Chapter: 7

VECTORS

F.Sc: F.A: Part: II

TAHIR
M.Sc. (MATH)

Contact No: 0345 6510779

VECTORS and SCALARS:-

"A physical quantity which is measured by means of magnitude, unit and direction is called Vector."

For example Force, Weight, torque, velocity, acceleration etc.

"A physical quantity which is measured by means of magntitude and unit is called Scalar."

For example work, length, Area, Volume, Speed etc.

Some Important Remarks on Vectors:-

(i) ⟹ The part of a vector in a particular direction is called its called its Component.

* A plane vector has two Components: $\vec{A} = A_1\hat{i} + A_2\hat{j}$
* A space vector has three Components: $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along x, y and z axis and form orthogonal unit vector system.

* A_1, A_2, A_3 are called direction ratios of vector $\vec{A}$.

(ii) ⟹ The vector whose magnitude (Norm, length or modulus) is zero is called Null or Zero vector having arbitrary direction and is denoted by $\vec{O}$. It is also called addative identity vector.

i.e. $\vec{A} + \vec{O} = \vec{O} + \vec{A} = \vec{A}$

(iii) ⟹ The vector use to denote the position of a vector is called position vector and usually denoted by $\vec{r}$ where

$$\vec{r} = \vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} = \vec{AP} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

where $\vec{OP}$ is position vector w.r.t. origion and $\vec{AP}$ w.r.t. A point.

* Components of a vector are scalar numbers.

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

(iv) ⇒ "The length of a vector is called its magnitude, norm or modulus." If $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ then magnitude of A is denoted and defined as $|\vec{A}| = \sqrt{(A_1)^2 + (A_2)^2 + (A_3)^2}$ where $|\vec{A}|$ is always positive and is greater than each component of the vector.

(v) ⇒ The vector whose magnitude is unity and use to denoted the direction of vector is called unit vector and is denoted as (for $\vec{A}$) $\hat{A}$ (A cap "read")

where $\hat{A} = \vec{A}/|\vec{A}|$

(vi) ⇒ The cosine angles that components of vector make with each axis are called direction Cosines and usually denoted as $l = \cos\alpha = \frac{A_1}{|A|}$, $m = \cos\beta = \frac{A_2}{|A|}$, $n = \cos\gamma = \frac{A_3}{|A|}$

For any vector where $0 \leq \alpha, \beta, \gamma \leq \pi$

$$l^2 + m^2 + n^2 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Challange: What is the difference b/w direction ratios and direction Cosines.

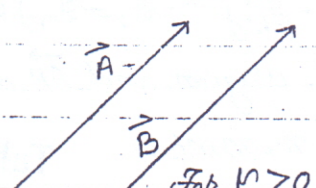
(vii) ⇒ Two (Coplaner) vectors are said to be parallel to each other if they are scalar multiple of each other.

i.e. $\vec{A}$ is parallel to $\vec{B}$ if $\vec{A} = \mu\vec{B}$ where $\mu \in \mathbb{R}$

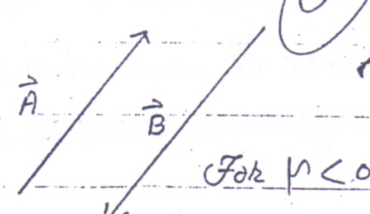
* Parallel vectors have same unit vectors.

* $\vec{A}$ and $\vec{B}$ have same directions if $\mu > 0$

* $\vec{A}$ and $\vec{B}$ have opposite directions if $\mu < 0$



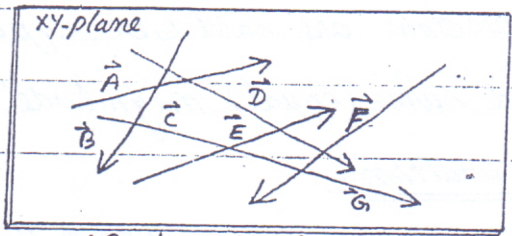
Parallel vectors



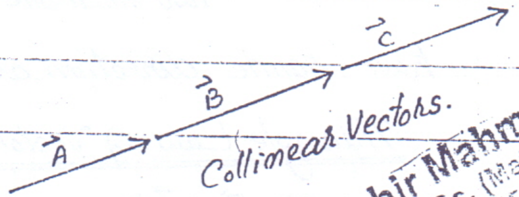
Anti-parallel vectors.

