}^9P_8 = \frac{9!}{(9-8)!} = \frac{9!}{1!}$

${}^9P_8 = 9.8.7.6.5.4.3.2$

${}^9P_8 = 362880$ Ans.

Q.2 Find the value of n when:

(i) ${}^nP_2 = 30$

$\frac{n!}{(n-2)!} = 30$

$\frac{n(n-1)(n-2)!}{(n-2)!} = 30$

$n^2 - n - 30 = 0$

$n^2 - 6n + 5n - 30 = 0$

$(n-6)(n+5) = 0$

$n-6=0 \quad \vee \quad n+5=0$

$n = 6, \quad n = -5$ (-ve so neglegible)

$\Rightarrow n = 6$ Ans.

(ii) ${}^{11}P_n = 11.10.9$

$\frac{11!}{(11-n)!} = 11.10.9$

$\frac{11.10.9.8!}{(11-n)!} = 11.10.9$

$\frac{8!}{(11-n)!} = 1$

$8! = (11-n)!$

$8 = 11-n \Rightarrow n = 11-8$

$n = 3$ Ans.

(iii) ${}^nP_4 : {}^{n-1}P_3 = 9 : 1$

$\frac{{}^nP_4}{{}^{n-1}P_3} = \frac{9}{1}$

$\frac{\frac{n!}{(n-4)!}}{\frac{(n-1)!}{(n-1-3)!}} = \frac{9}{1}$

$\frac{n!\,(n-4)!}{(n-1)!\,(n-4)!} = \frac{9}{1}$

$\frac{n(n-1)!}{(n-1)!} = \frac{9}{1}$

$n = 9$ Ans.

Q.3 Prove from first Principle:

(i) ${}^nP_r = n\,{}^{n-1}P_{r-1}$

RHS $= n \cdot {}^{n-1}P_{r-1}$

$= n \cdot \frac{(n-1)!}{(n-1-r+1)!}$

$= \frac{n(n-1)!}{(n-r)!}$

$= \frac{n!}{(n-r)!}$

$= {}^nP_r$ (Proved)

Imp

(ii) ${}^nP_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$

RHS $= {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$

$= \frac{(n-1)!}{(n-1-r)!} + \frac{r\,(n-1)!}{(n-1-r+1)!}$

$= \frac{(n-1)!}{(n-r-1)!} + \frac{r(n-1)!}{(n-r)!}$

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$$= \frac{(n-1)!}{(n-r-1)!} + \frac{r\,(n-1)!}{(n-r)(n-r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!}\left\{1 + \frac{r}{n-r}\right\}$$

$$= \frac{(n-1)!}{(n-r-1)!}\left\{\frac{n-r+r}{n-r}\right\} \Rightarrow \frac{(n-1)!}{(n-r-1)!}\,\frac{n}{(n-r)}$$

$$= \frac{n!}{(n-r)!} \quad \because n(n-1)! = n!$$

$$= {}^{n}P_{r} \qquad \text{(Proved)}$$

Q.4 No. of flags = 5

flags used at a time = 3

Number of Signals = ${}^{5}P_{3} = \frac{5!}{(5-3)!}$

$= \frac{5\cdot4\cdot3\cdot2!}{2!} = 5\cdot4\cdot3 = 60$ Ans.

Q.5 Total number of flags = 6

Number of Signals when 1 flag used = ${}^{6}P_{1} = 6$

Number of Signals when 2 flags used = ${}^{6}P_{2} = 30$

Number of Signals when 3 flags used = ${}^{6}P_{3} = 120$

Number of Signals when 4 flags used = ${}^{6}P_{4} = 360$

Number of Signals when 5 flags used = ${}^{6}P_{5} = 720$

Number of Signals when 6 flags used = ${}^{6}P_{6} = 720$

Total no. of Signals = $6+30+120+360+720+720$

$= 1956$ Ans.

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Q.6 How many words can be formed using all the letters

(i) PLANE

No. of letters = 5

No. of words using all letters = ${}^5P_5 = \frac{5!}{(5-5)!}$

$= \frac{5.4.3.2.1}{1} = 120$ Ans.

