Thus the Partialization is

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{5}{4(x+3)} + \frac{1}{12(x-1)} - \frac{4}{3(x+2)} + \frac{1}{(x+2)^2}$$

Q.12 $\dfrac{2x^4}{(x-3)(x+2)^2}$

$$= \frac{2x^4}{(x-3)(x^2+4x+4)}$$

$$= \frac{2x^4}{x^3+x^2-8x-12}$$

$$\begin{array}{r|l} & 2x-2 \\ \hline x^3+x^2-8x-12 & 2x^4 \\ & 2x^4+2x^3-16x^2-24x \\ \hline & -2x^3+16x^2+24x \\ & -2x^3-2x^2+16x+24 \\ \hline & 18x^2+8x-24 \end{array}$$

$$= 2x-2 + \frac{18x^2+8x-24}{(x-3)(x+2)^2}$$

Now

$$\frac{18x^2+8x-24}{(x-3)(x+2)^2} = \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

Multiplying both Sides by $(x-3)(x+2)^2$

$$18x^2+8x-24 = A(x+2)^2 + B(x-3)(x+2) + C(x-3)$$

Put $x-3=0 \Rightarrow x=3$

$$18(9)+8(3)-24 = A(5)^2+(0)B+(0)C$$

$$162+\cancel{24}-\cancel{24} = 25A \Rightarrow \boxed{A=\frac{162}{25}}$$

Put $x+2=0 \Rightarrow x=-2$

$$18(4)-16-24 = (0)A+(0)B+C(-5)$$

$$72-16-24 = -5C \Rightarrow \boxed{\frac{-32}{5}=C}$$

Comparing the Coefficients of

$$x^2 \Rightarrow 18 = A+B$$

$$B = 18-A = 18-\frac{162}{25}$$

$$B = \frac{450-162}{25} = \frac{288}{25}$$

$$\boxed{B=\frac{288}{25}}$$

Thus the Partialization is

$$\frac{2x^4}{(x-3)(x+2)^2} = 2x-2+\frac{162}{25(x-3)}+\frac{288}{25(x+2)}-\frac{32}{5(x+2)^2}$$

Type III In Case of non-repeated Quadratic (2nd degree) factors, write in the form of:

$$\frac{x}{(x^2+3)(x^2+5)} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+5} \text{ etc:}$$

Note: Quadratic factors here are those factors which are not reducible to Linear factors. **TAHIR**

Exercise 5.3

Q.1 $\dfrac{9x-7}{(x^2+1)(x+3)}$

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$$

Multiplying both Sides by $(x+3)(x^2+1)$

$$9x-7 = A(x^2+1) + B(x^2+3x) + C(x+3)$$

Put $x+3=0 \Rightarrow x=-3$

$$-27-7 = A(10)+0(B)+(0)C \Rightarrow A=\frac{-34}{10}$$

$$\boxed{A=\frac{-17}{5}}$$

Comparing the Coefficients of

$x^2 \Rightarrow 0 = A + B$

$B = -A = \frac{17}{5} \Rightarrow \boxed{B = \frac{17}{5}}$

$x \Rightarrow 9 = 3B + C$

$C = 9 - 3B = 9 - 3\left(\frac{17}{5}\right)$

$C = \frac{45-51}{5} = \frac{-6}{5}$

$\boxed{C = \frac{-6}{5}}$

Thus the Partialization is

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{-17}{5(x+3)} + \frac{\frac{17}{5}x - \frac{6}{5}}{x^2+1}$$

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{17x-6}{5(x^2+1)} - \frac{17}{5(x+3)}$$

Q.2 $\frac{1}{(x^2+1)(x+1)}$

$$\frac{1}{(x^2+1)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

Multiplying both Sides by $(x^2+1)(x+1)$

$1 = A(x^2+1) + B(x^2+x) + C(x+1)$

Put $x+1=0 \Rightarrow x=-1$

$1 = A(2) + (0)B + C(0)$

$\boxed{A = \frac{1}{2}}$

Comparing the Coefficients of

$x^2 \Rightarrow 0 = A + B \Rightarrow B = -A$

$\boxed{B = \frac{-1}{2}}$

$x \Rightarrow 0 = B + C \Rightarrow C = -B$

$\boxed{C = \frac{1}{2}}$

Thus Partialization is

$$\frac{1}{(x+1)(x^2+1)} = \frac{1}{2(x+1)} + \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+1}$$

$$\frac{1}{(x+1)(x^2+1)} = \frac{1}{2(x+1)} - \frac{x-1}{2(x^2+1)}$$

Q.3 $\frac{3x+7}{(x^2+4)(x+3)}$

$$\frac{3x+7}{(x+3)(x^2+4)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+4}$$

