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$$\tan\theta_1 = \frac{-2a^2 - 2ax_0}{x_0 y_0 + a y_0}$$

$$= \frac{-2a(a+x_0)}{y_0(x_0+a)}$$

$$\tan\theta_1 = -\frac{2a}{y_0} \quad \text{———} \quad (A)$$

Now

$$\tan\theta_2 = \frac{m_2 - m_1}{1 + m_2 m_1}$$

$$\tan\theta_2 = \frac{0 - \left(\frac{2a}{y_0}\right)}{1 + (0)\left(\frac{2a}{y_0}\right)} = \frac{-\frac{2a}{y_0}}{1}$$

$$\tan\theta_2 = -\frac{2a}{y_0} \quad \text{———} \quad (B)$$

From (A) and (B)

$$\tan\theta_1 = \tan\theta_2$$

Taking $\tan^{-1}$ on both Sides

$$\theta_1 = \theta_2$$

which is required.

Ellipse

"The Set of all those points in a plane whose sum of the distances from two fixed points remains Constant."

The fixed points are Called Foci

The mid point of Foci is called Centre of Ellipse.

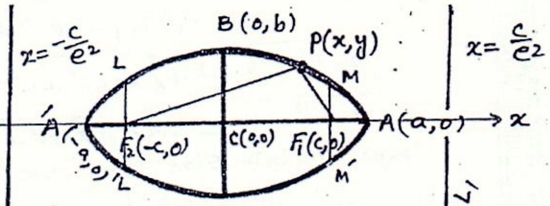
Major Axis

The line segment that joins the two points of Ellipse and passes through Foci is called Major Axis

NOTE:- Foci is the plural of Focus

Minor Axis

" The line segment that joins the two points of Ellipse, through centre and perpendicular to Major axis is called Minor Axis."



* $A'A$ is major axis of Ellipse having length $2a$.
* $B'B$ is minor axis of Ellipse having length $2b$.
* $F_1(c,0)$ and $F_2(-c,0)$ are Foci.
* $A(a,0)$, $A'(-a,0)$ are called Vertices and $B(0,b)$, $B'(0,-b)$ are Covertices.

Definations.

* The end points of Major axis are called Vertices.
* LL' and MM' are the focal chords through Foci perpendicular to Major axis and are called Latus Rectums or Latus Recta.
* Length of Each Latus rectum is $\frac{2b^2}{a}$.
* L_1 and L_2 are directrices of Ellipse having equations

$$L_1 : x = \frac{c}{e^2} \quad \text{and} \quad L_2 : x = \frac{-c}{e^2}$$

* The eccentricity of Ellipse is

$$e = \frac{c}{a} \quad \text{where } 0 < e < 1$$

So

$$L_1 : x = \frac{a}{e} \quad \text{and} \quad L_2 : x = -\frac{a}{e}$$

are also directrices of Ellipse.

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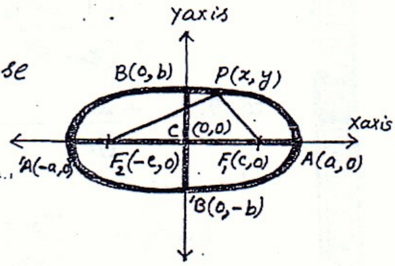
Equation of Ellipse in Standard Form:-



Let $P(x,y)$ be an arbitrary point on the ellipse then by defination

$$|PF_1|+|PF_2| = 2a \text{ (Constant)}$$

$$\sqrt{(x-c)^2+(y-0)^2}+\sqrt{(x+c)^2+(y-0)^2} = 2a$$

$$\Rightarrow \sqrt{(x+c)^2+y^2} = 2a-\sqrt{(x-c)^2+y^2}$$

Squaring both sides, we have

$$\Rightarrow (x+c)^2+y^2 = 4a^2+[(x-c)^2+y^2]-4a\sqrt{(x-c)^2+y^2}$$

$$\Rightarrow (x+c)^2-(x-c)^2+y^2-y^2-4a^2 = -4a\sqrt{(x-c)^2+y^2}$$

$$\Rightarrow x^2+c^2+2xc-x^2-c^2+2xc-4a^2 = -4a\sqrt{(x-c)^2+y^2}$$

$$\Rightarrow 4xc-4a^2 = -4a\sqrt{(x-c)^2+y^2} \Rightarrow xc-a^2 = -a\sqrt{x^2+c^2-2xc+y^2}$$

