(17)

LINEAR PROGRAMMING

"The mathematical technique which deals with the problems to get Optimization (maximization or minimization) of linear functions of variables is called Linear Programming."

OBJECTIVE FUNCTION:-

"A function which is to be maximized or minimized is called Objective function."

OPTIMAL SOLUTION:-

"The feasible Solution which maximizes or minimizes the objective function is called Optimal Solution."

Theorem of Linear Programming:-

"The maximum or minimum values of an objective function occure at the corner points of the feasible Solution region."

CRITERIA USED TO FIND OPTIMAL SOLUTION:-

Following steps should be adopt to find optimal Solution.

(i) Graph all the Constraints of inequalities to find feasible region.

(ii) Find all the Corner points of the feasible region.

(iii) Evaluate the objective function at every Corner point to find out the optimal Solution.

Tahir. Mahmood

(Math)

EXERCISE: 5.3

(18)

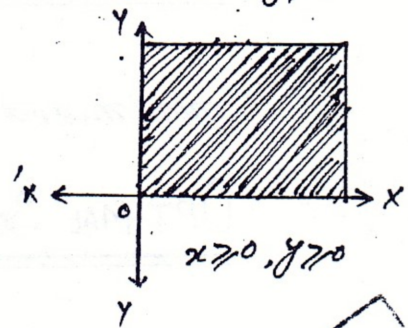
Q.1 Maximize $f(x,y) = 2x+5y$ subject to the Constraints

$2y - x \le 8$ $\quad x - y \le 4$ $\quad x \ge 0, y \ge 0$

The associated Equations are

$2y - x = 8$ $\qquad x - y = 4$

Firstly: $x \ge 0, y \ge 0$



$x \ge 0, y \ge 0$

To get x, y intercepts:

let $x = 0$ $\quad y = 0$ and $x = 0$ $\quad y = 0$.

$\Rightarrow y = 4$ $\quad \Rightarrow x = -8$ and $\Rightarrow y = -4$, $\Rightarrow x = 4$

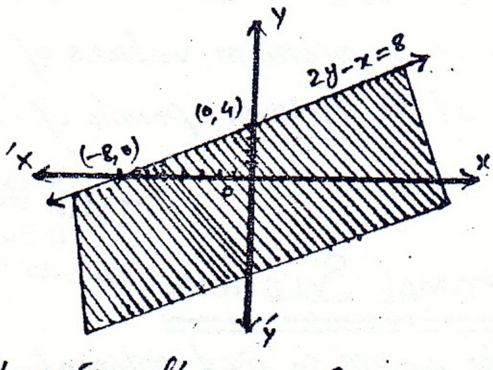
Thus ordered pairs are:

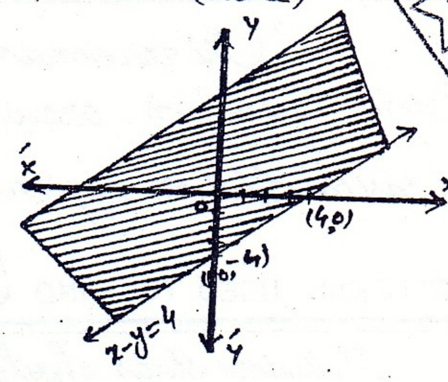
$(0,4), (-8,0)$ and $(0,-4), (4,0)$

Let us check whether $(0,0)$ is in Solution region or not:

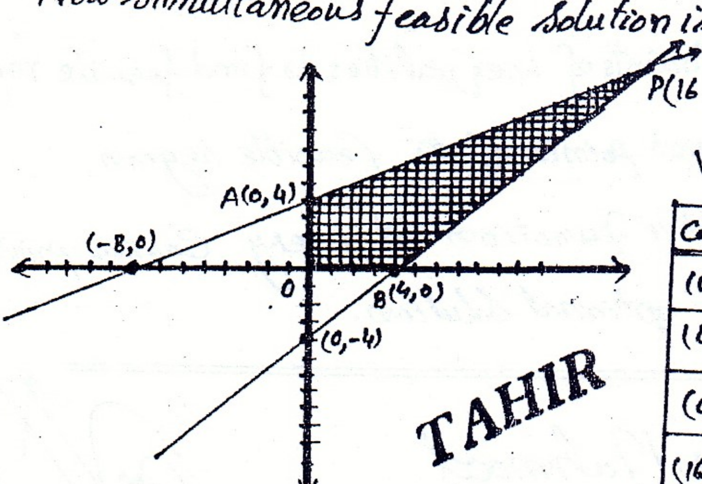
$2(0) - (0) \le 8$ and $(0) - (0) \le 4$