(viii) ⇒ The vectors lying in same plane are Coplaner vectors.

(ix) ⇒ The vectors lying on same line are called Colinear vectors.



(Coplaner Vectors)

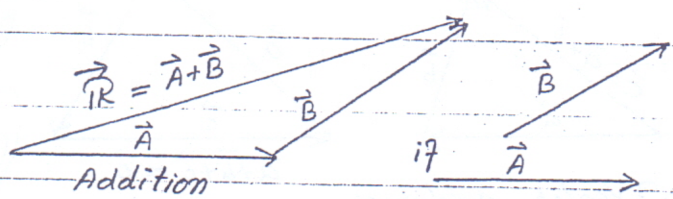


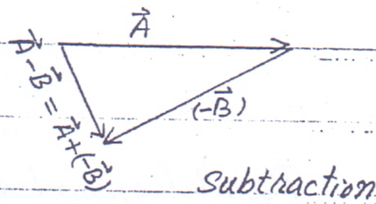
Addition and Subtraction of Vectors:-

"Two or more than two vector can be added and subtracted."

The subtraction is also addition by negative of vector.

The vector which gives resulting effect of two or more vectors is called Resultant Vector.





Properties of Vector Addition:

(i) $\vec{A}+\vec{B} = \vec{B}+\vec{A}$ (Commutative Law)

(iii) $\vec{A}+(\vec{B}+\vec{C}) = (\vec{A}+\vec{B})+\vec{C}$ (Associative Law)

(iii) $\vec{A}+\vec{0} = \vec{0}+\vec{A} = \vec{A}$ (Additive identity Law)

(iv) $\vec{A}+(-\vec{A}) = (-\vec{A})+\vec{A} = \vec{0}$ (Additive inverse Law)

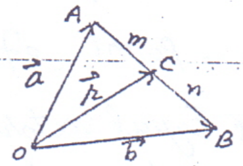
(v) $r(\vec{A}\pm\vec{B}) = r\vec{A}\pm r\vec{B}$ (Distribution of scalar multiplication over vector addition/Subtraction)

(vi) $\vec{A}(r\pm\lambda) = r\vec{A}\pm\lambda\vec{A}$ (Distribution of vector multiplication over scalar addition/Subtraction)

Ratio Formula:-

If $\vec{OA}$ and $\vec{OB}$ are the vectors and $\vec{OC}$ lies in the ratio $m:n$ then Position Vector of $\vec{OC} = \vec{r}$ is given by

$$\vec{r} = \frac{m\vec{b}+n\vec{a}}{m+n} \text{ if } \vec{OA} = \vec{a},\ \vec{OB} = \vec{b}.$$



Tahir Mahmood
M.Sc. Mathematics
Contact No:
0345 6510 779

TAHIR
TAHIR MEHMOOD
M.Sc. Math
0345-6510779

Equal Vectors:-

"Two or more vectors are said to be equal if they have same direction and same (equal) magnitude".

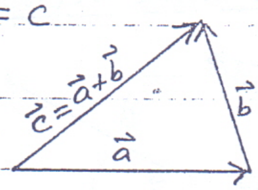
Triangle Law of Vector addition:-

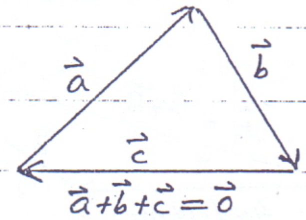
If $\vec{a}, \vec{b}, \vec{c}$ are sides of a triangle such that their order is same then

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

$$\vec{a}+\vec{b}+\vec{c}=\vec{0}$$

OR If $\vec{a}, \vec{b}, \vec{c}$ are sides of triangle such that terminal point of $\vec{a}$ coincide with initial point of $\vec{b}$ but not so with $\vec{c}$ then $\vec{a}+\vec{b}=\vec{c}$



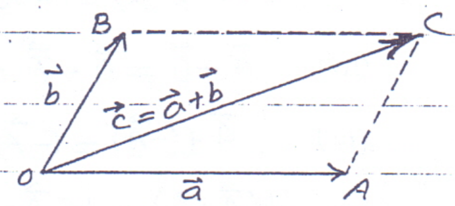


TAHIR
M.Sc. (Math)