Multiplying both Sides by $(x+3)(x^2+4)$

$3x+7 = A(x^2+4) + B(x^2+3x) + C(x+3)$

Put $x+3=0 \Rightarrow x=-3$

$-9+7 = A(9+4) + (0)B + 0(C)$

$-2 = A(13) \Rightarrow \boxed{A = \frac{-2}{13}}$

Comparing the Coefficients of

$x^2 \Rightarrow 0 = A + B \Rightarrow B = -A$

$\boxed{B = \frac{2}{13}}$ TAHIR

$x \Rightarrow 3 = 3B + C$

$C = 3 - 3B = 3 - 3\left(\frac{2}{13}\right)$

$C = \frac{39-6}{13} \Rightarrow \boxed{C = \frac{33}{13}}$

Thus the Partialization is

$$\frac{3x+7}{(x+3)(x^2+4)} = \frac{-2}{13(x+3)} + \frac{\frac{2}{13}x + \frac{33}{13}}{x^2+4}$$

$$\frac{3x+7}{(x+3)(x^2+4)} = \frac{2x+33}{13(x^2+4)} - \frac{2}{13(x+3}$$

Q.4 $\dfrac{x^2+15}{(x-1)(x^2+2x+5)}$

$$\frac{x^2+15}{(x-1)(x^2+2x+5)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+5}$$

~~Multiplying both Sides by $(x-1)(x^2+2x+5)$~~

$$x^2+15 = A(x^2+2x+5) + B(x^2-x) + C(x-1)$$

Put $x-1=0 \implies x=1$

$$1+15 = A(1+2+5) + (0)B + (0)C$$

$$16 = 8A \implies \boxed{A=2}$$

Comparing the Coefficients of

$x^2 \implies 1 = A+B$

$B = 1-A = 1-2$

$\boxed{B=-1}$

$x \implies 0 = 2A+C-B$

$C = B-2A = -1-2(2) = -5$

$\boxed{C=-5}$

Thus the Partialization is

$$\frac{x^2+15}{(x-1)(x^2+2x+5)} = \frac{2}{x-1} + \frac{-1x-5}{x^2+2x+5}$$

$$\frac{x^2+15}{(x-1)(x^2+2x+5)} = \frac{2}{x-1} - \frac{x+5}{x^2+2x+5}$$

Q.5 $\dfrac{x^2}{(x^2+4)(x+2)}$

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$$

Multiplying both Sides by $(x+2)(x^2+4)$

$$x^2 = A(x^2+4) + B(x^2+2x) + C(x+2)$$

Put $x+2=0 \implies x=-2$

$$4 = A(4+4) + (0)B + (0)C$$

$$8A = 4 \implies \boxed{A=\tfrac{1}{2}}$$

Comparing the Coefficients of

$x^2 \implies 1 = A+B \implies B = 1-A$

$B = 1-\frac{1}{2} = \frac{2-1}{2}$

$\boxed{B=\tfrac{1}{2}}$

$x \implies 0 = 2B+C$

$C = -2B \implies C = -2\cdot\frac{1}{2}$

$\boxed{C=-1}$

Thus the Partialization is

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{1}{2(x+2)} + \frac{\frac{1}{2}x-1}{(x^2+4)}$$

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{1}{2(x+2)} + \frac{x-2}{2(x^2+4)}$$

