Eq. of tangent for $A(x_1, y_1)$ is

$$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$$

$$\Rightarrow xx_1 + yy_1 - 2(x+x_1) + 3(y+y_1) + \frac{21}{2} = 0$$

$\because (4,5)$ lies on tangent so must satisfy it.

$$\Rightarrow 4x_1 + 5y_1 - 2(4+x_1) + 3(5+y_1) + \frac{21}{2} = 0$$

$$\Rightarrow 8x_1 + 10y_1 - 4(4+x_1) + 6(5+y_1) + 21 = 0$$

$$\Rightarrow 8x_1 + 10y_1 - 16 - 4x_1 + 30 + 6y_1 + 21 = 0$$

$$\Rightarrow 4x_1 + 16y_1 + 35 = 0 \quad \text{--------} \ (1)$$

Now Equation of tangent for $B(x_2, y_2)$ is

$$xx_2 + yy_2 + g(x+x_2) + f(y+y_2) + c = 0$$

$$\Rightarrow xx_2 + yy_2 - 2(x+x_2) + 3(y+y_2) + \frac{21}{2} = 0$$

$\because (4,5)$ lies on tangent so must satisfy it.

$$\Rightarrow 4x_2 + 5y_2 - 2(4+x_2) + 3(5+y_2) + \frac{21}{2} = 0$$

$$\Rightarrow 8x_2 + 10y_2 - 4(4+x_2) + 6(5+y_2) + 21 = 0$$

$$\Rightarrow 8x_2 + 10y_2 - 16 - 4x_2 + 30 + 6y_2 + 21 = 0$$

$$\Rightarrow 4x_2 + 16y_2 + 35 = 0 \quad \text{--------} \ (2)$$

From (1) and (2) both A and B lies on $4x + 16y + 35 = 0$

So Equation of Chord is $4x + 16y + 35 = 0$

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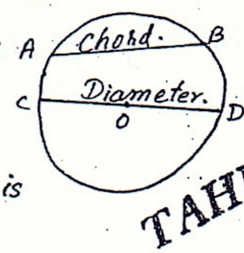
Analytic Problems about Circle

Chord:-

A line segment that joins the two points lying on the circum-ference of the circle is called Chord of Circle.



Diameter:-

The greatest chord of the circle which passes through the Centre of the Circle is called diameter of the Circle.

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From the Figure $\overline{AB}$ is Chord and $\overline{CD}$ is diameter.

* Length of the diameter of the Circle is twice of radius.
* Orthogonal Circles are those circles whose tangents inscribed an angle of $90°$.

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Analytic Problems relating to Circle:

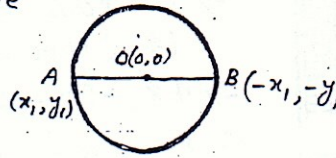
The problems of circle, dicussed analytically i.e. by means of co-ordinates and points are called Analytic problems of Circle.

Some of them are proved below:-

Theorem (1):- Length of the diameter of a circle $x^2+y^2=a^2$ is "2a".

Proof:- Let AOB be the diameter of the circle

$$x^2+y^2=a^2 \quad \text{——— (1)}$$



Let A has co-ordinates (x_1, y_1).

The equation of AOB by using two points form

$$\frac{y-0}{y_1-0} = \frac{x-0}{x_1-0} \Rightarrow y = \frac{x}{x_1}y_1 \quad \text{——— (2)}$$

