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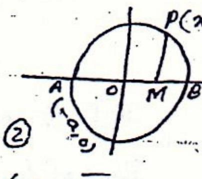
Q.4:- Prove that perpendicular dropped from a point of Circle on the diameter is mean (Geometric) proportional between the segments into which it divides the diameter.



Proof:- Consider a circle $x^2+y^2=a^2$ —①

Let $P(x_1,y_1)$ be any point on it so $x_1^2+y_1^2=a^2$ —②

Let $A(-a,0)$, $B(a,0)$ be the end points of diameter $\overline{AB}$

Let $\overline{PM}$ be perpendicular on $\overline{AB}$ so $M(x_1,0)$

Now $|\overline{AM}| = \sqrt{(x_1+a)^2+(0-0)^2} = x_1+a$

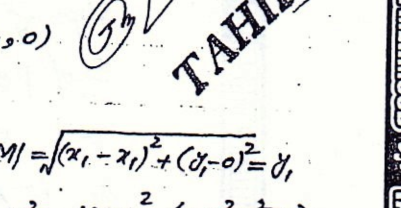
$|\overline{BM}| = \sqrt{(a-x_1)^2+(0-0)^2} = a-x_1$ and $|PM| = \sqrt{(x_1-x_1)^2+(y_1-0)^2} = y_1$

Consider $|\overline{AM}|.|\overline{BM}| = (a+x_1)(a-x_1) = a^2-x_1^2 = y_1^2 = |PM|^2 \quad (\because a^2-x_1^2=y_1^2)$

$|\overline{AM}|\cdot|\overline{BM}| = |PM|\cdot|PM| \Rightarrow \dfrac{|\overline{AM}|}{|\overline{PM}|} = \dfrac{|\overline{PM}|}{|\overline{BM}|}$

so $|\overline{PM}|$ is mean proportional to $|\overline{AM}|$ and $|\overline{BM}|$. (Proved)

Parabola:-



"The set of all those points whose distance from a fixed point and a fixed line remains same is called parabola."

*Fixed point is called Focus and fixed line is called Directrix.

*Fixed point does not lie on the fixed line (Directrix).

Coordinates of focus are $F(a,0)$ and Eq of directrix is $x+a=0$.

The line through focus and perpendicular to directrix is called Axis of parabola. The point where axis meet parabola is called Vertex.

The line that joins two points of parabola is called Chord.

The chords which passes through focus are called Focal Chords.

A focal chord which is perpendicular to the axis of parabola is called Latus Reactum. (LL') having length $|LL'| = 4a$.

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Eccentricity:-

"It is the ratio of the distance of a point of Curve to Fixed point (Focus) to distance of a point of Curve to a Fixed line (directrix) denoted by "e" where e is +ve real number

If M is a point of fixed line L and F is focus and P being point of the Curve then $\frac{|\overline{PF}|}{|\overline{PM}|} = e$.

Note:- (i) If $e = 1$ then Conics is parabola.

(ii) If $e < 1$ then curve (Conics) is Ellipse.

(iii) If $e > 1$ then Curve (Conics) is Hyperbola.

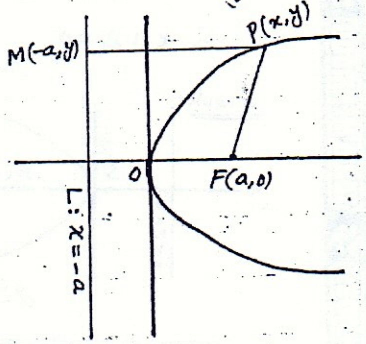
Equation of Parabola:-

Let $P(x,y)$ be any point on parabola with focus $F(a,0)$ and Directrix $x = -a$

Draw $PM \perp L: x+a=0$ so $M(-a, y)$

By defination $|\overline{PM}| = |\overline{PF}|$

$$\sqrt{(x+a)^2+(y-y)^2} = \sqrt{(x-a)^2+(y-0)^2}$$



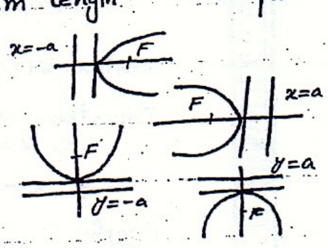
Squaring $\Rightarrow (x+a)^2 + 0^2 = (x-a)^2 + y^2$

$$x^2+a^2+2ax = x^2+a^2-2ax+y^2 \Rightarrow y^2 = 2ax+2ax$$

$\boxed{y^2 = 4ax}$ which is standard Eq of parabola.