$\Rightarrow 0 \le 8$ (True) $\qquad 0 \le 4$ (True)





Now Simultaneous feasible Solution is:



The Corner points are $(0,0), (0,4), (4,0), (16,12)$

Value of $f(x,y)$ at Corner points:

Corner Pts.	$f(x,y) = 2x+5y$
$(0,0)$	$f(0,0) = 2(0)+5(0) = 0$
$(4,0)$	$f(4,0) = 2(4)+5(0) = 8$
$(0,4)$	$f(0,4) = 2(0)+5(4) = 20$
$(16,12)$	$f(16,12) = 2(16)+5(12) = 92$

Solving $2y - x = 8$ and $x - y = 4$

$x = 16$, $y = 12$ $\Rightarrow$ Pt$(16,12)$

The maximum value at $(16,12)$.

(19)

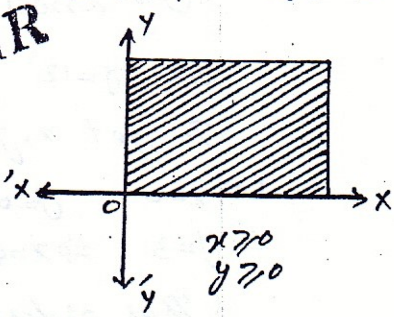
Q.2:- Maximize $f(x,y) = x+3y$ Subject to Constraints

$2x+5y \le 30 \qquad 5x+4y \le 20 \qquad x \ge 0,\ y \ge 0$

The associated Equations are:

$2x+5y = 30 \qquad 5x+4y = 20$

The graph of $x \ge 0, y \ge 0$ is



To get x, y intercepts:

Let $x=0$ $\quad y=0$ $\quad$ and $\quad x=0$ $\quad y=0$

$\Rightarrow y=6 \quad \Rightarrow x=15 \quad$ and $\Rightarrow y=5 \quad \Rightarrow x=4$

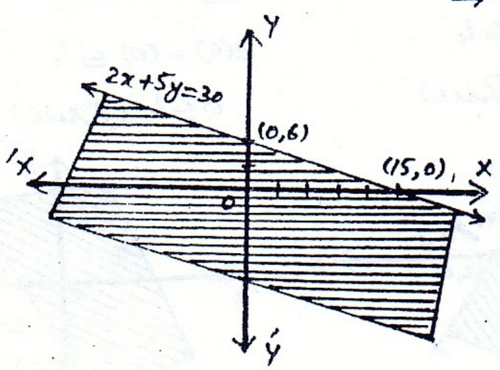
The ordered pairs are:

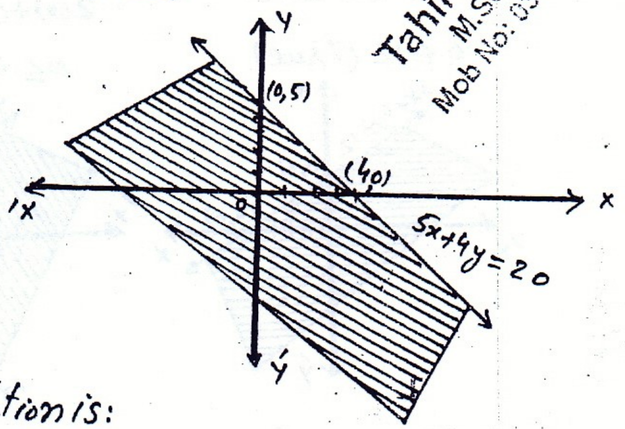
$(0,6), (15,0)$ and $(0,5), (4,0)$

Now Let's check whether $(0,0)$ is in Solution region or not:

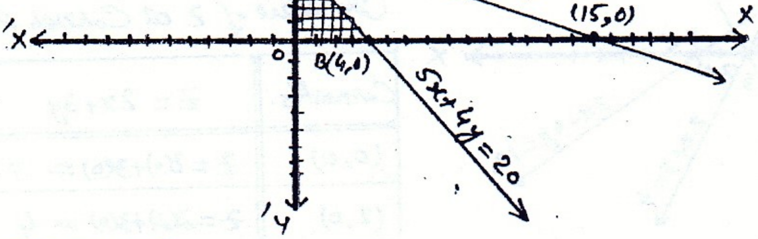
$2(0)+5(0) \le 30$ and $5(0)+4(0) \le 20$

$\Rightarrow 0 \le 30$ (True) $\qquad \Rightarrow 0 \le 20$ (True)





The feasible Simultaneous solution is:



Thus Corner points are:

$(0,0), (0,5), (4,0)$

Value of $f(x,y)$ at Corner points:

Corner Pts.	$f(x,y) = x+3y$
$(0,0)$	$f(0,0) = 0+3(0) = 0$
$(4,0)$	$f(4,0) = 4+3(0) = 4$
$(0,5)$	$f(0,5) = 0+3(5) = 15$

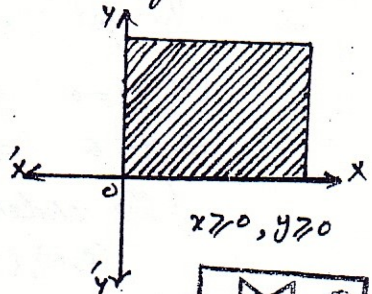
Thus $f(x,y)$ has maximum value 15 at the Corner point $(0,5)$.

Q.3 Maximize $Z = 2x+3y$ Subject to the Constraints: (20)

$3x+4y \le 12$, $2x+y \le 4$, $2x-y \le 4$, $x \ge 0$, $y \ge 0$ (20)

The associated Equations are:

Firstly



$3x+4y=12$, $2x+y=4$, $2x-y=4$

To get x, y intercepts

Let $x=0$ $\Rightarrow y=3$; $y=0$ $\Rightarrow x=4$, $x=0$ $\Rightarrow y=4$; $y=0$ $\Rightarrow x=2$, $x=0$ $\Rightarrow y=-4$; $y=0$ $\Rightarrow x=2$

Thus ordered pairs are:

$(0,3), (4,0)$ and $(0,4), (2,0)$ and $(0,-4), (2,0)$

Let's check whether $(0,0)$ is in solution region or not:

$3(0)+4(0) \le 12$

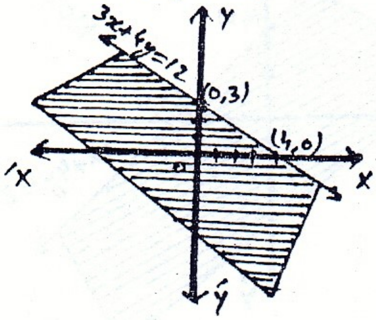
$0 \le 12$ (True)

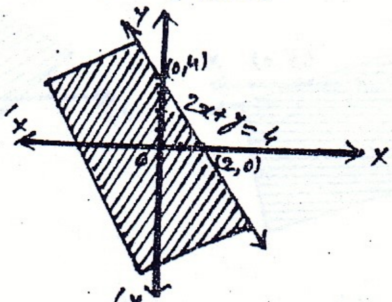
$2(0)+(0) \le 4$

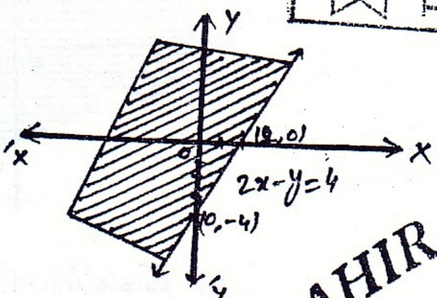
$0 \le 4$ (True)