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Putting this value in (1), we have

$$x^2+\frac{x^2}{x_1^2}y_1^2=a^2 \Rightarrow x^2x_1^2+x^2y_1^2=a^2x_1^2$$

$$\Rightarrow x^2(x_1^2+y_1^2)=a^2x_1^2 \qquad \because x_1^2+y_1^2=a^2 \text{ as A lies on (1)}$$

$$\Rightarrow x^2a^2=a^2x_1^2 \Rightarrow x^2=x_1^2 \Rightarrow x=\pm x_1$$

If $x=x_1 \Rightarrow y=y_1$ so are co-ordinates of A

If $x=-x_1 \Rightarrow y=-y_1$ so are co-ordinates of B

Now length of diameter $\overline{AB} = \sqrt{[x_1-(-x_1)]^2+[y_1-(-y_1)]^2}$

$$= \sqrt{(2x_1)^2+(2y_1)^2} = \sqrt{4x_1^2+4y_1^2}$$

$$= \sqrt{4(x_1^2+y_1^2)} = \sqrt{4a^2} \qquad \because x_1^2+y_1^2=a^2$$

$$= |2a| = 2a$$

Thus length of diameter $\overline{AB} = 2a$ where a is radial length.

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* Perpendicular dropped from centre of circle on a chord bisect the chord.
* Congruent chords of a circle are equidistant from its centre.
* Tangents drawn from external point to a circle are equal in length.

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* Tangent Circles:-

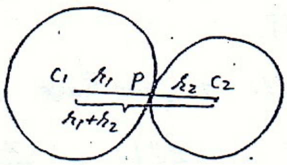
"Two circles are said to be tangent circles if they intersect each other at just one common point."

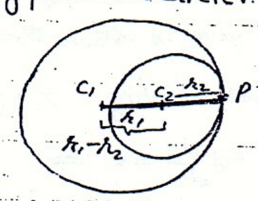
There are two types of tangent circles.

(i) When they touch externally at one common point.

(ii) When they touch internally at one common point.

(iii) Two tangents can be drawn from any point to a circle.





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* Distance between the centres of externally touching circles is equal to the sum of radii of circles. i.e. $|C_1C_2| = r_1+r_2$

* Distance between the centres of internally touching circles is equal to the difference of radii of circles. i.e. $|C_1C_2| = r_1-r_2$ if $r_1>r_2$

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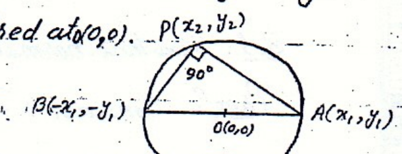
Theorem (2):- An angle inscribed in semicircle is a right angle.

Proof:- Let $x^2+y^2=a^2$ be the circle centred at $(0,0)$.

Let AOB be the diameter of circle with end points $A(x_1,y_1)$ and $B(-x_1,-y_1)$.



Suppose $P(x_2,y_2)$ be any point on $x^2+y^2=a^2$

To show that $m\angle APB=90°$, we have to show $\overline{AP}\perp\overline{BP}$

Slope of $\overline{AP}$ $(m_1) = \dfrac{y_2-y_1}{x_2-x_1}$ and Slope of $\overline{BP}$ $(m_2) = \dfrac{y_2+y_1}{x_2+x_1}$

Consider $m_1\cdot m_2 = \dfrac{(y_2-y_1)}{(x_2-x_1)}\cdot\dfrac{(y_2+y_1)}{(x_2+x_1)} = \dfrac{y_2^2-y_1^2}{x_2^2-x_1^2}$

$\because$ A, P lies on circle so

$x_1^2+y_1^2=a^2$ —(1)

$x_2^2+y_2^2=a^2$ —(2)

(2)−(1) $\Rightarrow$ $x_2^2-x_1^2+y_2^2-y_1^2=0$

$\Rightarrow (y_2^2-y_1^2) = -(x_2^2-x_1^2)$

$m_1\cdot m_2 = \dfrac{-(x_2^2-x_1^2)}{(x_2^2-x_1^2)} = -1$

Thus $m_1\cdot m_2=-1$

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So $\overline{AP}\perp\overline{BP}$

Thus $m\angle APB=90°$ (Proved)

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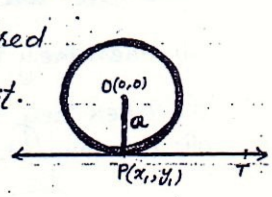
<u>Theorem</u> ③ :- The tangent to a circle at any point of the circle is perpendicular to the radial segment at that point.