Squaring again $\Rightarrow x^2c^2+a^4-2xca^2 = a^2x^2+a^2c^2-2xa^2c+a^2y^2$

$$\Rightarrow a^4-a^2c^2 = x^2a^2-x^2c^2+a^2y^2 \Rightarrow a^2(a^2-c^2) = x^2(a^2-c^2)+a^2y^2$$

Dividing throughout by $a^2(a^2-c^2)$, we have

$$1 = \frac{x^2}{a^2}+\frac{y^2}{a^2-c^2} \quad \text{or} \quad \boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1} \quad \text{where } b^2=a^2-c^2$$

which is known as standard Equation of ellipse for $a>b$ with "a" as semi major axis and "b" semi minor axis.

* Circle is a Special Case of Ellipse for $a=b$

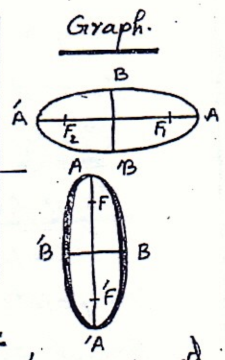
If $a=b$ then $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow x^2+y^2=a^2$ with Centre (0,0) and radius "a".

* Parametric Equations of Ellipse are $x=a\cos\theta$, $y=b\sin\theta$.

Standard Ellipses

Equations	Foci	Vertices	Latus Reactum	Directrices
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ for $a>b$	$F_1(c,0)$ $F_2(-c,0)$	$A(a,0)$ $'A(-a,0)$	$\frac{2b^2}{a}$	$x=\pm\frac{c}{e^2}=\pm\frac{a}{e}$
$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$ for $a>b$	$F_1(0,c)$ $F_2(0,-c)$	$A(0,a)$ $'A(0,-a)$	$\frac{2b^2}{a}$	$y=\pm\frac{c}{e^2}=\pm\frac{a}{e}$

Graph.



When centre of Ellipse is (h,k) then Equations reduces to

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \quad \text{for } a>b \qquad \text{and} \qquad \frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1 \quad \text{for } a>b$$

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EXERCISE 6.5

Q.1 Find an Equation of ellipse with given data and graph it:

(i) Foci $(\pm 3, 0)$

Length of minor axis $= 10$

Centre $(C) = (0, 0)$

$\because 2b = 10 \Rightarrow b = 5$

$a^2 = b^2 + c^2 = (5)^2 + (3)^2$

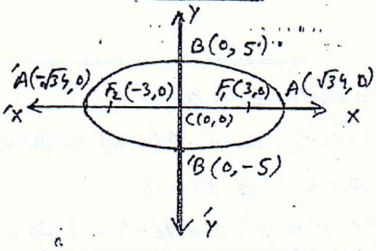
$= 25 + 9 \Rightarrow a^2 = 34$

$a = \sqrt{34}$

Thus Eq of Ellipse is

$$\frac{(x-0)^2}{a^2} + \frac{(y-0)^2}{b^2} = 1$$

$$\Rightarrow \boxed{\frac{x^2}{34} + \frac{y^2}{25} = 1}$$



(ii) $F_1(0, -1)$ and $F_2(0, -5)$

Length of major axis $= 6$

Centre $(C) = \left(\frac{0+0}{2}, \frac{-1-5}{2}\right) = (0, -3)$

$\because 2a = 6 \Rightarrow a = 3$

$2c = |F_1F_2| = \sqrt{(0-0)^2 + (-1+5)^2} = \sqrt{16}$

$2c = 4 \Rightarrow c = 2$

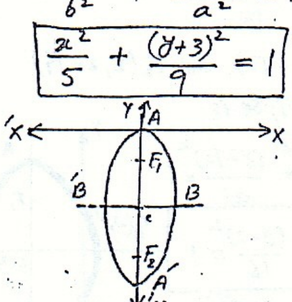
$b^2 = a^2 - c^2 = (3)^2 - (2)^2 = 9 - 4 = 5$