Standard Parabolas

Equation	Focus	Vertex	Axis	Directrix	LatusReactum Length	Graph.
$y^2 = 4ax$	$F(a,0)$	$O(0,0)$	$y=0$	$x+a=0$	$4a$	
$y^2 = -4ax$	$F(-a,0)$	$O(0,0)$	$y=0$	$x-a=0$	$4a$	
$x^2 = 4ay$	$F(0,a)$	$O(0,0)$	$x=0$	$y+a=0$	$4a$	
$x^2 = -4ay$	$F(0,-a)$	$O(0,0)$	$x=0$	$y-a=0$	$4a$	



* Parametric Equations of parabola are $x = at^2$ $y = 2at$ where $-\infty < t < \infty$.

* If (h,k) is vertex then equation of parabola is $(y-k)^2 = 4a(x-h)$

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Exercise 6.4

Q.1 Find focus, vertex and directrix of parabola and Sketch the graph.

(i) $y^2 = 8x$

Comparing with $y^2 = 4ax$

$4a = 8 \Rightarrow a = 2$

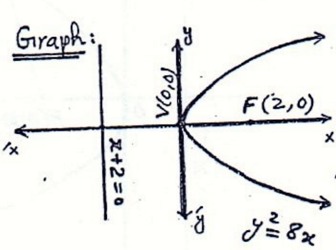
$F(a,0) = F(2,0)$ is focus.

Vertex: $V(0,0)$

Directrix: $x = -a$
$\Rightarrow x = -2$

$\Rightarrow x + 2 = 0$

Graph:



(ii) $x^2 = -16y$

Comparing with $x^2 = -4ay$

$4a = 16 \Rightarrow a = 4$

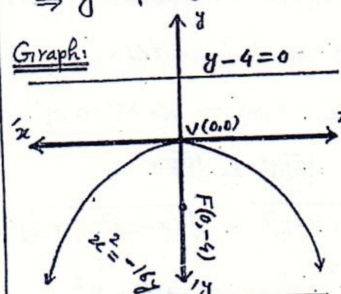
Focus: $F(0,-a) = F(0,-4)$

Vertex: $V(0,0)$

Directrix: $y = a$
$\Rightarrow y = 4$

$\Rightarrow y - 4 = 0$

Graph:



(iii) $x^2 = 5y$

Comparing with $x^2 = 4ay$

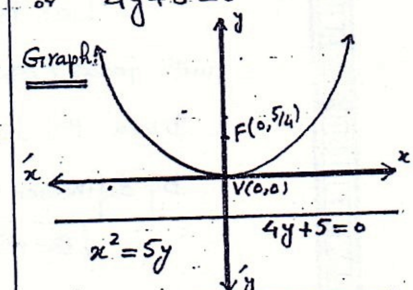
$4a = 5 \Rightarrow a = 5/4$

Focus: $F(0,a) = F(0, 5/4)$

Vertex: $V(0,0)$

Directrix: $y = -a$
$\Rightarrow y = -5/4$
or $4y + 5 = 0$

Graph:



(iv) $y^2 = -12x$

Comparing with $y^2 = 4ax$

$\Rightarrow 4a = 12 \Rightarrow a = 3$

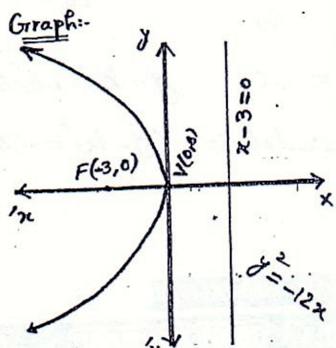
Focus: $F(-a,0) = F(-3,0)$

Vertex: $V(0,0)$

Directrix: $x = a$
$\Rightarrow x = 3$

$\Rightarrow x - 3 = 0$

Graph:-



(v) $x^2 = 4(y-1)$

$\Rightarrow (x-0)^2 = 4(y-1)$

Comparing with $(x-h)^2 = 4a(y-k)$

Vertex: $V(h,k) = V(0,1)$

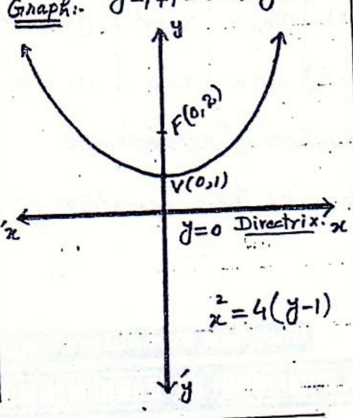
$4a = 4 \Rightarrow a = 1$

Now focus: $F(h+0, k+a)$
$F(0,2)$

Directrix: $(y-1) = -a$

$y - 1 = -1$

Graph:- $y - 1 + 1 = 0 \Rightarrow y = 0$



(vi) $y^2 = -8(x-3)$

$\Rightarrow (y-0)^2 = -8(x-3)$

Comparing with $(y-k)^2 = -4a(x-h)$

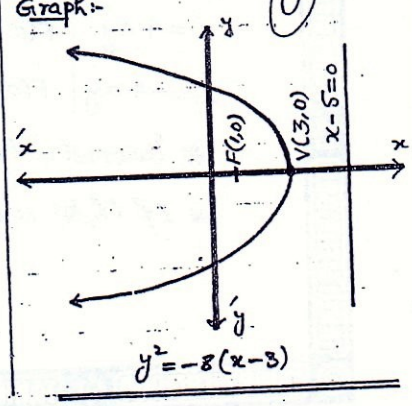
Vertex: $V(h,k) = V(3,0)$

$4a = 8 \Rightarrow a = 2$

Now focus: $F(h-a, k+0)$
$F(3-2, 0) = F(1,0)$

Directrix: $x - 3 = 2$
$\Rightarrow x - 5 = 0$

Graph:-



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(vii) $(x-1)^2 = 8(y+2)$

Comparing with $(x-h)^2 = 4a(y-k)$

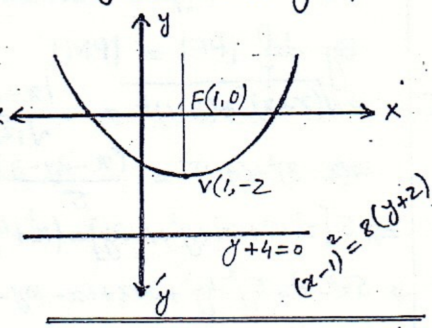
Vertex: $V(h,k) = V(1,-2)$

$4a = 8 \Rightarrow a = 2$

Now focus: $F(h+0, k+a)$: $F(1, 2-2) = F(1,0)$

Directrix: $(y+2) = -a$

$y+2 = -2$ or $y+4=0$



(viii) $y = 6x^2 - 1$

$\Rightarrow 6x^2 = y+1$ or $x^2 = \frac{1}{6}(y+1)$

or $(x-0)^2 = \frac{1}{6}(y-(-1))$

Comparing with $(x-h)^2 = 4a(y-k)$

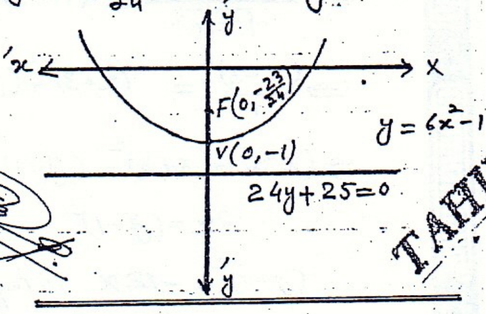
$4a = \frac{1}{6}$ or $a = \frac{1}{24}$ $\quad F(h+0, k+a)$