Q.6 $\dfrac{x^2+1}{x^3+1}$ $\quad \because a^3+b^3 = (a+b)(a^2-ab+b^2)$

$$= \frac{x^2+1}{(x+1)(x^2-x+1)}$$

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

Multiplying both Sides by $(x+1)(x^2-x+1)$

$$x^2+1 = A(x^2-x+1) + B(x^2+x) + C(x+1)$$

Put $x+1=0 \implies x=-1$

$$2 = A(1+1+1) + 0(B) + C(0)$$

$$2 = 3A \implies \boxed{A=\tfrac{2}{3}}$$

Comparing the Coefficients of

$x^2 \Rightarrow 1 = A+B$

$B = 1 - A = 1 - \frac{2}{3} = \frac{3-2}{3}$

$\boxed{B = \frac{1}{3}}$

$x \Rightarrow 0 = -A + B + C$

$C = A - B$

$C = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

$\boxed{C = \frac{1}{3}}$

Thus the Partialization is

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{2}{3(x+1)} + \frac{\frac{1}{3}x+\frac{1}{3}}{x^2-x+1}$$

$$\frac{x^2+1}{x^3+1} = \frac{2}{3(x+1)} + \frac{x+1}{3(x^2-x+1)}$$

Q.7 $\dfrac{x^2+2x+2}{(x^2+3)(x+1)(x-1)}$

$$\frac{x^2+2x+2}{(x-1)(x+1)(x^2+3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+3}$$

Multiplying by $(x-1)(x+1)(x^2+3)$

$x^2+2x+2 = A(x+1)(x^2+3) + B(x-1)(x^2+3) + Cx(x-1)(x+1) + D(x-1)(x+1)$

$x^2+2x+2 = A(x^3+x^2+3x+3) + B(x^3-x^2+3x-3) + C(x^3-x) + D(x^2-1)$

Put $x-1=0 \Rightarrow x=1$

$1+2+2 = A(1+1+3+3) + B(0) + C(0) + D(0)$

$5 = 8A \Rightarrow \boxed{A = \frac{5}{8}}$

Put $x+1=0 \Rightarrow x=-1$

$1-2+2 = (0)A + B(-2)(4) + C(0) + D(0)$

$1 = -8B \Rightarrow \boxed{B = \frac{-1}{8}}$

Comparing the Coefficients of

$x^3 \Rightarrow 0 = A+B+C$ — ①

$x^2 \Rightarrow 1 = A - B + D$ — ②

From ① $C = -A - B$

$C = \frac{-5}{8} + \frac{1}{8} = \frac{-4}{8} = \frac{-1}{2}$

$\boxed{C = \frac{-1}{2}}$

From ② $D = B - A + 1$

$D = 1 - \frac{1}{8} - \frac{5}{8} = \frac{8-1-5}{8}$

$D = \frac{2}{8} = \frac{1}{4}$

$\boxed{D = 1/4}$

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Thus the Partialization is

$$\frac{x^2+2x+2}{(x-1)(x+1)(x^2+3)} = \frac{5}{8(x-1)} - \frac{1}{8(x+1)} + \frac{-\frac{1}{2}x+\frac{1}{4}}{x^2+3}$$

$$\frac{x^2+2x+2}{(x-1)(x+1)(x^2+3)} = \frac{5}{8(x-1)} - \frac{1}{8(x+1)} - \frac{2x-1}{4(x^2+3)}$$

Q.8 $\dfrac{1}{(x-1)^2(x^2+2)}$

$$\frac{1}{(x-1)^2(x^2+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+2}$$

Multiplying by $(x-1)^2(x^2+2)$

$1 = A(x-1)(x^2+2) + B(x^2+2) + Cx(x-1)^2 + D(x-1)^2$

$1 = A(x^3-x^2+2x-2) + B(x^2+2) + C(x^3-2x^2+x) + D(x^2-2x+1)$

Put $x-1=0 \Rightarrow x=1$

$1=(0)A+B(1+2)+(0)C+(0)D$

$3B=1 \Rightarrow \boxed{B=\frac{1}{3}}$

Comparing the Coefficients of

$x^3 \Rightarrow A+C=0$ —— (1)

$x^2 \Rightarrow 0=-A+B-2C+D$ —— (2)

$x \Rightarrow 0=2A+C-2D$ —— (3)

Constt $\Rightarrow 1=-2A+2B+D$ —— (4)

$2A-D=2B-1$

$2A-D=\frac{2}{3}-1=\frac{-1}{3}$

$2A-D=-1/3$

$2A-B=-1/3$ —— (5)

From (2)

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$A+2C-D=B$

$A+2C-D=1/3$ —— 6

Multiplying by 2

$2\cancel{A}+4C-2\cancel{D}=\frac{2}{3}$

$-\cancel{2A}\pm C\mp\cancel{2D}=0$ —— (3)