$b = \sqrt{5}$

The Eq of Ellipse is

$$\frac{(x-0)^2}{b^2} + \frac{(y+3)^2}{a^2} = 1$$

$$\boxed{\frac{x^2}{5} + \frac{(y+3)^2}{9} = 1}$$



(iii) Vertices: $A(6, 0)$ and $A'(-6, 0)$

$F_2(-3\sqrt{3}, 0)$ and so $F_1(3\sqrt{3}, 0)$

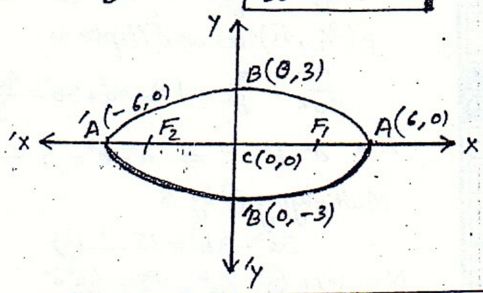
Centre $(C) = \left(\frac{6-6}{2}, \frac{0+0}{2}\right) = (0, 0)$

$a = 6 \quad c = 3\sqrt{3}$

$b^2 = a^2 - c^2 = 36 - 27 = 9 \Rightarrow b = 3$

Thus Equation of Ellipse is

$$\frac{(x-0)^2}{a^2} + \frac{(y-0)^2}{b^2} = 1 \Rightarrow \boxed{\frac{x^2}{36} + \frac{y^2}{9} = 1}$$



(iv) Vertices: $A(5, 1)$ and $A'(-1, 1)$

Foci: $F_1(4, 1)$ and $F_2(0, 1)$

Centre $(C) = \left(\frac{5-1}{2}, \frac{1+1}{2}\right) = (2, 1) \quad \because c = |CF_2|$

Now

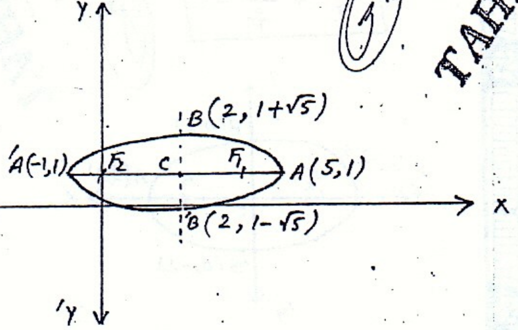
$a = |AC| = 3$ and $c = \sqrt{(4-2)^2 + (1-1)^2} = 2$

$b^2 = a^2 - c^2 = 9 - 4 = 5 \Rightarrow b = \sqrt{5}$

So Equation of Ellipse is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\Rightarrow \boxed{\frac{(x-2)^2}{9} + \frac{(y-1)^2}{5} = 1}$$



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(v) $F_1(\sqrt{5},0)$ and $F_2(-\sqrt{5},0)$

$P(3/2,\sqrt{3})$ lies on ellipse

Centre (C) $= \left(\frac{\sqrt{5}-\sqrt{5}}{2}, \frac{0+0}{2}\right)$

$= (0,0)$

$\because$ Foci lie on x axis so major axis along x-axis

Let Equation of Ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{—(1)}$$

$\because p(3/2,\sqrt{3})$ lies on Ellipse so

$$\frac{9}{4a^2}+\frac{3}{b^2}=1 \Rightarrow 4a^2+3b^2=\frac{4a^2b^2}{3} \quad \text{—(2)}$$

Also $a^2-b^2=c^2 \Rightarrow a^2-b^2=5$ —(3)

Multiplying (3) by 3

$3a^2-3b^2=15$ —(4)

