Thus the Partialization is

$$\frac{x^2-2x+3}{x^4+x^2+1} = \frac{1x+\frac{5}{2}}{x^2+x+1} + \frac{-1x+\frac{1}{2}}{x^2-x+1}$$

$$\frac{x^2-2x+3}{x^4+x^2+1} = \frac{2x+5}{2(x^2+x+1)} + \frac{1-2x}{2(x^2-x+1)}$$

These are the Partial fractions.

Type-IV In Case of repeated quadratic factors which are not reducible to linear are written as:

$$\frac{x^2}{(4x^2+5)^2} = \frac{Ax+B}{4x^2+5} + \frac{Cx+D}{(4x^2+5)^2} + \dots \text{etc.}$$

Exercise 5.4

Q.1 $\dfrac{x^3+2x+2}{(x^2+x+1)^2}$

$$\frac{x^3+2x+2}{(x^2+x+1)^2} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{(x^2+x+1)^2}$$

Multiplying both Sides by $(x^2+x+1)^2$

$x^3+2x+2 = Ax(x^2+x+1)+B(x^2+x+1)+Cx+D$

$x^3+2x+2 = A(x^3+x^2+x)+B(x^2+x+1)+Cx+D$

Comparing the Coefficients of

$x^3 \Rightarrow \boxed{1 = A}$

$x^2 \Rightarrow 0 = A+B$

$B = -A \Rightarrow \boxed{B=-1}$

$x \Rightarrow 2 = A+B+C$

$C = 2-A-B \Rightarrow C = 2-1+1$

$\boxed{C = 2}$

Const $\Rightarrow 2 = B+D$

$D = 2-B$

$D = 2+1 = 3$

$\boxed{D = 3}$

Hence the Partial fractions are

$$\frac{x^3+2x+2}{(x^2+x+1)^2} = \frac{x-1}{x^2+x+1} + \frac{2x+3}{(x^2+x+1)^2}$$

Q.2 $\dfrac{x^2}{(x^2+1)^2(x-1)}$

$$\frac{x^2}{(x-1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

Multiplying by $(x^2+1)^2(x-1)$ on both Sides.

$x^2 = A(x^2+1)^2+Bx(x-1)(x^2+1)+C(x-1)(x^2+1)$

$+Dx(x-1)+E(x-1)$

$x^2 = A(x^4+2x^2+1)+B(x^4-x^3+x^2-x)+D(x^2-x)$

$+C(x^3-x^2+x-1)+E(x-1)$

Put $x-1=0 \Rightarrow x=1$

$1 = A(1+2+1)+(0)B+(0)C+(0)D+(0)E$

$\boxed{A = \frac{1}{4}}$

Comparing the Coefficients of

$x^4 \Rightarrow 0 = A+B$

$B = -A \Rightarrow \boxed{B = -\frac{1}{4}}$

$x^3 \Rightarrow 0 = -B+C \Rightarrow C = B$

$\boxed{C = -\frac{1}{4}}$

$x^2 \Rightarrow 1 = 2A + B + D - C$

$D = 1 + 2A - B + C$

$D = 1 - \frac{\cancel{2}^{1}}{\cancel{4}_{2}} + \frac{1}{4} - \frac{1}{4} = \frac{2-1}{2} = \frac{1}{2}$

$\boxed{D = 1/2}$

$x \Rightarrow 0 = -B + C - D + E$

$E = B - C + D$

$E = -\frac{1}{4} + \frac{1}{4} + \frac{1}{2}$

$\boxed{E = \frac{1}{2}}$

Thus the Partialization is

$$\frac{x^2}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} + \frac{-\frac{1}{4}x + (-\frac{1}{4})}{x^2+1} + \frac{\frac{1}{2}x + \frac{1}{2}}{(x^2+1)^2}$$

$$\frac{x^2}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} - \frac{x+1}{4(x^2+1)} + \frac{x+1}{2(x^2+1)^2}$$

Q.3 $\dfrac{2x-5}{(x^2+2)^2(x-2)}$

$$\frac{2x-5}{(x-2)(x^2+2)^2} = \frac{A}{x-2} + \frac{Bx+C}{x^2+2} + \frac{Dx+E}{(x^2+2)^2}$$