Vertex: $V(0,-1)$ and Focus: $F\left(0, -1+\frac{1}{24}\right)$

$F\left(0, \frac{-23}{24}\right)$

Directrix: $y+1 = -a \Rightarrow y+1 = -\frac{1}{24}$

$y+1+\frac{1}{24} = 0 \Rightarrow 24y+25=0$



(ix) $x+8-y^2+2y=0$

$\Rightarrow y^2-2y = x+8 \Rightarrow y^2-2y+1 = x+9$

$(y-1)^2 = 1.(x+9)$

Comparing with $(y-k)^2 = 4a(x-h)$

Vertex: $V(h,k) = V(-9,1)$

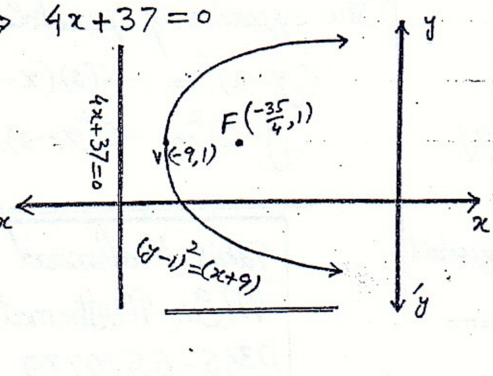
$4a = 1 \Rightarrow a = 1/4$

so focus: $F(-9+1/4, 1) = F\left(\frac{-35}{4}, 1\right)$

Directrix: $(x-h) = -a$

$\Longrightarrow \quad x+9 = -\frac{1}{4} \Rightarrow x+9+1/4 = 0$

$\Rightarrow 4x+37=0$



(x) $x^2-4x-8y+4=0$

$\Rightarrow x^2-4x+4 = 8y \Rightarrow (x-2)^2 = 8(y-0)$

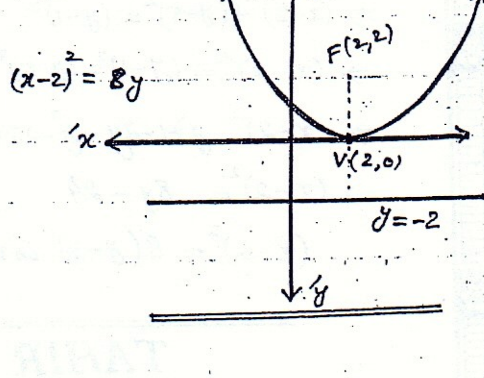
Comparing with $(x-h)^2 = 4a(y-k)$

Vertex: $V(h,k) = V(2,0)$ $\quad F(h+0, k+a)$

$4a = 8 \Rightarrow a = 2$ so Focus: $F(2, 0+2)$

Focus: $F(2,2)$

Directrix: $y-k = -a \Rightarrow y = -2$



Q: Write the Equation of the Parabola with given elements:

(i) Focus $(-3,1)$, Directrix: $x=3$

Sol:- Let $F(-3,1)$ and $L: x-3=0$

Let $P(x,y)$ be the point on parabola and M be a point on directrix L.

By def. $|PM| = |PF|$

$$\Rightarrow \frac{|x-3|}{\sqrt{1^2+0^2}} = \sqrt{(x+3)^2+(y-1)^2}$$

$$\Rightarrow \frac{(x-3)^2}{1} = (x+3)^2+(y-1)^2$$

$$\Rightarrow (x-3)^2-(x+3)^2 = (y-1)^2$$

$$\Rightarrow -12x = (y-1)^2$$

$(y-1)^2 = -12x$ is required.

(ii) Focus $(2,5)$, Directrix: $y=1$

Sol:- Let $F(2,5)$ and $L: y-1=0$

Let $P(x,y)$ be a point on parabola and M be a point on directrix L.

By def. $|PF| = |PM|$

$$\Rightarrow \sqrt{(x-2)^2+(y-5)^2} = \frac{|y-1|}{\sqrt{0^2+1^2}}$$

$$\Rightarrow (x-2)^2+(y-5)^2 = (y-1)^2$$

$$\Rightarrow (x-2)^2 = (y-1)^2-(y-5)^2$$

$$\Rightarrow (x-2)^2 = y^2+1-2y-y^2-25+10y$$

$$\Rightarrow (x-2)^2 = 8y-24$$

$(x-2)^2 = 8(y-3)$ is required.

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(iii) Focus $(-3,1)$, Directrix: $x-2y-3=0$

Sol:- Let $F(-3,1)$ and $L: x-2y-3=0$

Suppose $P(x,y)$ be a point on parabola and M be a point on directrix L.

By def. $|PF| = |PM|$

$$\Rightarrow \sqrt{(x+3)^2+(y-1)^2} = \frac{|x-2y-3|}{\sqrt{1^2+2^2}}$$

$$\Rightarrow (x+3)^2+(y-1)^2 = \frac{(x-2y-3)^2}{5}$$

$$\Rightarrow 5\left[x^2+9+6x+y^2+1-2y\right] = \left(x^2+4y^2+9-4xy+12y-6x\right)$$

$$\Rightarrow 5x^2-x^2+5y^2-4y^2+30x+6x-10y-12y+4xy+50-9=0$$

$$\Rightarrow 4x^2+y^2+36x-22y+4xy+41=0$$

is the required Equation of parabola.

(iv) Focus $F(1,2)$ and Vertex $V(3,2)$

Sol:- $\because$ Focus lies left to Vertex at $y=2$ line (Axis)

So parabola opens left Side

Now suppose $(y-2)^2 = -4a(x-3)$ be Equation

Here $a = \sqrt{(3-1)^2+(2-2)^2} = |VF|$

$a = \sqrt{4+0} \Rightarrow a = 2$

Thus equation of parabola is

$$(y-2)^2 = -4(2)(x-3)$$

$$(y-2)^2 = -8(x-3) \quad \text{(Required)}$$

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(v) Focus: $F(-1,0)$, Vertex: $V(-1,2)$.

Sol:- $\because$ Focus lies below the vertex with axis $x=-1$ so parabola opens downward.

Let Equation is $(x+1)^2=-4a(y-2)$

Now $a=\sqrt{(-1+1)^2+(2-0)^2}=\sqrt{4}=2$

Thus $(x+1)^2=-4(2)(y-2)$

$(x+1)^2=-8(y-2)$ is required.