Now (2)+(4) $7a^2=15+\frac{4a^2b^2}{3}$

$\because a^2-b^2=5 \Rightarrow b^2=a^2-5$

$\Rightarrow 7a^2=15+\frac{4a^2(a^2-5)}{3}$

$21a^2=45+4a^4-20a^2$

$4a^4-41a^2+45=0$

$\Rightarrow a^2=9$ or $a^2=5/4$

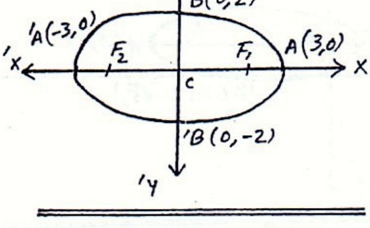
If $a^2=5/4$ then $b^2=5/4-5$

$b^2=\frac{-15}{4}$ (Not possible)

So $a^2=9$ and $b^2=9-5=4$

Hence Eq of Ellipse is

$$\boxed{\frac{x^2}{9}+\frac{y^2}{4}=1}$$



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(vi) Vertices: $A(0,5)$ and $'A(0,-5)$

$\Rightarrow a=5$ and $e=3/5$

$\because c=ae=5\cdot\frac{3}{5}=3$

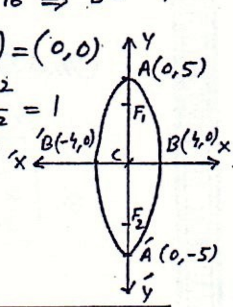
So Foci: $F_1(0,3)$ and $F_2(0,-3)$

Now $b^2=a^2-c^2=25-9=16 \Rightarrow b=4$

Centre (C) $=\left(\frac{0+0}{2},\frac{5-5}{2}\right)=(0,0)$

So Eq of Ellipse is $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

$$\Rightarrow \boxed{\frac{x^2}{16}+\frac{y^2}{25}=1}$$



(vii) Centre (C) $=(0,0)$

$F_2(0,-3)$ and $A(0,4)$ so $'A(0,-4)$

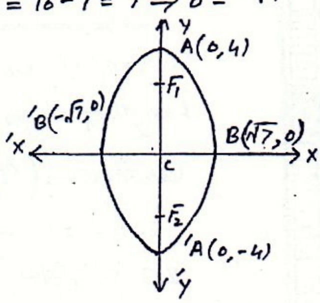
$\Rightarrow |a|=4$ and $|c|=3$

Now $b^2=a^2-c^2=16-9=7 \Rightarrow b=\sqrt{7}$

So Eq of Ellipse is

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

$$\Rightarrow \boxed{\frac{x^2}{7}+\frac{y^2}{16}=1}$$



(viii) Centre $(2,2)$

Major axis $\parallel$ to y axis and $2a=8 \Rightarrow a=4$

Minor axis $\parallel$ to x-axis and $2b=6 \Rightarrow b=3$

Now $c^2=a^2-b^2=16-9=7$

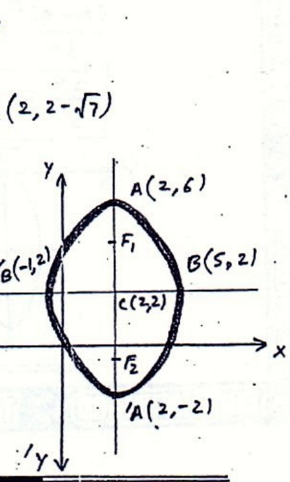
$\Rightarrow c=\sqrt{7}$

So $F_1(2,2+\sqrt{7})$ and $F_2(2,2-\sqrt{7})$

So Eq of Ellipse is

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

$$\boxed{\frac{(x-2)^2}{9}+\frac{(y-2)^2}{16}=1}$$



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(ix) Centre $C = (0,0)$

$\because$ Ellipse is Symmetric about both axes

so let its Equation is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{——} (1)$$

Also $(2,3)$ and $(6,1)$ lies on it so

$$\frac{4}{a^2} + \frac{9}{b^2} = 1 \quad \text{——} (2) \quad \text{and} \quad \frac{36}{a^2} + \frac{1}{b^2} = 1 \quad \text{——} (3)$$

using (3) $\frac{1}{b^2} = 1 - \frac{36}{a^2}$

So (2) becomes $\frac{4}{a^2} + 9\left(1 - \frac{36}{a^2}\right) = 1$

$\Rightarrow \frac{4}{a^2} + 9 - \frac{324}{a^2} = 1 \Rightarrow \frac{4}{a^2}(1-81) = -8$