Multiplying both Sides by $(x-2)(x^2+2)^2$

$2x-5 = A(x^2+2)^2 + Bx(x-2)(x^2+2)$
$\quad + C(x^2+2)(x-2) + Dx(x-2) + E(x-2)$

$2x-5 = A(x^4+4x^2+4) + B(x^4 - 2x^3 + 2x^2 - 4x)$
$\quad + C(x^3 - 2x^2 + 2x - 4) + D(x^2 - 2x) + E(x-2)$

Put $x - 2 = 0 \Rightarrow x = 2$

$4 - 5 = A(36) + (0)B + (0)C + (0)D + (0)E$

$\boxed{A = \frac{-1}{36}}$

Comparing the Coefficients of

$x^4 \Rightarrow 0 = A + B \Rightarrow B = -A$

$\boxed{B = \frac{1}{36}}$

$x^3 \Rightarrow 0 = -2B + C$

$C = 2B = \frac{2}{36} = \frac{1}{18}$

$\boxed{C = \frac{1}{18}}$

$x^2 \Rightarrow 0 = 4A + 2B - 2C + D$

$D = 2C - 4A - 2B$

$D = \frac{2}{18} + \frac{\cancel{4}^{2}}{\cancel{36}_{18}} - \frac{\cancel{2}}{\cancel{36}_{18}}$

$D = \frac{2+2-1}{18} = \frac{\cancel{3}^{1}}{\cancel{18}_{6}}$

$\boxed{D = \frac{1}{6}}$

$x \Rightarrow 2 = -4B + 2C - 2D + E$

$E = 2 + 4B - 2C + 2D$

$E = 2 + \frac{\cancel{4}}{\cancel{36}_{9}} - \frac{\cancel{2}}{\cancel{18}_{9}} + \frac{\cancel{2}}{\cancel{6}_{3}}$

$= \frac{18+1-1+3}{9}$

$E = \frac{\cancel{21}}{\cancel{9}} \frac{7}{3} \Rightarrow \boxed{E = \frac{7}{3}}$

The Partial Fractions are:

$$\frac{2x-5}{(x-2)(x^2+2)^3} = \frac{-1}{36(x-2)} + \frac{\frac{1}{36}x + \frac{1}{18}}{x^2+2} + \frac{\frac{1}{6}x + \frac{7}{3}}{(x^2+2)^2}$$

$$= \frac{-1}{36(x-2)} + \frac{x+2}{36(x^2+2)} + \frac{(x+14)}{6(x^2+2)^2}$$

Q.4 $\dfrac{8x^2}{(x^2+1)^2(1-x^2)}$

$= \dfrac{8x^2}{(1-x)(1+x)(x^2+1)^2}$

$\dfrac{8x^2}{(1-x)(1+x)(x^2+1)^2} = \dfrac{A}{1-x} + \dfrac{B}{1+x} + \dfrac{Cx+D}{x^2+1}$