(vi) Directrix: $x=-2$ Focus $(2,2)$

Sol:- Let $p(x,y)$ be a point on parabola and M be a point on directrix L.

By def. $|PF|=|PM|$

$\Rightarrow \sqrt{(x-2)^2+(y-2)^2}=\frac{|x+2|}{\sqrt{1^2+0^2}}$

$\Rightarrow (x-2)^2+(y-2)^2=(x+2)^2$

$\Rightarrow (y-2)^2=(x+2)^2-(x-2)^2=4(x)(2)$

$\Rightarrow (y-2)^2=8x$ is required.

(vii) Directrix: $y=3$ and Vertex $(2,2)$

Sol:- Let Equation is $(x-2)^2=4a(y-2)$

Now Distance between Vertex and Directrix = Distance b/w focus and Vertex

Thus $|a|=|3-2|=1$

$\because$ vertex is b/ow the directrix so $a=-1$

Hence $(x-2)^2=4(-1)(y-2)$

$(x-2)^2=-4(y-2)$

is required parabola.

(viii) Directrix: $y=1$ Length of latus reactum $=8$ open downward.

Sol:- $\because 4a=8 \Rightarrow a=2$

$\because$ Parabola opens downward so $a=2$

Suppose (h,k) be the vertex. Here $k=-1$

So Eq is $(x-h)^2=-4(2)(y+1)$

$\Rightarrow (x-h)^2=-8(y+1)$ Required.

(ix) Axis $y=0$ through $(2,1)$ and $(11,-2)$.

Sol:- The equation is for $y=0$ being axis

$(y-k)^2=4a(x-h)$

$\because y=0$ is axis so $k=0$

$\Rightarrow y^2=4a(x-h)$ — (A)

Now $(2,1)$ and $(11,-2)$ lies on (1)

so $(1)^2=4a(2-h)$ — (1) $\Rightarrow 8a-4ah=1$

$(-2)^2=4a(11-h)$ — (2) $\Rightarrow 44a-4ah=4$

$-36a=-3$

$\Rightarrow a=\frac{3}{36}=\frac{1}{12}$

so $\boxed{a=\frac{1}{12}}$

Now putting $a=1/12$ in (1)

$8(\frac{1}{12})-4(\frac{1}{12})h=1$

$\frac{2}{3}-\frac{1}{3}h=1 \Rightarrow -\frac{1}{3}h=1-\frac{2}{3}$

$-\frac{1}{3}h=\frac{1}{3} \Rightarrow h=-1$

So Eq of parabola is

$y^2=4(\frac{1}{12})(x+1)$

$y^2=\frac{1}{3}(x+1)$ required.

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(x) Axis ∥ to y axis and through (0,3), (3,4), (4,11).

Soln:- Since axis of parabola is ∥ to y axis so let its equation be

$$(x-h)^2 = 4a(y-k) \quad \text{——} ①$$

$\because$ (0,3), (3,4), (4,11) lies on parabola so must satisfy its equation.