$\Rightarrow -\frac{80}{a^2} = -2 \Rightarrow a^2 = 40$

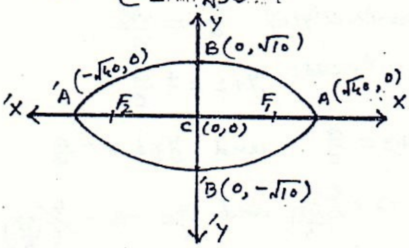
or $a = \sqrt{40}$

Now $\frac{1}{b^2} = 1 - \frac{36}{40} = \frac{4}{40}$

$\Rightarrow b^2 = 10$ and $b = \sqrt{10}$

Now $c^2 = a^2 - b^2 = 40 - 10 = 30$

or $c = \sqrt{30}$



(x) Centre $(C) = (0,0)$

Major Axis along x-axis.

Let the Equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\because (3,1)$ and $(4,0)$ lies on it so

$$\frac{9}{a^2} + \frac{1}{b^2} = 1 \quad \text{——} (1) \quad \text{and} \quad \frac{16}{a^2} + 0 = 1 \quad \text{——} (2)$$

$(2) \Rightarrow a^2 = 16$ or $a = 4$

Now $(1) \Rightarrow \frac{9}{16} + \frac{1}{b^2} = 1$

$\frac{1}{b^2} = 1 - \frac{9}{16} = \frac{7}{16}$ or $b^2 = \frac{16}{7}$

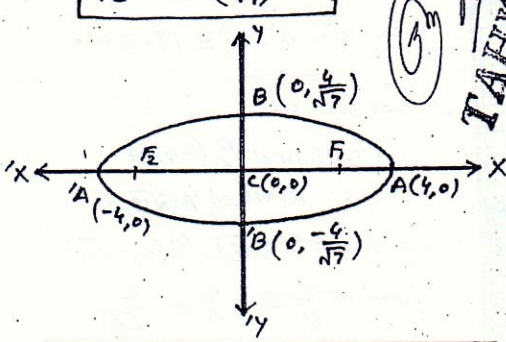
or $b = \frac{4}{\sqrt{7}}$

and $c^2 = a^2 - b^2 = 16 - \frac{16}{7} = \frac{112-16}{7}$

$c^2 = \frac{96}{7}$ or $c = \frac{4\sqrt{6}}{7}$

So

$$\boxed{\frac{x^2}{16} + \frac{y^2}{(16/7)} = 1}$$



Q.2 Find Centre, Foci, Eccentricity vertices and directrices of Ellipse.

(i) $x^2 + 4y^2 = 16$

Dividing by 16 throughout

$$\frac{x^2}{16} + \frac{y^2}{4} = 1 \quad \text{or} \quad \frac{(x-0)^2}{4^2} + \frac{(y-0)^2}{2^2} = 1$$

$\Rightarrow$ Centre $C = (0,0)$

$a^2 = 4^2$ or $a = 4$ so vertices $A(4,0)$ and $A'(-4,0)$

$b^2 = 2^2$ or $b = 2$ so covertices $B(0,2)$ and $B'(0,-2)$

Now $c^2 = a^2 - b^2 = 16 - 4 = 12$

$c = 2\sqrt{3}$

So foci are

$F_1(2\sqrt{3}, 0)$ and $F_2(-2\sqrt{3}, 0)$

Eccentricity $(e) = \frac{c}{a} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

Directrices: $x = \pm\frac{a}{e} = \frac{\pm 4}{\sqrt{3}/2} = \pm\frac{8}{\sqrt{3}}$

Thus directrices are

$$\boxed{x = \frac{8}{\sqrt{3}}} \quad \text{and} \quad \boxed{x = \frac{-8}{\sqrt{3}}}$$

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(ii) $9x^2+y^2=18$

Dividing by 18 throughout

$$\frac{x^2}{2}+\frac{y^2}{18}=1 \text{ or } \frac{(x-0)^2}{(\sqrt{2})^2}+\frac{(y-0)^2}{(\sqrt{18})^2}=1$$

So Centre (C) = (0,0)