$+ \dfrac{Ex+F}{(x^2+1)^2}$

$8x^2 = A(1+x)(x^2+1)^2 + B(1-x)(1+x^2)^2$

$+Cx(1-x^2)(x^2+1) + D(1-x^2)(x^2+1)$

$+Ex(1-x^2) + F(1-x^2)$

$8x^2 = A(1+x)(x^4+2x^2+1) + B(1-x)(1+x^4+2x^2)$

$+C(x-x^5) + D(1-x^4) + E(x-x^3) + F(1-x^2)$

$8x^2 = A(x^5+2x^3+x^4+2x^2+x+1) + B(x^4-x^5$

$+2x^2+1-2x^3-x) + C(x-x^5) + D(1-x^4)$

$+E(x-x^3) + F(1-x^2)$

Put $1-x=0 \Rightarrow 1=x$

$8 = A(1+2+1+2+1+1) + (0)B + (0)C + (0)D$

$+(0)E + (0)F$

$8 = 8A + 0 \Rightarrow \boxed{A=1}$

Put $x+1=0 \Rightarrow x=-1$

$8 = (0)A + B(-2)(2)^2 + (0)C + (0)D + (0)E$
$+(0)F$

$8 = -8B \Rightarrow \boxed{B=-1}$

Comparing the Coefficients of

$x^5 \Rightarrow 0 = A - B - C$

$C = A - B = 1+1 = 2$

$\boxed{C=2}$

$x^4 \Rightarrow 0 = A + B - D$

$D = A + B = 1 - 1 = 0$

$\boxed{D=0}$

$x^3 \Rightarrow 0 = 2A - 2B - E$

$E = 2(1) - 2(-1) = 2+2$

$\boxed{E=4}$

$x^2 \Rightarrow 8 = 2A + 2B - F$

$F = 2A + 2B - 8$

$F = 2(1) + 2(-1) - 8 = 2 - 2 - 8$

$F = -8 \Rightarrow \boxed{F=-8}$

Thus the Partialization is

$\dfrac{8x^2}{(1-x^2)(x^2+1)^2} = \dfrac{1}{(1-x)} - \dfrac{1}{(1+x)} + \dfrac{2x+0}{(x^2+1)}$

$+ \dfrac{4x-8}{(x^2+1)^2}$

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Thus

$\dfrac{8x^2}{(1-x^2)(1+x^2)^2} = \dfrac{1}{(1-x)} - \dfrac{1}{(1+x)} + \dfrac{2x}{x^2+1}$

$+ \dfrac{4x-8}{(x^2+1)^2}$

Q.5 $\dfrac{4x^4+3x^3+6x^2+5x}{(x-1)(x^2+x+1)^2}$

This is proper fraction so

$\dfrac{4x^4+3x^3+6x^2+5x}{(x-1)(x^2+x+1)^2} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+x+1}$

$+ \dfrac{Dx+E}{(x^2+x+1)^2}$

Multiplying both Sides by $(x-1)(x^2+x+1)^2$

$$4x^4+3x^3+6x^2+5x = A(x^2+x+1)^2+Bx(x-1)(x^2+x+1)$$
$$+C(x-1)(x^2+x+1)+Dx(x-1)+E(x-1)$$
$$4x^4+3x^3+6x^2+5x = A(x^4+3x^2+2x^3+2x+1)+B(x^4-x)$$
$$+C(x^3-1)+D(x^2-x)+E(x-1)$$

Put $x-1=0 \Rightarrow x=1$

$$4+3+6+5 = A(1+3+2+2+1)+(0)B+(0)C+(0)D+(0)E$$
$$18=9A \Rightarrow \boxed{A=2}$$

Comparing the Coefficients of

$$x^4 \Rightarrow 4=A+B \Rightarrow B=4-A$$
$$B=4-2 \Rightarrow \boxed{B=2}$$
$$x^3 \Rightarrow 3=2A+C \Rightarrow C=3-2A$$
$$C=3-2(2) \Rightarrow C=3-4 \Rightarrow \boxed{C=-1}$$
$$x^2 \Rightarrow 6=3A+D \Rightarrow D=6-3A$$
$$D=6-3(2) \Rightarrow D=6-6 \Rightarrow \boxed{D=0}$$
$$x \Rightarrow 5=2A-B-D+E$$
$$E=5-2A+B+D$$

$$E=5-2(2)+2+0$$
$$E=5-4+2 \Rightarrow \boxed{E=3}$$

Thus the Partial Fractions are

$$\frac{4x^4+3x^3+6x^2+5x}{(x-1)(x^2+x+1)^2} = \frac{2}{x-1}+\frac{2x-1}{(x^2+x+1)}+\frac{(0)x+3}{(x^2+x+1)^2}$$
$$=\frac{2}{x-1}+\frac{2x-1}{(x^2+x+1)}+\frac{3}{(x^2+x+1)^2}$$

Q.6 $\dfrac{2x^4-3x^3-4x}{(x^2+2)^2(x+1)^2}$

$$\frac{2x^4-3x^3-4x}{(x+1)^2(x^2+2)^2} = \frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{Cx+D}{x^2+2}+\frac{Ex+F}{(x^2+2)^2}$$

Multiplying both Sides by $(x+1)^2(x^2+2)^2$

$$2x^4-3x^3-4x = A(x+1)(x^2+2)^2+B(x^2+2)^2$$
$$+Cx(x+1)^2(x^2+2)+D(x^2+2)(x+1)^2$$
$$+Ex(x+1)^2+F(x+1)^2$$
$$2x^4-3x^3-4x = A(x+1)(x^4+4x^2+4)+B(x^4+4x^2+4)$$
$$+Cx(x^2+2x+1)(x^2+2)+D(x^2+2)(x^2+2x+1)$$
$$+Ex(x^2+2x+1)+F(x^2+2x+1)$$
$$\Rightarrow 2x^4-3x^3-4x = A(x^5+x^4+4x^3+4x^2+4x$$
$$+4)+B(x^4+4x^2+4)+C(x^5+2x^4+3x^3+4x^2$$
$$+2x)+D(x^4+2x^3+3x^2+4x+2)$$
$$+E(x^3+2x^2+x)+F(x^2+2x+1)$$