$$(0,3) \Rightarrow h^2 = 4a(3-k) \quad \text{——} ②$$

$$(3,4) \Rightarrow (3-h)^2 = 4a(4-k) \quad \text{——} ③$$

$$(4,11) \Rightarrow (4-h)^2 = 4a(11-k) \quad \text{——} ④$$

③ can be written as

$$(3-h)^2 = 4a(3-k+1)$$

$$(3-h)^2 = 4a(3-k) + 4a$$

$$(3-h)^2 = h^2 + 4a \quad \text{by ②} \quad \text{——} ⑤$$

④ can be written as

$$(4-h)^2 = 4a(3-k+8)$$

$$(4-h)^2 = 4a(3-k) + 32a$$

$$(4-h)^2 = h^2 + 32a \quad \text{by ②} \quad \text{——} ⑥$$

Now ⑥ − 8×⑤

$$\cancel{32a} + h^2 = 16 + h^2 - 8h$$

$$\underline{-\cancel{32a} \pm 8h^2 = 72 + 8h^2 \mp 48h}$$

$$-7h^2 = -56 - 7h^2 + 40h$$

$$\Rightarrow 40h = 56$$

$$\Rightarrow \boxed{h = \frac{56}{40} = \frac{7}{5}}$$

$$⑤ \Rightarrow 4a + (7/5)^2 = (3 - 7/5)^2$$

$$4a + \frac{49}{25} = \left(\frac{15-7}{5}\right)^2$$

$$4a + \frac{49}{25} = \frac{64}{25}$$

$$4a = \frac{64-49}{25} = \frac{15}{25} = 3/5$$

$$\boxed{a = \frac{3}{20}} \text{ or } \boxed{4a = 3/5}$$

Now by ② $(7/5)^2 = \frac{3}{5}(3-k)$

$$\frac{49}{25} = \frac{3}{5}(3-k)$$

$$3-k = \frac{49}{25} \times \frac{5}{3} = \frac{49}{15}$$

$$\text{or } 3 - \frac{49}{15} = k$$

$$\text{or } k = \frac{45-49}{15} = \frac{-4}{15}$$

So ① becomes

$$\left(x - \frac{7}{5}\right)^2 = \frac{3}{5}\left(y + \frac{4}{15}\right)$$

which is required.

Q.3 Focus: F(0,0)

(i) Directrix parallel to x-axis.

Sol:- Since directrix is parallel to x-axis so let $y = a$ be directrix

May be written as $y - a = 0$

By def $|PM| = |PF|$

where P(x,y) be any point of parabola and M lies on the directrix.

$$\sqrt{(x-0)^2 + (y-0)^2} = \frac{|y-a|}{\sqrt{0^2+1^2}}$$

$$\sqrt{x^2+y^2} = |y-a|$$

On squaring $x^2 + y^2 = (y-a)^2$

$$\Rightarrow x^2 + y^2 = y^2 + a^2 - 2ay \Rightarrow x^2 = a^2 - 2ay$$

or $x^2 = -2a\left(y - \frac{a}{2}\right)$ is required Eq.

(ii) Directrix parallel to y axis

Sol:- Since directrix is ∥ to y axis so let $x = a$ or $x - a = 0$ be its equation

By def $|PM| = |PF|$

where P(x,y) being arbitrary point parabola and M lies on directrix.

$$\sqrt{(x-0)^2 + (y-0)^2} = \frac{|x-a|}{\sqrt{1^2+0^2}}$$

squaring

$$\Rightarrow x^2 + y^2 = (x-a)^2 \Rightarrow x^2 + y^2 = x^2 + a^2 - 2ax$$

$$\Rightarrow y^2 = a^2 - 2ax$$

$$\Rightarrow y^2 = -2a\left(x - \frac{a}{2}\right)$$

which is required Equation.

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Q.4 Let Focus $F(a\cos\alpha, a\sin\alpha)$ and $L: x\cos\alpha + y\sin\alpha + a = 0$ be directrix

Let $P(x,y)$ be any arbitrary point on parabola and M be a point of L then by defination of parabola $|PM| = |PF|$

$$\sqrt{(x-a\cos\alpha)^2+(y-a\sin\alpha)^2} = \frac{|x\cos\alpha + y\sin\alpha + a|}{\sqrt{\cos^2\alpha+\sin^2\alpha}}$$

Squaring both sides, we have $\quad (\because \sin^2\alpha + \cos^2\alpha = 1)$

$$(x-a\cos\alpha)^2+(y-a\sin\alpha)^2 = (x\cos\alpha + y\sin\alpha + a)^2$$

$$x^2+a^2\cos^2\alpha+y^2+a^2\sin^2\alpha - 2xa\cos\alpha - 2ya\sin\alpha = x^2\cos^2\alpha + y^2\sin^2\alpha + a^2 + 2xy\sin\alpha\cos\alpha + 2ya\sin\alpha + 2ax\cos\alpha$$

$$\Rightarrow x^2+y^2+a^2(\cos^2\alpha+\sin^2\alpha) - 4xa\cos\alpha - 4ya\sin\alpha = x^2\cos^2\alpha + y^2\sin^2\alpha + a^2 + 2xy\sin\alpha\cos\alpha$$

$$\Rightarrow x^2(1-\cos^2\alpha) + y^2(1-\sin^2\alpha) + a^2 - a^2 - 4a(x\cos\alpha + y\sin\alpha) = 2xy\sin\alpha\cos\alpha$$

$$\Rightarrow x^2\sin^2\alpha + y^2\cos^2\alpha - 2xy\sin\alpha\cos\alpha = 4a(x\cos\alpha + y\sin\alpha)$$

$$\Rightarrow (x\sin\alpha - y\cos\alpha)^2 = 4a(x\cos\alpha + y\sin\alpha)$$

which is the Equation of parabola.