$b=\sqrt{2}$ and $a=\sqrt{18}$

Now $c^2=a^2-b^2=18-2=16$

$c=\pm 4$

Thus foci are

$F_1(4,0)$ and $F_2(-4,0)$

Vertices: $A(\sqrt{18},0)$, $A'(-\sqrt{18},0)$

Covertices: $B(0,\sqrt{2})$, $B'(0,-\sqrt{2})$

Ecentricity(e) $=\frac{c}{a}=\frac{4}{\sqrt{18}}$

Directrices:

$$y=\pm\frac{a}{e}=\frac{\pm\sqrt{18}}{(4/\sqrt{18})}$$

$$y=\frac{18}{4}=\frac{9}{2} \text{ and } y=\frac{-18}{4}=-\frac{9}{2}$$

(iii) $25x^2+9y^2=225$

Divinding throughout by 225

$$\frac{x^2}{9}+\frac{y^2}{25}=1 \text{ or } \frac{(x-0)^2}{3^2}+\frac{(y-0)^2}{5^2}=1$$

Centre C : (0,0)

$a=5$ and $b=3$

$c^2=a^2-b^2=25-9=16$

$c=4$

So Foci: $F_1(0,4)$, $F_2(0,-4)$

Vertices: $A(0,5)$, $A'(0,-5)$

Covertices: $B(3,0)$, $B'(-3,0)$

Ecentricity(e) $=\frac{c}{a}=\frac{4}{5}$

Directrices:

$$y=\pm\frac{a}{e}=\frac{\pm 5}{(4/5)}$$

$$y=\pm\frac{25}{4}$$

So $\boxed{y=\frac{25}{4}}$ and $\boxed{y=-\frac{25}{4}}$

(iv) $\frac{(2x-1)^2}{4}+\frac{(y+2)^2}{16}=1$

$$\frac{2^2(x-\frac{1}{2})^2}{4}+\frac{[y-(-2)]^2}{16}=1$$

or $$\frac{(x-\frac{1}{2})^2}{1^2}+\frac{(y-(-2))^2}{4^2}=1$$

Centre(C): $(\frac{1}{2},-2)$

$a=4$ and $b=1$

$c^2=a^2-b^2=16-1=15$ or $c=\sqrt{15}$

So Foci are $F_1(h,k+a)$ and $F_2(h,k-a)$

$F_1(\frac{1}{2},-2+\sqrt{15})$ and $F_2(\frac{1}{2},-2-\sqrt{15})$

Vertices: $A(\frac{1}{2},-2+4)$, $A'(\frac{1}{2},-2-4)$ $\because A(h,k\pm a)$

$A(\frac{1}{2},2)$, $A'(\frac{1}{2},-6)$

Covertices: $B(\frac{1}{2}+1,-2)$, $B'(\frac{1}{2}-1,-2)$ $\because B(h\pm b, k)$