$2(0)-(0) \le 4$

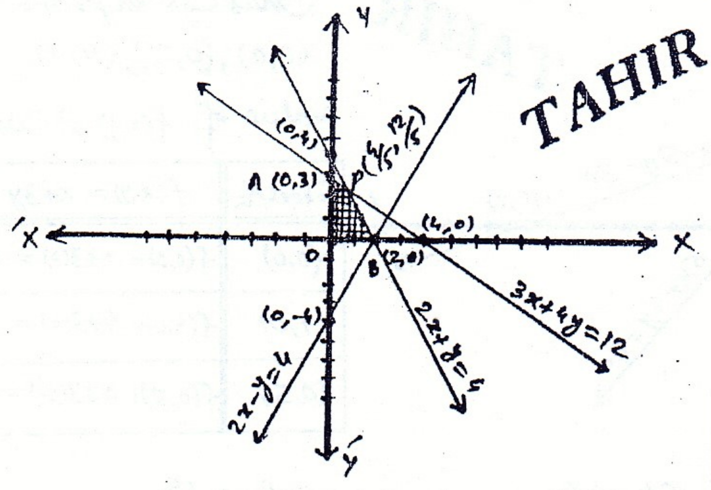
$0 \le 4$ (True)







The Simultaneous feasible solution is:



The Corner points are given by:

$(0,0), (2,0), (0,3), (\frac{4}{5}, \frac{12}{5})$

The value of Z at Corner points are:

Corner Pts.	$Z = 2x+3y$
$(0,0)$	$Z = 2(0)+3(0) = 0$
$(2,0)$	$Z = 2(2)+3(0) = 4$
$(0,3)$	$Z = 2(0)+3(3) = 9$
$(\frac{4}{5}, \frac{12}{5})$	$Z = 2(\frac{4}{5})+3(\frac{12}{5}) = \frac{44}{5}$

Solving $3x+4y=12$ & $2x+y=4$

$x = \frac{4}{5}$ $y = \frac{12}{5}$

Pt $(\frac{4}{5}, \frac{12}{5})$

The maximum value of Z is 9 at the Corner point $(0,3)$

Q.4:- Minimize $Z = 2x + y$ Subject to the Constraints

$x + y \geq 3$ $\quad 7x + 5y \leq 35$ $\quad x \geq 0, y \geq 0$

The associated Equations are:

$x + y = 3$ $\quad 7x + 5y = 35$

To get x, y intercepts

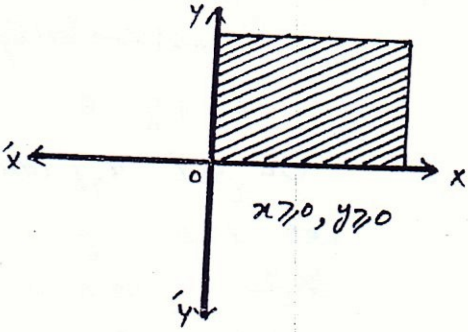
Let $x = 0$ $\quad y = 0$ $\quad$ and $\quad x = 0$ $\quad y = 0$

$\Rightarrow y = 3$ $\quad \Rightarrow x = 3$ $\quad$ and $\Rightarrow y = 7$ $\quad \Rightarrow x = 5$

The ordered pairs are:

$(0,3), (3,0)$ and $(0,7), (5,0)$

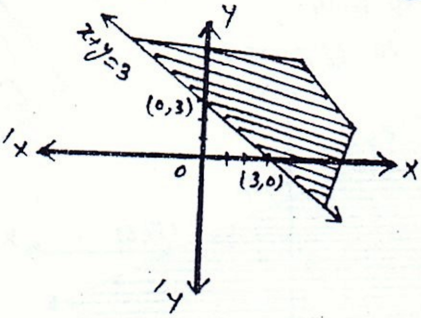
The graph of $x \geq 0, y \geq 0$ is

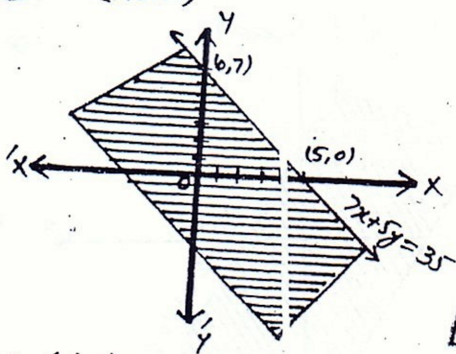


Let's Check whether $(0,0)$ is in the Solution region or not:

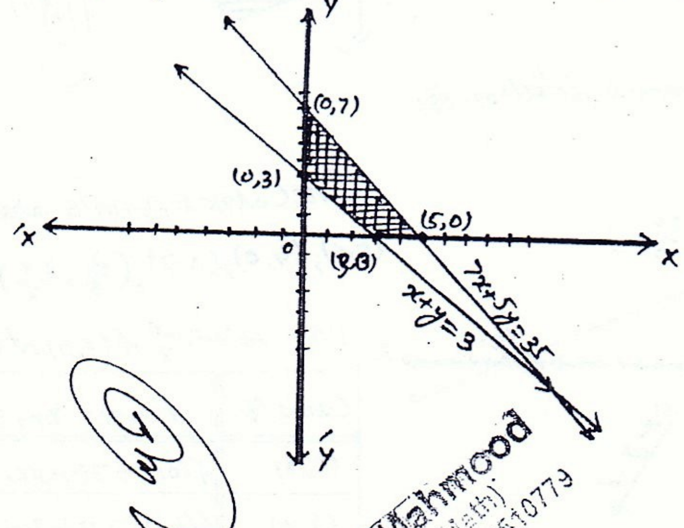
$0 + 0 \geq 3$ $\quad$ and $\quad 7(0) + 5(0) \leq 35$

$\Rightarrow 0 \geq 3$ (false) $\quad \Rightarrow 0 \leq 35$ (True)





The feasible Simultaneous Solution is:



The Corner Points are:

$(0,3), (3,0), (0,7), (5,0)$

The value of Z at Corner points is:

Corner Pts.	$Z = 2x + y$
$(0,3)$	$Z = 2(0) + 3 = 3$
$(3,0)$	$Z = 2(3) + 0 = 6$
$(0,7)$	$Z = 2(0) + 7 = 7$
$(5,0)$	$Z = 2(5) + 0 = 10$

Thus Z is minimum 3 at Corner Point $(0,3)$

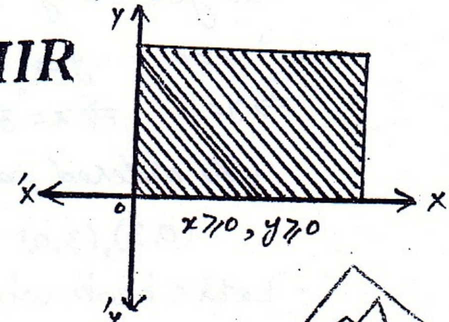
Q.5 Maximize $f(x,y) = 2x+3y$ Subject to the Constraints (22)

$2x+y \le 8 \qquad x+2y \le 14 \qquad x \ge 0, y \ge 0$

The associated Equations are:

$2x+y=8 \qquad x+2y=14$

The graph of $x \ge 0, y \ge 0$ is



To get x, y intercepts

Let $x=0 \quad y=0$ and $x=0 \quad y=0$

$\Rightarrow y=8 \quad \Rightarrow x=4$ and $\Rightarrow y=7 \quad \Rightarrow x=14$

The ordered pairs are:

$(0,8),(4,0)$ and $(0,7),(14,0)$

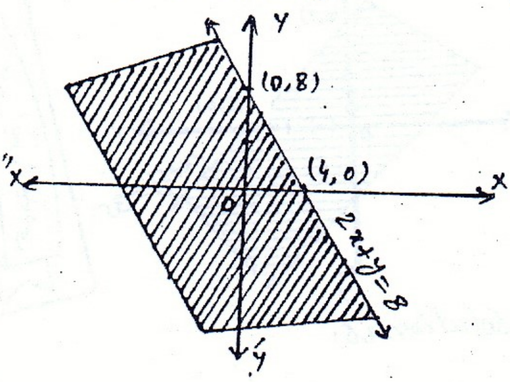
Now Let's check whether $(0,0)$ is in solution region or not:

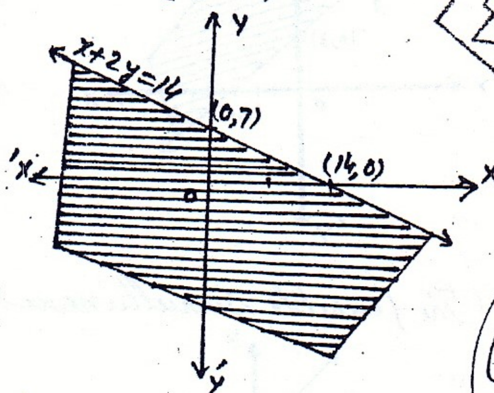
$2(0)+(0) \le 8$

$\Rightarrow 0 \le 8$ (True)