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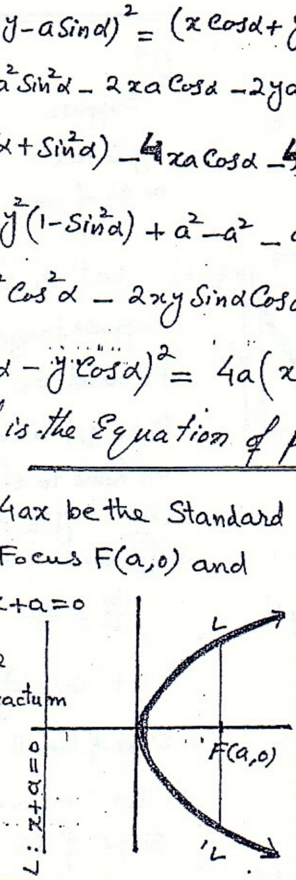
Q.5 Let $y^2 = 4ax$ be the Standard parabola with Focus $F(a,0)$ and directrix $x + a = 0$



$\because 4a = |LL'|$ is the length of latus reactum

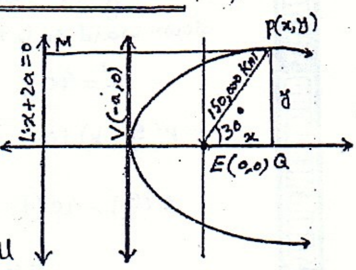
If $P(x,y)$ be any Point on $y^2 = 4ax$ then

$$y = \pm\sqrt{(4a)(x)}$$

$$y = \pm\sqrt{(\text{length of latus reactum})(\text{Abscissa of } P)}$$

which shows that ordinate is mean proportional to the length of latus reactum and abscissa of the point P.

Q.6



Let $E(0,0)$ be the origion being focus for orbit.

So vertex will be $V(-a, 0)$ (say)

then $L: x + 2a = 0$ be directrix.

By def. $|PM| = |PF|$

For $P(x,y)$ being arbitrary point of parabola and M lies on L.

$$\sqrt{(x-0)^2+(y-0)^2} = \frac{|x+2a|}{\sqrt{1^2+0^2}}$$

$$x^2 + y^2 = (x+2a)^2 \quad \text{--- (1)}$$

From ΔEPQ $\quad x^2 + y^2 = (150,000)^2 \quad \text{--- (2)}$

Comparing (1) and (2)

$$(x+2a)^2 = (150,000)^2$$

$$\Rightarrow x + 2a = 150,000 \quad \text{--- (3)}$$

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From ΔEPQ $\cos 30^\circ = \dfrac{x}{150{,}000}$

or $x = 150{,}000 \times \dfrac{\sqrt{3}}{2} = 75000\sqrt{3}$

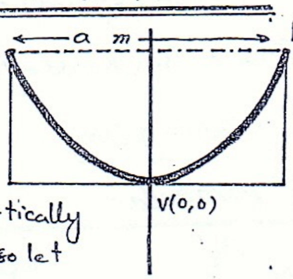
(3) $\Rightarrow$ $2a = 150{,}000 - 75000\sqrt{3}$

$$a = \frac{75000}{2}\left[2 - \sqrt{3}\right]$$

$$a = 37500(2 - \sqrt{3})$$

Thus Comet (سیارہ) will be closest to earth if it is $37500(2 - \sqrt{3})$ Km from the Earth.

Q.7



Clearly Cable can be hung vertically down ward so let

$x^2 = 4cy$ be its equation.

$\because P(\frac{a}{2}, b)$ lies on it so satisfy it

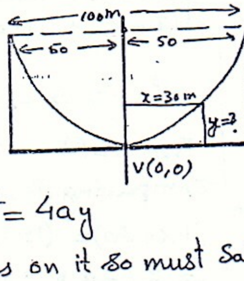
$\Rightarrow (\frac{a}{2})^2 = 4c(b) \Rightarrow \dfrac{a^2}{4} = 4bc$

or $c = \dfrac{a^2}{16b}$

Hence $x^2 = 4\left[\dfrac{a^2}{16b}\right]y$

$\therefore x^2 = \dfrac{a^2}{4b}y$ is required Eq:

Q.8



By def. Eq of parabola in this case will be $x^2 = 4ay$

$\because P(50,25)$ lies on it so must satisfy it

$\Rightarrow (50)^2 = 4a(25)$

$\Rightarrow 2500 = 100a$

$\boxed{a = 25}$

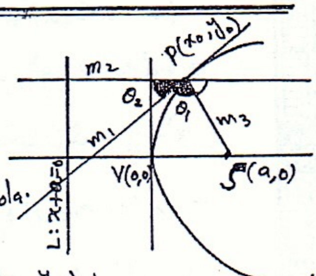
Thus $x^2 = 100y$ $\quad \because a = 25$

Now $y = ?$ for $x = 30$ m

So $(30)^2 = 100y$

or $900 = 100y$ or $\boxed{y = 9\text{ m}}$

Q.9



Suppose $y^2 = 4ax$ be Eq of parabola.