(ii) OBJECT

No. of letters = 6

No. of words using all letters = ${}^6P_6 = \frac{6!}{(6-6)!}$

$= \frac{6.5.4.3.2.1}{1} = 720$ Ans.

(iii) FASTING

No. of letters = 7

No. of words using all the letters = ${}^7P_7 = \frac{7!}{(7-7)!}$

$= \frac{7.6.5.4.3.2.1}{1} = 5040$ Ans.

Q.7 How many 3-digit numbers can be form using 2, 3, 5, 7, 9 once?

No. of digits = 5

Digits forming each number = 3

3-digit numbers = ${}^5P_3 = \frac{5!}{(5-3)!}$

$= \frac{5.4.3.2!}{2!} = 5.4.3$

$= 60$ Ans.

Q.8 Imp. To find numbers greater than 23000 using 1,2,3,4,5 once.

The numbers greater than 23000 are form when

$23\otimes\otimes\otimes = {}^3P_3 = 6$

$24\otimes\otimes\otimes = {}^3P_3 = 6$

$25\otimes\otimes\otimes = {}^3P_3 = 6$

$3\otimes\otimes\otimes\otimes = {}^4P_4 = 24$

$4\otimes\otimes\otimes\otimes = {}^4P_4 = 24$

$5\otimes\otimes\otimes\otimes = {}^4P_4 = 24$

Total nos. greater than 23000 = 24+24+24+6+6+6 = 90 Ans.

Q.9 Imp. To find 5-digit numbers using 1,2,4,6,8 once.

Total 5 digit numbers = ${}^5P_5 = 120$

(i) The numbers when 2 and 8 are next together

Permutation when 28 together = ${}^4P_4 = 24$

Permutation when 82 together = ${}^4P_4 = 24$

Total numbers containing 2 and 8 together = 24+24 = 48 Ans.

(ii) The numbers when 2 and 8 are not next together.

Numbers with out 2,8 together = 120 − 48 = 72 Ans.

Q.10 To find 6-digit numbers using 0,1,2,3,4,5 once.

The 6-digit numbers will be form if 0 is not at the extreme left position so five posibilities are available out of 6

Thus total number of 6-digits = $5 \cdot {}^5P_5 = \frac{5 \cdot 5!}{0!}$

$= 5 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 600$ Ans.

The numbers containing 0 at tens place $\otimes\otimes\otimes\otimes 0 \otimes = {}^5P_5$

TAHIR $= \frac{5!}{0!} = 120$ numbers. Ans.

Q.11 The 5-digit numbers using 2,3,5,7,9 once

Total 5-digit numbers = ${}^5P_5 = 120$

The numbers will be multiple of 5 if the number extreme right will be 5 so 4 digits are left and 1 is fixed

The 5 digit numbers multiple of 5 = ${}^4P_4 = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1} = 24$ Ans.

Q.12 Total number of books = 8

Let E_1 and E_2 be the two English books.

Total ways of arranging 8 books = ${}^8P_8 = 40320$

Permutation containing E_1E_2 together = ${}^7P_7 = 5040$

Permutation Containing E_2E_1 together = ${}^7P_7 = 5040$

Total no. of ways containing E_1, E_2 together = $5040 + 5040 = 10080$

The ways without E_1, E_2 together = $40320 - 10080 = 30240$ Ans.

Q.13 Total books = 8

Let E denotes English books and U denoted urdu books

Permutation containing EEEUUUUU = ${}^3P_3 \cdot {}^5P_5 = 6 \times 120 = 720$

Permutation Containing UUUUUEEE = ${}^5P_5 \cdot {}^3P_3 = 120 \times 6 = 720$

Total arrangements containing books of same subject together = $720 + 720 = 1440$ Ans.

Q.14 Total no. of persons = 9

Permutation Containing the ways of 5 boys = ${}^5P_5 = 120$

Permutation Containing the ways of 4 girls = ${}^4P_4 = 24$

Permutation Containing all the boys and girls = ${}^5P_5 \cdot {}^4P_4$

$= 120 \times 24 = 2880$ Ans.