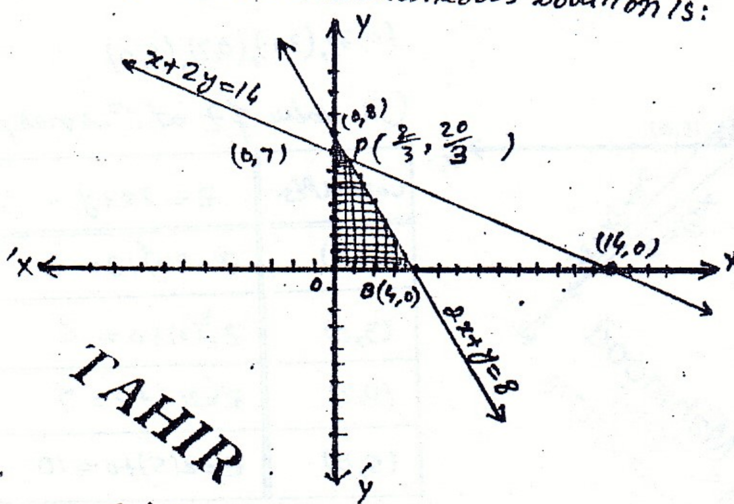
$(0)+2(0) \le 14$

$\Rightarrow 0 \le 14$ (True)





The feasible Simultaneous solution is:



The Corner points are:

$(0,0), (4,0), (0,7), \left(\frac{2}{3}, \frac{20}{3}\right)$

The Value of $f(x,y)$ at Corner Points:

Corner Pts.	$f(x,y) = 2x+3y$
$(0,0)$	$f(0,0) = 2(0)+3(0) = 0$
$(4,0)$	$f(4,0) = 2(4)+3(0) = 8$
$(0,7)$	$f(0,7) = 2(0)+3(7) = 21$
$\left(\frac{2}{3}, \frac{20}{3}\right)$	$f\left(\frac{2}{3}, \frac{20}{3}\right) = 2\left(\frac{2}{3}\right) + 3\left(\frac{20}{3}\right) = 64/3$

Solving $x+2y=14$ and $2x+y=8$

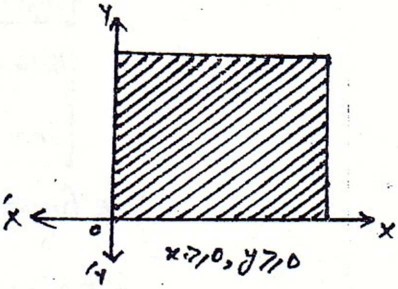
$x = \frac{2}{3} \qquad y = \frac{20}{3}$

So Pt $\left(\frac{2}{3}, \frac{20}{3}\right)$

Thus maximum value of $f(x,y)$ is $64/3$ at Corner point $\left(\frac{2}{3}, \frac{20}{3}\right)$

Q.6 Minimize $z = 3x + y$ subject to Constraints

$3x + 5y \geq 15$ $\quad x + 3y \geq 9$ $\quad x \geq 0, y \geq 0$



The associated Equations are:

$3x + 5y = 15$ $\quad x + 3y = 9$

To get x, y intercepts

Let $x = 0$ $\quad y = 0$ $\quad$ and $\quad x = 0$ $\quad y = 0$

$\Rightarrow y = 3$ $\quad \Rightarrow x = 5$ $\quad$ and $\Rightarrow y = 3$ $\quad \Rightarrow x = 9$

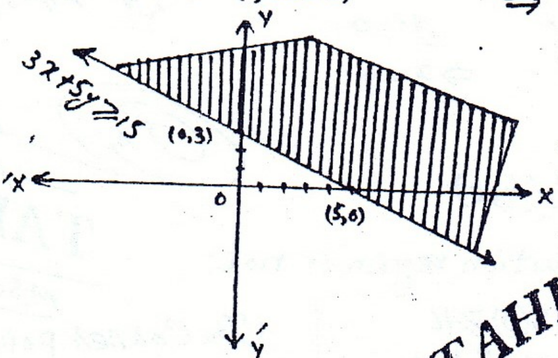
The ordered pairs are:

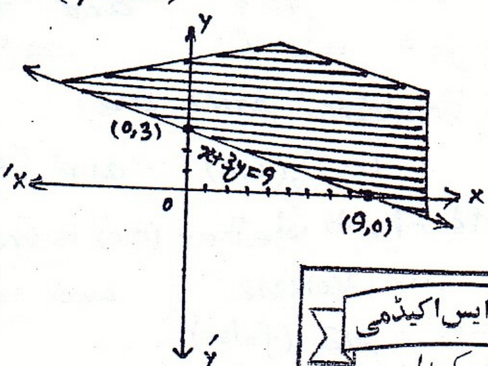
$(0,3), (5,0)$ and $(0,3), (9,0)$

Let's check whether $(0,0)$ is in Solution region or not:

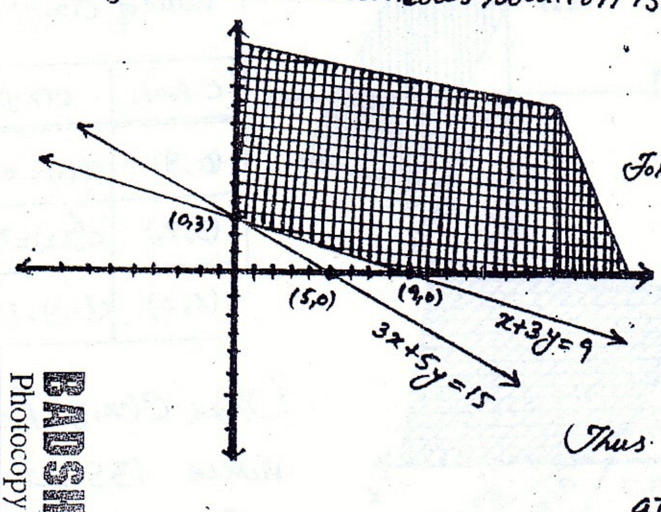
$3(0) + 5(0) \geq 15$ $\quad (0) + 3(0) \geq 9$

$\Rightarrow 0 \geq 15$ (false) $\quad \Rightarrow 0 \geq 9$ (false)





The feasible Simultaneous Solution is:



The Corner points are:

$(0,3), (9,0)$

For The minimum value of $f(x,y) = z$

Corner Pts.	$z = 3x + y$
$(0,3)$	$z = 3(0) + 3 = 3$
$(9,0)$	$z = 3(9) + 0 = 27$

Thus z has minimum value 3 at Corner point $(0,3)$

Q.7

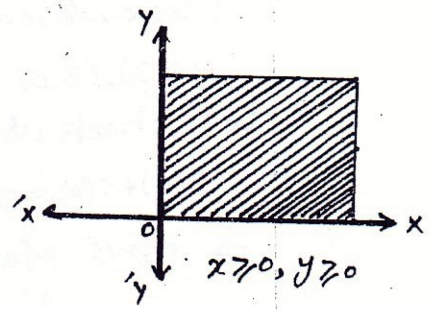
Let x units of X food and y units of Y food be fed to each animal so Cost function is $C(x,y) = 25x+30y$

Now using the given data:

Ingr.	X Food	Y Food	units Required	Constraints
Protien	$2x$	$3y$	12	$2x+3y \geq 12$
Iron	$4x$	$2y$	16	$4x+2y \geq 16$

We have to minimize $C(x,y) = 25x+30y$ using above Constraints:

$2x+3y \geq 12$ $\qquad$ $4x+2y \geq 16$



The associated Equations are:

$2x+3y=12$ $\qquad$ $4x+2y=16$

To get x, y intercepts

Let $x=0$ $\quad y=0$ $\quad$ and $x=0$ $\quad y=0$

$\Rightarrow y=4$ $\quad \Rightarrow x=6$ $\qquad \Rightarrow y=8$ $\quad \Rightarrow x=4$