Parallelogram Law of Vector addition:-

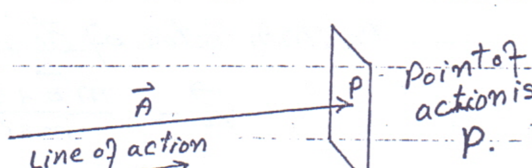
If $\vec{a}=\vec{OA}$ and $\vec{b}=\vec{OB}$ are adjecent sides of a parallelogram then sum of sides $\vec{a}$ and $\vec{b}$ is equal to the length vector of diagonal $\vec{OC}=\vec{c}$

$$\vec{OC}=\vec{OA}+\vec{OB} \Rightarrow \vec{a}+\vec{b}=\vec{c}$$



---

* The path along which a vector moves is called Line of action of the vector.
* The point at which at which a vector reacts is called point of action of the vector.
* Equal vectors are always parallel to each other.



Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

Exercise : 7·1

Q.1 Write the vector $\overrightarrow{PQ}$ in the form $x\hat{i}+y\hat{j}$:

(i) $P(2,3)$, $Q(6,-2)$

Let $\vec{r}=\overrightarrow{PQ}$ be P.Vector

$\vec{r}=\overrightarrow{PQ}=(6-2)\hat{i}+(-2-3)\hat{j}$

$\vec{r}=4\hat{i}-5\hat{j}$ Ans.

(ii) $P(0,5)$, $Q(-1,-6)$

let $\vec{r}=\overrightarrow{PQ}$ be position vector

$\vec{r}=\overrightarrow{PQ}=(-1-0)\hat{i}+(-6-5)\hat{j}$

$\vec{r}=-\hat{i}-11\hat{j}$ Ans.

Q.2 Find the magnitude of the vector $\vec{u}$:

(i) $\vec{u}=2\hat{i}-7\hat{j}$

$|\vec{u}|=\sqrt{(2)^2+(-7)^2}$

$|\vec{u}|=\sqrt{4+49}$

$|\vec{u}|=\sqrt{53}$ Ans.

(ii) $\vec{u}=\hat{i}+\hat{j}$

$|\vec{u}|=\sqrt{(1)^2+(1)^2}$

$|\vec{u}|=\sqrt{1+1}$

$|\vec{u}|=\sqrt{2}$ Ans.

(iii) $\vec{u}=3\hat{i}-4\hat{j}$

$|\vec{u}|=\sqrt{(3)^2+(-4)^2}$

$|\vec{u}|=\sqrt{9+16}=\sqrt{25}$

$|\vec{u}|=5$ Ans.

Q.3 If $\vec{u}=2\hat{i}-7\hat{j}$, $\vec{v}=\hat{i}-6\hat{j}$ and $\vec{w}=-\hat{i}+\hat{j}$. Find the following:

(i) $\vec{u}+\vec{v}-\vec{w}=(2\hat{i}-7\hat{j})+(\hat{i}-6\hat{j})-(-\hat{i}+\hat{j})$

$\vec{u}+\vec{v}-\vec{w}=(2i+i+i)+(-7\hat{j}-6\hat{j}-\hat{j})$

$\vec{u}+\vec{v}+\vec{w}=4\hat{i}-14\hat{j}$ Ans.

(ii) $2\vec{u}-3\vec{v}+4\vec{w}=2(2\hat{i}-7\hat{j})-3(\hat{i}-6\hat{j})+4(-\hat{i}+\hat{j})$

$2\vec{u}-3\vec{v}+4\vec{w}=(4i-3i-4i)+(-14\hat{j}+18\hat{j}+4\hat{j})$

$2\vec{u}-3\vec{v}+4\vec{w}=-3\hat{i}+8\hat{j}$ Ans.

(iii) $\frac{1}{2}\vec{u}+\frac{1}{2}\vec{v}+\frac{1}{2}\vec{w}=\frac{1}{2}(2\hat{i}-7\hat{j})+\frac{1}{2}(\hat{i}-6\hat{j})+\frac{1}{2}(-\hat{i}+\hat{j})$

$\frac{1}{2}\vec{u}+\frac{1}{2}\vec{v}+\frac{1}{2}\vec{w}=\left(\hat{i}+\frac{\hat{i}}{2}-\frac{\hat{i}}{2}\right)+\left(\frac{-7}{2}\hat{j}-3\hat{j}+\frac{1}{2}\hat{j}\right)$

$\frac{1}{2}\vec{u}+\frac{1}{2}\vec{v}+\frac{1}{2}\vec{w}=\hat{i}-6\hat{j}$ Ans.