$B(\frac{3}{2},-2)$, $B'(-\frac{1}{2},-2)$

Ecentricity(e) $=\frac{c}{a}=\frac{\sqrt{15}}{4}$

Directrices: $y+2=\pm\frac{a}{e}$

$y+2=\frac{a}{e}$ and $y+2=-\frac{a}{e}$

$y=-2+4/(\frac{\sqrt{15}}{4})$ and $y=-2-4/(\frac{\sqrt{15}}{4})$

$y=-2+\frac{16}{\sqrt{15}}$ and $y=-2-\frac{16}{\sqrt{15}}$

(v) $x^2+16x+4y^2-16y+76=0$

$\Rightarrow x^2+16x+4y^2-16y=-76$

$x^2+16x+(8)^2+4y^2-16y+(4)^2=-76+8^2+4^2$

$(x+8)^2+4(y-2)^2=-76+64+16=4$

Dividing by 4

$$\frac{(x-(-8))^2}{4}+\frac{4(y-2)^2}{4}=1$$

$$\frac{[x-(-8)]^2}{2^2}+\frac{(y-2)^2}{1^2}=1$$

Centre C : $(-8,2)$

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$a = 2 \qquad b = 1$

$c^2 = a^2 - b^2 = 4 - 1 = 3$

$c = \sqrt{3}$

Foci: $F_1(-8+\sqrt{3}, 2)$, $F_2(-8-\sqrt{3}, 2)$

Vertices: $A(-8+2, 2)$, $A'(-8-2, 2)$

$A(-6, 2)$ and $A'(-10, 2)$

Covertices: $B(-8, 2+1)$, $B'(-8, 2-1)$

$B(-8, 3)$, $B'(-8, 1)$

Ecentricity$(e) = \frac{c}{a} = \frac{\sqrt{3}}{2}$

Directrices: $x + 8 = \pm \frac{a}{e}$

$x + 8 = \frac{a}{e}$ and $x + 8 = -\frac{a}{e}$

$x + 8 = \frac{2}{(\sqrt{3}/2)}$ and $x + 8 = \frac{-2}{(\sqrt{3}/2)}$

$x = -8 + \frac{4}{\sqrt{3}}$ and $x = -8 - \frac{4}{\sqrt{3}}$

(Similarly (vi) do yourself.)

Q.3/ is already solved as proof of Equation of Ellipse.

Q.4 $F_1(0,0)$ and $F_2(1,1)$ and $2a = 2$

So $|PF_1| + |PF_2| = 2$

$\sqrt{(x-0)^2 + (y-0)^2} + \sqrt{(x-1)^2 + (y-1)^2} = 2$

$\sqrt{x^2+y^2} + \sqrt{x^2+y^2+2-2x-2y} = 2$

Let $x^2 + y^2 = t$

$\sqrt{t} + \sqrt{t-2x-2y+2} = 2$

Squaring $(\sqrt{t} + \sqrt{t-2x-2y+2})^2 = (2)^2$

$t + t - 2x - 2y + 2 + 2\sqrt{t(t-2x-2y+2)} = 4$

$t - x - y + 1 + \sqrt{t^2 - 2xt - 2yt + 2t} = 2$

$\sqrt{t^2 - 2xt - 2yt + 2t} = 2 - 1 - t + x + y$

$\sqrt{t^2 - 2xt - 2yt + 2t} = (1-t) + (x+y)$

Squaring again, we have

$t^2 - 2xt - 2yt + 2t = (1-t)^2 + (x+y)^2 + 2(1-t)(x+y)$

$\cancel{t^2} - \cancel{2xt} - \cancel{2yt} + 2t = 1 + \cancel{t^2} - 2t + x^2 + y^2 + 2xy + 2x + 2y - \cancel{2tx} - \cancel{2ty}$

$\Rightarrow 2t = 1 - 2t + t + 2xy + 2x + 2y \quad \because x^2 + y^2 = t$

$2t + 2t - t - 2x - 2y - 2xy - 1 = 0$

$3t - 2x - 2y - 2xy - 1 = 0$

$3(x^2 + y^2) - 2x - 2y - 2xy - 1 = 0$

$3x^2 + 3y^2 - 2x - 2y - 2xy - 1 = 0$

which is required Equation of Ellipse.

Q:5

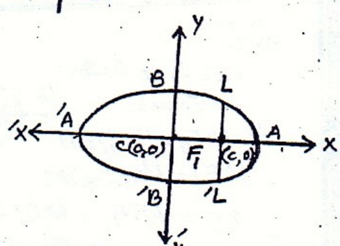
Latus Rectum:-

The chord through foci and perpendicular to the axis of Ellipse is called Latus Rectum of Ellipse.

Let

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

be Ellipse and LF_1L' be the Latus Rectum



Co-ordinates of L and L' are

$x = c \Rightarrow \frac{y^2}{b^2} + \frac{c^2}{a^2} = 1 \Rightarrow \frac{y^2}{b^2} = 1 - \frac{c^2}{a^2}$

$\frac{y^2}{b^2} = \frac{a^2 - c^2}{a^2} = \frac{b^2}{a^2} \quad \because c^2 = a^2 - b^2, \; b^2 = a^2 - c^2$

$\Rightarrow y^2 = \frac{b^4}{a^2}$

So $y = \pm \frac{b^2}{a}$

So $L\left(c, \frac{b^2}{a}\right)$ and $L'\left(c, -\frac{b^2}{a}\right)$

$|LF_1L'| = \sqrt{(c-c)^2 + \left(\frac{b^2}{a} + \frac{b^2}{a}\right)^2}$

$= \sqrt{0^2 + \frac{4b^4}{a^2}} = \sqrt{\frac{4b^4}{a^2}}$

$|LF_1L'| = \frac{2b^2}{a}$

which is Length of Latus Rectum.