Put $x+1=0 \Rightarrow x=-1$

$$2-3(-1)-4(-1) = (0)A+B(1+4+4)$$
$$+(0)C+(0)D+(0)E+(0)F$$
$$9=9B \Rightarrow \boxed{B=1}$$

Comparing the Coefficients of

$$x^5 \Rightarrow 0=A+C \quad \text{— (1)}$$
$$x^4 \Rightarrow 2=A+B+2C+D$$
$$A+2C+D=2-1$$
$$A+2C+D=1 \quad \text{— (2)}$$

$x^3 \Rightarrow -3 = 4A+3C+2D+E$ ___ (3)

$x^2 \Rightarrow 0 = 4A+4B+4C+3D+2E+F$ ___ (4)

$x \Rightarrow -4 = 4A+2C+4D+E+2F$ ___ (5)

Constt $\Rightarrow 0 = 4A+4B+2D+F$ ___ (6)

From (2)

$A+2C+D+B = 2$

$A+C+C+D = 2-1 \quad (\because B=1)$

$(A+C)+C+D = 1$

From (1) $A+C=0$

$0+C+D=1$

$C+D=1$ ___ (7)

From (3) we have

$4A+3C+2D+E=-3$

$4A+4C+2C+2D-3C+E=-3$

$4(A+C)+2(C+D)-3C+E=-3$

$\because A+C=0 \wedge C+D=1$

$4(0)+2(1)-3C+E=-3$

$2-3C+E=-3$

$3C-E=3+2$

$3C-E=5$ ___ (8)

From (4), we have

$4A+4C+3D+2E+F=-4B$

$4(A+C)+3D+2E+F=-4 \quad (\because B=1)$

$4(0)+3D+2E+F=-4$

$3D+2E+F=-4$ ___ (9)

From (5), we have

$4A+2C+4D+E+2F=-4$

$4A+4C+4C+4D-6C+E+2F=-4$

$4(A+C)+4(C+D)-6C+E+2F=-4$

$0+4-6C+E+2F=-4$

$-6C+E+2F=-8$ ___ (10)

From (6), we have

$4A+2D+F=-4B$

$4A+2D+F=-4 \quad (\because B=1)$

$\because A+C=0 \Rightarrow A=-C$

$-4C+2D+F=-4$

$-4C-4D+6D+F=-4$

$-4(C+D)+6D+F=-4$

$-4(1)+6D+F=-4$

$6D+F=-4+4$

$6D+F=0$ ___ (11)

Multiplying (9) by 2

$6D+4E+\cancel{2F}=-\cancel{8}$ ___ (9)

$\mp 6C \pm E \pm \cancel{2F} = \mp \cancel{8}$ ___ (10)

$6C+6D+3E=0$

$6(C+D)+3E=0$

$6(1)+3E=0$

$3E=-6 \Rightarrow \boxed{E=-2}$

From (8) $3C-E=5$

$3C=5+E \Rightarrow 3C=5-2$

$3C=3 \Rightarrow \boxed{C=1}$

Now

$$A+C=0 \Rightarrow A=-C \Rightarrow \boxed{A=-1}$$

Also $C+D=1 \Rightarrow D=1-C \Rightarrow D=1-1=0 \Rightarrow \boxed{D=0}$

$\because 6D+F=0 \Rightarrow F=-6D \Rightarrow \boxed{F=0}$ $(\because D=0)$

Thus partial Fractions are

$$\frac{2x^4-3x^3-4x}{(x+1)^2(x^2+2)^2} = \frac{1}{(x+1)^2} - \frac{1}{(x+1)} + \frac{x+0}{x^2+2} + \frac{-2x+0}{(x^2+2)^2}$$

$$= \frac{1}{(x+1)^2} - \frac{1}{x+1} + \frac{x}{x^2+2} - \frac{2x}{(x^2+2)^2}$$