"Nothing is good or bad but our thinkings make it so!"

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

Q.4 Find Sum of $\vec{AB}$ and $\vec{CD}$ where $A(1,-1)$, $B(2,0)$, $C(-1,3)$, $D(-2,2)$

Soln:- Firstly $\vec{AB} = (2-1)\hat{i} + (0+1)\hat{j} = \hat{i} + \hat{j}$

$\vec{CD} = (-2+1)\hat{i} + (2-3)\hat{j} = -\hat{i} - \hat{j}$

Now $\vec{AB} + \vec{CD} = (\hat{i}+\hat{j}) + (-\hat{i}-\hat{j}) = 0\hat{i} + 0\hat{j}$

$\vec{AB} + \vec{CD} = \vec{O}$ Ans.

Q.5: Find Vector from Point A to origion $\vec{AB} = 4\hat{i} - 2\hat{j}$ and $B(-2,5)$

Soln:- let co-ordinates of A are $A(x,y)$

then $\vec{AB} = (-2-x)\hat{i} + (5-y)\hat{j}$ ——— (1)

also $\vec{AB} = 4\hat{i} - 2\hat{j}$ ——— (2)

From (1) and (2), Comparing the Coefficients of $\hat{i}$ and $\hat{j}$

$4 = -2 - x$ and $-2 = 5 - y$

$\Rightarrow x = -2 - 4 = -6$ and $y = 5 + 2 \Rightarrow x = -6$ & $y = 7$

Thus $A(x,y) = A(-6,7)$

Thus vector from A to Origion is $\vec{AO} = (0+6)\hat{i} + (0-7)\hat{j}$

$\vec{AO} = 6\hat{i} - 7\hat{j}$ Ans.

Q.6 Find a unit vector in the direction of:

(i) $\vec{v} = 2\hat{i} - \hat{j}$

$|\vec{v}| = \sqrt{(2)^2 + (-1)^2}$

$|\vec{v}| = \sqrt{4+1} = \sqrt{5}$

Thus unit vector is

$\hat{v} = \frac{\vec{v}}{|\vec{v}|}$

$\hat{v} = \frac{2\hat{i} - \hat{j}}{\sqrt{5}}$

$\hat{v} = \frac{2}{\sqrt{5}}\hat{i} - \frac{1}{\sqrt{5}}\hat{j}$ Ans.

(ii) $\vec{v} = \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$

$|\vec{v}| = \sqrt{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$

$|\vec{v}| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

Thus unit vector is

$\hat{v} = \frac{\vec{v}}{|\vec{v}|}$

$\hat{v} = \frac{\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}}{1}$

$\hat{v} = \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$ Ans.

(iii) $\vec{v} = -\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}$

$|\vec{v}| = \sqrt{(-\sqrt{3}/2)^2 + (-1/2)^2}$

$|\vec{v}| = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$

Thus unit vector is

$\hat{v} = \frac{\vec{v}}{|\vec{v}|}$

$\hat{v} = \frac{-\sqrt{3}/2\,\hat{i} - 1/2\,\hat{j}}{1}$

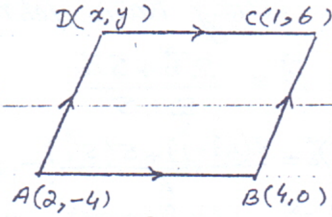
$\hat{v} = -\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}$ Ans.

Q.7 If $A(2,-4)$, $B(4,0)$, $C(1,6)$ then find D if (i) $ABCD$ is a parallelogram (ii) $ADBC$ is a parallelogram.