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Q.6 Length of major axis $(2a) = 4\sqrt{2}$

$\Rightarrow 2a = 4\sqrt{2}$ or $a = 2\sqrt{2}$

Suppose Equation of Ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{—— ①}$$

Distance b/w foci = length of minor axis

$\Rightarrow \quad 2c = 2b \Rightarrow b = c$

$\because |F_1F_2| = 2c =$ Distance b/w Foci.

$\because \; c^2 = a^2 - b^2$

$\Rightarrow b^2 = (2\sqrt{2})^2 - b^2$

$2b^2 = 8$ or $b^2 = 4$

$\Rightarrow \; b = 2$

So ① reduces to

$$\boxed{\frac{x^2}{8} + \frac{y^2}{4} = 1}$$

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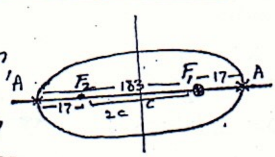
Q.7

Let F_1 be the Sun as focus.



So $|AF_1| = |'AF_2| = 17$

$|AF_2| = |'AF_1| = 183$

$2c = |F_1F_2| = |AF_2| - |AF_1|$
$= 183 - 17 = 166$

$\therefore c = 83$

Now $a = |AC| = |CF_1| + |AF_1|$

but $|CF_1| = |CF_2| = \frac{|F_1F_2|}{2} = \frac{166}{2} = 83$

So $a = 83 + 17 = 100 \Rightarrow a^2 = 10000$

Now $b^2 = a^2 - c^2 = (100)^2 - (83)^2$

$b^2 = 10000 - 6889 = 3111$

So Equation of orbit is

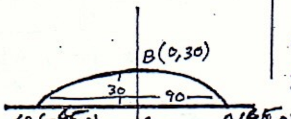
$$\boxed{\frac{x^2}{10000} + \frac{y^2}{3111} = 1}$$

Q.8

$\because 2a = 90\,m$

$a = 45\,m$

and $b = 30\,m$



So Eq of Ellipse is

$$\frac{x^2}{45^2} + \frac{y^2}{30^2} = 1$$

Now $y = 20\sqrt{2}$ m and $x = ?$

So $\frac{x^2}{(45)^2} + \frac{(20\sqrt{2})^2}{(30)^2} = 1$

$\Rightarrow \frac{x^2}{(45)^2} = 1 - \frac{800}{900} = \frac{100}{900} = \frac{1}{9}$

$x^2 = \frac{(45)^2}{9}$ or $x^2 = \left(\frac{45}{3}\right)^2 = (15)^2$

$x = 15\,m.$

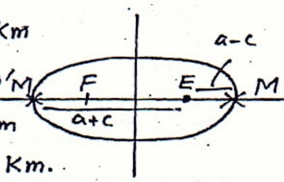
Q.9:-

$2a = 768806$ Km

$a = 384403$ Km

$2b = 767746$ Km

$b = 383873$ Km.



Equation of orbit is

$$\frac{x^2}{(384403)^2} + \frac{y^2}{(383873)^2} = 1$$

$c^2 = a^2 - b^2 = (384403)^2 - (383873)^2$

$= 147765666409 - 147358480129$

$c^2 = 407186280$

$c = 20178.86$ Km.

Greatest distance: $|MF| = |'ME| = a + c$

$= 384403 + 20178.86$

$= 404581.86$ Km

≈ 404582 Km Ans.

Least distance: $|ME|$ or $|'MF| = a - c$

$= 384403 - 20178.86$

$= 364224.14$ Km

≈ 364224 Km. Ans.

Hyperbola:-

"The set of all those points in a plane whose difference of the distances from two fixed points remains Constant."

The fixed points are called Foci.

The mid point of foci is called Centre of Hyperbola.

The distance between foci is "2c".

$|F_1F_2| = 2c$

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