<u>Proof</u>:- Let $x^2+y^2=a^2$ be the circle centred at $O(0,0)$. Let $P(x_1, y_1)$ be any point on it.



Let $\overline{PT}$ be the tangent to the circle.

We have to show that $\overline{OP} \perp \overline{PT}$.

Differentiating $x^2+y^2=a^2$ w.r.t. x, we have

$$2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-x}{y} \qquad \left.\frac{dy}{dx}\right|_{at\,P} = \frac{-x_1}{y_1}$$

Thus (m_1) Slope of tangent at $P = \frac{-x_1}{y_1}$

(m_2) Slope of radial segment $(\overline{OP}) = \frac{y_1-0}{x_1-0} = y_1/x_1$

Consider $m_1 \cdot m_2 = \frac{-x_1}{y_1} \cdot \frac{y_1}{x_1} = -1$

Thus $m_1 \cdot m_2 = -1 \Rightarrow \overline{PT} \perp \overline{OP}$ which is required.

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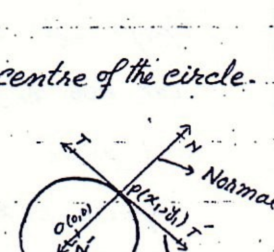
<u>Exercise : 6.3</u>

<u>Q.1</u>: Show that normal lines of a circle pass through centre of the circle.

<u>Proof</u>:- Consider $x^2+y^2=a^2$ —① be the circle centred at origion. let $P(x_1,y_1)$ be any point on it so $x_1^2+y_1^2=a^2$ —②



Diff. ① w.r.t. x, we have $2x+2y\frac{dy}{dx}=0$

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$$\Rightarrow \frac{dy}{dx} = \frac{-x}{y} \quad \text{or} \quad \left.\frac{dy}{dx}\right|_{at\,P} = \frac{-x_1}{y_1}$$

so Slope of tangent at $P = \frac{-x_1}{y_1}$ also slope of normal at $P = y_1/x_1$

Now Eq. of normal is $y-y_1 = \frac{y_1}{x_1}(x-x_1) \Rightarrow x_1(y-y_1) = y_1(x-x_1)$

$$\Rightarrow x_1y - x_1y_1 = xy_1 - x_1y_1 \Rightarrow x_1y = xy_1 \text{ —③}$$

Clearly ③ is satisfied by $O(0,0)$ so normal lines of the circle pass through origion which is centre of circle.

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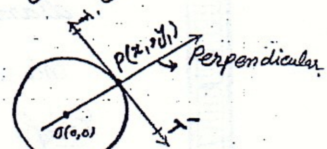
Q.2: Prove that the straight line drawn from the centre of a circle perpendicular to a tangent pass through point of tangency.

Proof:- Consider the circle $x^2+y^2=a^2$ —— ①



Let $P(x_1, y_1)$ be any point on it so

$x_1^2+y_1^2=a^2$ —— ②

Diff. ① w.r.t. x, we have $2x+2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{x}{y}$

Slope of tangent at $P=\frac{-x_1}{y_1}$

Slope of perpendicular line $=\frac{-1}{\left(\frac{-x_1}{y_1}\right)}=y_1/x_1$

Eq of perpendicular through centre is $y-0=\frac{y_1}{x_1}(x-0)$

$\Rightarrow$ $x_1 y=x y_1$ which is satisfied by $P(x_1, y_1)$

so it passes through tangency point.

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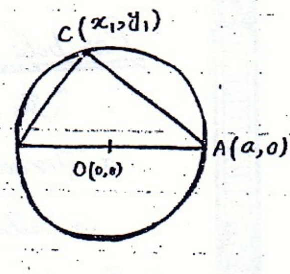
Q.3:- Prove that mid point of the hypotenuse of a right triangle is circumcentre of the triangle.