Soln:- Let Co-ordinates of D are $D(x,y)$ then

For ABCD parallelogram

$\because$ ABCD is Parallelogram so opposite sides must be equal



Let $\vec{AB} = \vec{DC}$ (In same direction)

$(4-2)\hat{i}+(0+4)\hat{j} = (1-x)\hat{i}+(6-y)\hat{j}$

$2\hat{i}+4\hat{j} = (1-x)\hat{i}+(6-y)\hat{j}$

Comparing the Coeff. of $\hat{i}$ and $\hat{j}$

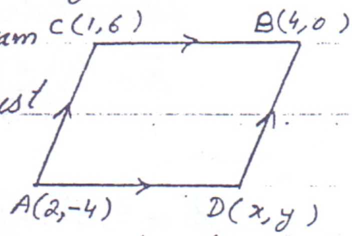
$2 = 1-x$ and $4 = 6-y$

$x = -1$ $\quad y = 2$

Thus $D(x,y) = D(-1,2)$

For ADBC parallelogram

$\because$ ADBC is parallelogram so opposite sides must be equal



Let $\vec{AD} = \vec{CB}$ (In same direction)

$(x-2)\hat{i}+(y+4)\hat{j} = (4-1)\hat{i}+(0-6)\hat{j}$

$(x-2)\hat{i}+(y+4)\hat{j} = 3\hat{i}-6\hat{j}$

Comparing the Coeff. of $\hat{i}$ and $\hat{j}$

$x-2 = 3$ and $y+4 = -6$

$x = 3+2$ and $y = -6-4$

$x = 5$ $\quad y = -10$

Thus $D(x,y) = D(5,-10)$

Q.8 and Q.10 should be done by students themselves.

Q.9 If $\vec{OP} = \vec{AB}$ let $P(x,y) = ?$, $O(0,0)$, $A(-3,7)$, $B(1,0)$

Sol:- Given that $\vec{OP} = \vec{AB} \Rightarrow (x-0)\hat{i}+(y-0)\hat{j} = (1+3)\hat{i}+(0-7)\hat{j}$

$\Rightarrow x\hat{i}+y\hat{j} = 4\hat{i}-7\hat{j}$

Comparing the Coeff. of $\hat{i}$ and $\hat{j}$ $\Rightarrow x = 4$ and $y = -7$

Thus $P(x,y) = P(4,-7)$ Ans.

Q.11 $\vec{AB} = \vec{CD}$ $\quad A(x,y) = ?$ $\quad B(1,2)$, $C(-2,5)$, $D(4,11)$

Sol:- $\vec{AB} = \vec{CD} \Rightarrow (1-x)\hat{i}+(2-y)\hat{j} = (4+2)\hat{i}+(11-5)\hat{j}$

$\Rightarrow (1-x)\hat{i}+(2-y)\hat{j} = 6\hat{i}+6\hat{j}$ Comparing Coeff. of $\hat{i}$ and $\hat{j}$

$1-x = 6$ and $2-y = 6 \Rightarrow x = 1-6$ and $y = 2-6$

$\Rightarrow x = -5$ and $y = -4$ Thus $A(x,y) = A(-5,-4)$ Ans.

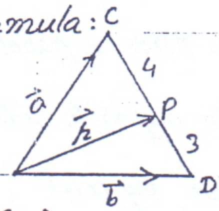
Q:12 Find the P.vector divided by given ratio of vectors:

(i) $m:n = 4:3$ Let $\vec{r} = ?$

Let $\vec{a} = 2\hat{i} - 3\hat{j}$ & $\vec{b} = 3\hat{i} + 2\hat{j}$

Using Ratio Formula:

$$\vec{r} = \frac{4\vec{b} + 3\vec{a}}{4+3}$$



$$\vec{r} = \frac{4(3\hat{i}+2\hat{j}) + 3(2\hat{i}-3\hat{j})}{7}$$

$$\vec{r} = \frac{(12\hat{i}+6\hat{i}) + (8\hat{j}-9\hat{j})}{7}$$

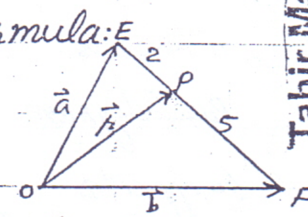
$$\vec{r} = \frac{18}{7}\hat{i} - \frac{1}{7}\hat{j} \quad \text{Ans.}$$

(ii) $m:n = 2:5$ Let $\vec{r} = ?$

Let $\vec{a} = 5\hat{i}$ & $\vec{b} = 4\hat{i} + \hat{j}$

Using Ratio Formula:

$$\vec{r} = \frac{2\vec{b} + 5\vec{a}}{2+5}$$



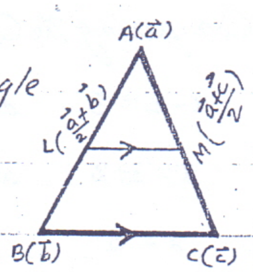
$$\vec{r} = \frac{2(4\hat{i}+\hat{j}) + 5(5\hat{i})}{7}$$

$$\vec{r} = \frac{(8\hat{i}+25\hat{i}) + 2\hat{j}}{7}$$

$$\vec{r} = \frac{33}{7}\hat{i} + \frac{2}{7}\hat{j} \quad \text{Ans.}$$

Q.13:

Let ABC be a triangle with p.v's $\vec{a}, \vec{b}, \vec{c}$ respectively.