Let $P(x_0, y_0)$ be any point on it. Suppose tangent makes θ_1 angle with PS and θ_2 angle with line through $P(x_0, y_0)$ and $\parallel$ to axis of parabola. We have to show that $\theta_1 = \theta_2$

Take $y^2 = 4ax$

Diff w.r.t. $x \Rightarrow 2y\dfrac{dy}{dx} = 4a$

$\dfrac{dy}{dx} = \dfrac{2a}{y}$ or $\left(\dfrac{dy}{dx}\right)_{P(x_0,y_0)} = \dfrac{2a}{y_0}$

$\Rightarrow$ let $m_1 = \dfrac{2a}{y_0}$ —— (1) (slope of tangent at P)

$\because$ Slope of Line $\parallel$ to axis of parabola is zero

so let $m_2 = 0$ —— (2)

Slope of PS $= \dfrac{y_0 - 0}{x_0 - a}$

let $m_3 = \dfrac{y_0}{x_0 - a}$ —— (3)

Let us use the formula

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

So $\tan\theta_1 = \dfrac{m_1 - m_3}{1 + m_1 m_3}$

$$\tan\theta_1 = \frac{\left(\frac{2a}{y_0}\right) - \left(\frac{y_0}{x_0 - a}\right)}{1 + \left(\frac{2a}{y_0}\right)\left(\frac{y_0}{x_0 - a}\right)}$$

$$\tan\theta_1 = \frac{2a(x_0 - a) - y_0^2}{y_0(x_0 - a) + 2a y_0}$$

$$= \frac{2ax_0 - 2a^2 - (4ax_0)}{x_0 y_0 - a y_0 + 2a y_0}$$

$\because y_0^2 = 4ax_0$ as p lies on $y^2 = 4ax$

$$\tan\theta_1 = \frac{-2a^2 - 2ax_0}{x_0 y_0 + a y_0}$$

$$= \frac{-2a(a+x_0)}{y_0(x_0+a)}$$

$$\tan\theta_1 = -\frac{2a}{y_0} \quad \text{——— (A)}$$

Now

$$\tan\theta_2 = \frac{m_2 - m_1}{1 + m_2 m_1}$$

$$\tan\theta_2 = \frac{0 - \left(\frac{2a}{y_0}\right)}{1 + (0)\left(\frac{2a}{y_0}\right)} = \frac{-\frac{2a}{y_0}}{1}$$

$$\tan\theta_2 = -\frac{2a}{y_0} \quad \text{——— (B)}$$

From (A) and (B)

$$\tan\theta_1 = \tan\theta_2$$

Taking $\tan^{-1}$ on both Sides

$$\theta_1 = \theta_2$$

which is required.

Ellipse

"The Set of all those points in a plane whose sum of the distances from two fixed points remains Constant."

The fixed points are Called Foci

The mid point of Foci is called Centre of Ellipse.

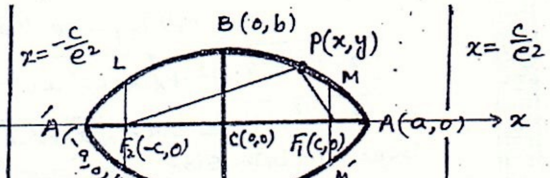
Major Axis

The line segment that joins the two points of Ellipse and passes through Foci is called Major Axis

NOTE: Foci is the plural of Focus

Minor Axis

"The line segment that joins the two points of Ellipse through centre and perpendicular to Major axis is called Minor Axis."



* $A'A$ is major axis of Ellipse having length $2a$.
* $B'B$ is minor axis of Ellipse having length $2b$.
* $F_1(c,0)$ and $F_2(-c,0)$ are Foci.
* $A(a,0)$, $A'(-a,0)$ are called Vertices and $B(0,b)$, $B'(0,-b)$ are Covertices.

Definations.

* The end points of Major axis are called Vertices.
* $L'L$ and $M'M$ are the focal chords through Foci perpendicular to Major axis and are called Latus Rectums or Latus Recta.
* Length of Each Latus rectum is $\frac{2b^2}{a}$.
* L_1 and L_2 are directrices of Ellipse having equations

$L_1: x = \frac{c}{e^2}$ and $L_2: x = \frac{-c}{e^2}$

* The eccentricity of Ellipse is

$e = \frac{c}{a}$ where $0 < e < 1$

So

$L_1: x = \frac{a}{e}$ and $L_2: x = -\frac{a}{e}$

are also directrices of Ellipse.