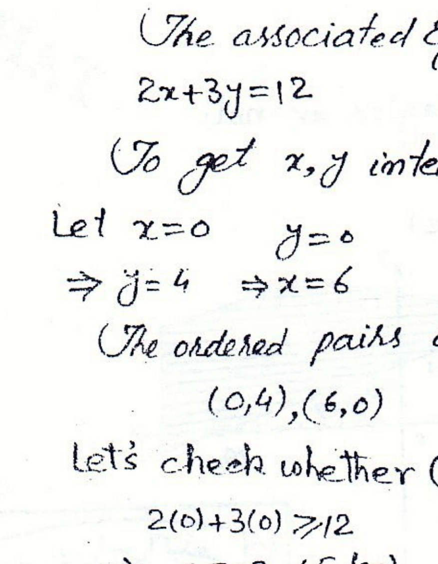
The ordered pairs are:

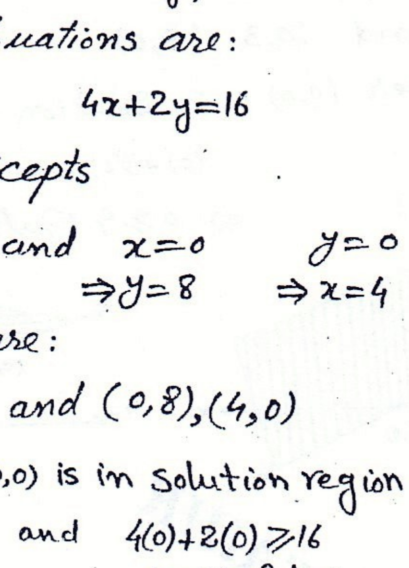
$(0,4),(6,0)$ and $(0,8),(4,0)$

Let's check whether $(0,0)$ is in solution region or not:

$2(0)+3(0) \geq 12$ $\qquad$ and $4(0)+2(0) \geq 16$

$\Rightarrow 0 \geq 12$ (false) $\qquad 0 \geq 16$ (false)





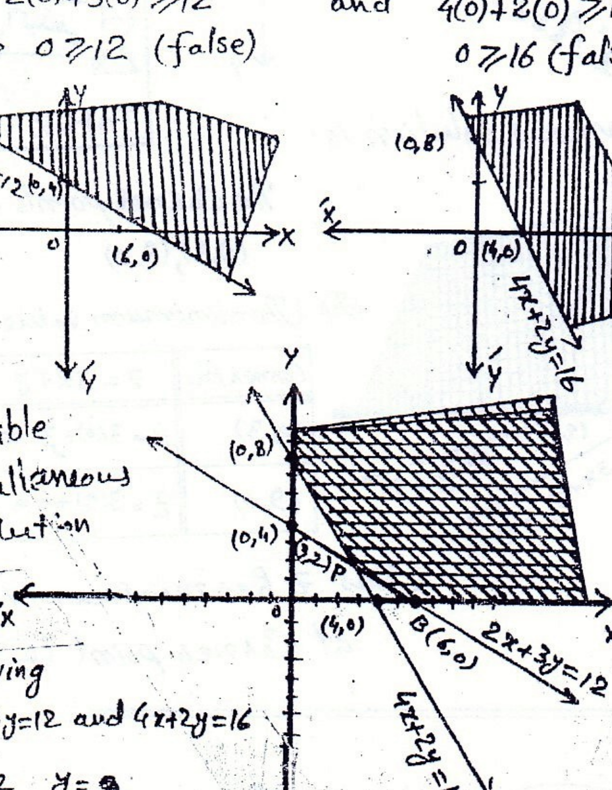
The Corner points are:

$(0,8),(3,2),(6,0)$

Value of $C(x,y)$ at Corner pts.

C. Points	$C(x,y)=25x+30y$
$(0,8)$	$C(0,8) = 0+240 = 240$
$(3,2)$	$C(3,2) = 75+60 = 135$
$(6,0)$	$C(6,0) = 150+0 = 150$

Feasible Simultaneous Solution



Solving

$2x+3y=12$ and $4x+2y=16$

$x=3$ $\quad y=2$

Pt$(3,2)$

Thus $C(x,y)$ has minimum value 135 at $(3,2)$ Corner point.

THE END