Proof:- Consider a circle $x^2+y^2=a^2$ —— ①



Let $A(a,0)$, $B(-a,0)$, $C(x_1,y_1)$ be the vertices of right triangle with $m\angle C=90°$ and $\overline{AB}$ being hypotenuse.

$O(0,0)$ being mid point of hypotenuse.

(m_1) Slope of $\overline{AC}=\frac{y_1-0}{x_1-a}=\frac{y_1}{x_1-a}$

(m_2) Slope of $\overline{BC}=\frac{y_1-0}{x_1+a}=\frac{y_1}{x_1+a}$

$\because m\angle C=90° \Rightarrow \overline{AC}\perp\overline{BC}$ so $m_1 m_2=-1 \Rightarrow \left(\frac{y_1}{x_1-a}\right)\left(\frac{y_1}{x_1+a}\right)=-1$

$\Rightarrow y_1^2=-(x_1^2-a^2) \Rightarrow y_1^2=-x_1^2+a^2 \Rightarrow a^2=x_1^2+y_1^2$ —— ②

Now $|\overline{OA}|=\sqrt{(a-0)^2+(0-0)^2}=a$

$|\overline{OB}|=\sqrt{(-a-0)^2+(0-0)^2}=a$

$|\overline{OC}|=\sqrt{(x_1-0)^2+(y_1-0)^2}=\sqrt{x_1^2+y_1^2}=\sqrt{a^2}=a$

$\because |\overline{OA}|=|\overline{OB}|=|\overline{OC}|$ so O is the circumcentre of ΔABC.

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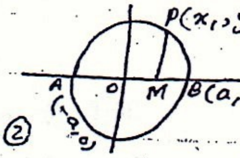
Q.4:- Prove that perpendicular dropped from a point of Circle on the diameter is mean (Geometric) proportional between the segments into which it divides the diameter.



Proof:- Consider a circle $x^2+y^2=a^2$ — ①

Let $P(x_1, y_1)$ be any point on it so $x_1^2+y_1^2=a^2$ — ②

Let $A(-a,0)$, $B(a,0)$ be the end points of diameter $\overline{AB}$

Let $\overline{PM}$ be perpendicular on $\overline{AB}$ so $M(x_1, 0)$

Now $|\overline{AM}| = \sqrt{(x_1+a)^2+(0-0)^2} = x_1+a$

$|\overline{BM}| = \sqrt{(a-x_1)^2+(0-0)^2} = a-x_1$ and $|PM| = \sqrt{(x_1-x_1)^2+(y_1-0)^2} = y_1$

Consider $|\overline{AM}|.|\overline{BM}| = (a+x_1)(a-x_1) = a^2-x_1^2 = y_1^2 = |PM|^2$ $(\because a^2-x_1^2=y_1^2)$

$|\overline{AM}|\cdot|\overline{BM}| = |PM|\cdot|PM| \Rightarrow \dfrac{|\overline{AM}|}{|\overline{PM}|} = \dfrac{|\overline{PM}|}{|\overline{BM}|}$

so $|\overline{PM}|$ is mean proportional to $|\overline{AM}|$ and $|\overline{BM}|$. (Proved)

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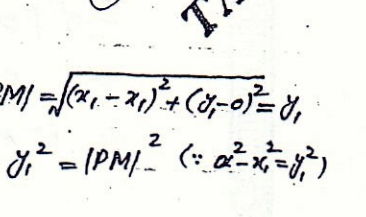
Parabola:-

"The set of all those points whose distance from a fixed point and a fixed line remains same is called parabola."



*Fixed point is called Focus and Fixed line is called Directrix.

*Fixed point does not lie on the fixed line (Directrix).

Coordinates of focus are $F(a,0)$ and Eq of directrix is $x+a=0$

The line through focus and perpendicular to directrix is called Axis of parabola. The point where axis meet parabola is called Vertex.

The line that joins two points of parabola is called Chord.

The chords which passes through focus are called Focal Chords.

A focal chord which is perpendicular to the axis of parabola is called Latus Reactum. (LL') having length $|LL'| = 4a$.

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