Let L, M be the mid points of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ then

P.V. of $L = \dfrac{\vec{a}+\vec{b}}{2}$

P.V. of $M = \dfrac{\vec{a}+\vec{c}}{2}$

Take $\overrightarrow{LM} =$ P.V of M − P.V of L

$$\overrightarrow{LM} = \frac{\vec{a}+\vec{c}}{2} - \frac{\vec{a}+\vec{b}}{2} = \frac{\vec{a}+\vec{c}-\vec{a}-\vec{b}}{2}$$

$$\overrightarrow{LM} = \frac{\vec{c}-\vec{b}}{2} \quad \text{——— (i)}$$

Now $\overrightarrow{BC} =$ P.V of $\vec{C}$ − P.V of B

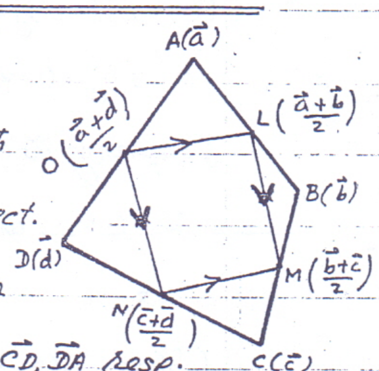
$$\overrightarrow{BC} = \vec{c} - \vec{b} \quad \text{——— (ii)}$$

From (i) and (ii)

$\overrightarrow{LM} = \frac{1}{2}\overrightarrow{BC}$ and $\overrightarrow{LM} \parallel \overrightarrow{BC}$

Thus $\overrightarrow{LM}$ is half of $\overrightarrow{BC}$ but Parallel to $\overrightarrow{BC}$. (Proved)

Q.14: Let ABCD be a quadrilateral with P.V's $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respect.



Let L, M, N, O are the mid points of $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DA}$ resp.

P.V. of $L = \dfrac{\vec{a}+\vec{b}}{2}$ P.V. of $M = \dfrac{\vec{b}+\vec{c}}{2}$

P.V. of $N = \dfrac{\vec{c}+\vec{d}}{2}$ P.V. of $O = \dfrac{\vec{a}+\vec{d}}{2}$

To prove LMNO a parallelogram,

Take $\overrightarrow{LM} =$ P.V. of M − P.V. of L

$$\overrightarrow{LM} = \frac{\vec{b}+\vec{c}}{2} - \frac{\vec{a}+\vec{b}}{2} = \frac{\vec{c}-\vec{a}}{2} \quad \text{—— ①}$$

$$\overrightarrow{ON} = \text{P.V. of } N - \text{P.V. of } O = \frac{\vec{c}+\vec{d}}{2} - \frac{\vec{a}+\vec{d}}{2} = \frac{\vec{c}-\vec{a}}{2} \quad \text{—— ②}$$

$$\overrightarrow{OL} = \text{P.V. of } L - \text{P.V. of } O = \frac{\vec{a}+\vec{b}}{2} - \frac{\vec{a}+\vec{d}}{2} = \frac{\vec{b}-\vec{d}}{2} \quad \text{—— ③}$$

$$\overrightarrow{NM} = \text{P.V. of } M - \text{P.V. of } N = \frac{\vec{b}+\vec{c}}{2} - \frac{\vec{c}+\vec{d}}{2} = \frac{\vec{b}-\vec{d}}{2} \quad \text{—— ④}$$

From ①, ②, ③, ④

$\overrightarrow{LM} = \overrightarrow{ON}$ and $\overrightarrow{OL} = \overrightarrow{NM}$

$\overrightarrow{LM} \parallel \overrightarrow{ON}$ and $\overrightarrow{OL} \parallel \overrightarrow{NM}$

Thus LMNO is a parallelogram. (Proved)