$3C=2/3$

$\boxed{C=\frac{2}{9}}$

$A+C=0 \Rightarrow \boxed{A=-\frac{2}{9}}$

$A=-C \Rightarrow$

$\because 2A-D=\frac{-1}{3}$

$2A+\frac{1}{3}=D \Rightarrow D=\frac{-4}{9}+\frac{1}{3}$

$D=\frac{-4+3}{9}=\frac{-1}{9} \Rightarrow \boxed{D=\frac{-1}{9}}$

Thus Partialization

$$\frac{1}{(x-1)^2(x^2+2)}=\frac{-2}{9(x-1)}+\frac{1}{3(x-1)^2}+\frac{\frac{2}{9}x-\frac{1}{9}}{x^2+2}$$

$$\frac{1}{(x-1)^2(x^2+2)}=\frac{-2}{9(x-1)}+\frac{1}{3(x-1)^2}+\frac{2x-1}{9(x^2+2)}$$

Q.9 $\dfrac{x^4}{1-x^4}$

$$1-x^4 \overline{\big)\, x^4 \,}\; -1$$
$$\underline{-x^4 \mp 1}$$
$$1$$

$=\dfrac{x^4}{1-x^4}=-1+\dfrac{1}{1-x^4}$

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$=-1+\dfrac{1}{(1-x^2)(1+x^2)}$

$=-1+\dfrac{1}{(1-x)(1+x)(1+x^2)}$

$=\dfrac{1}{(1-x)(1+x)(1+x^2)}=\dfrac{A}{1-x}+\dfrac{B}{1+x}+\dfrac{Cx+D}{1+x^2}$

Multiplying by $(1-x^4)$, we get

$1=A(1+x)(1+x^2)+B(1-x)(1+x^2)+Cx(1-x)(1+x)$
$+D(1-x)(1+x)$

$1=A(1+x^2+x+x^3)+B(1+x^2-x-x^3)$
$+C(x-x^3)+D(1-x^2)$

Put $1-x=0 \Rightarrow x=1$

$1=A(1+1+1+1)+(0)B+(0)C+(0)D$

$4A=1 \Rightarrow \boxed{A=\frac{1}{4}}$

Put $1+x=0 \Rightarrow x=-1$

$1=(0)A+B(1+1+1+1)+(0)C+(0)D$

$\boxed{B=\frac{1}{4}}$

Comparing the Coefficients of

$x^3 \Rightarrow 0 = A - B - C$

$C = A - B = \frac{1}{4} - \frac{1}{4} = 0$

$\boxed{C = 0}$

$x^2 \Rightarrow 0 = A + B - D$

$D = A + B = \frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4}$

$\boxed{D = \frac{1}{2}}$

Thus the Partialization is

$$\frac{x^4}{1-x^4} = -1 + \frac{1}{4(1-x)} + \frac{1}{4(1+x)} + \frac{0x + \frac{1}{2}}{(1+x^2)}$$

$$\frac{x^4}{1-x^4} = \frac{1}{4(1-x)} + \frac{1}{4(1+x)} + \frac{1}{2(x^2+1)} - 1$$

Q.10 $\dfrac{x^2 - 2x + 3}{x^4 + x^2 + 1}$

$$= \frac{x^2 - 2x + 3}{x^4 + 2x^2 + 1 - x^2}$$

$$= \frac{x^2 - 2x + 3}{(x^2+1)^2 - x^2}$$

$$= \frac{x^2 - 2x + 3}{(x^2+1-x)(x^2+x+1)}$$

$$\frac{x^2 - 2x + 3}{(x^2+x+1)(x^2-x+1)} = \frac{Ax+B}{(x^2+x+1)} + \frac{Cx+D}{(x^2-x+1)}$$

Multiplying by (x^4+x^2+1)

$x^2 - 2x + 3 = (Ax+B)(x^2-x+1) + (Cx+D)(x^2+x+1)$

$x^2 - 2x + 3 = A(x^3 - x^2 + x) + B(x^2 - x + 1) + C(x^3 + x^2 + x) + D(x^2 + x + 1)$

Comparing the Coefficients of

$x^3 \Rightarrow A + C = 0$ —— (1)

$x^2 \Rightarrow 1 = -A + C + B + D$ —— (2)

$x \Rightarrow -2 = A + C - B + D$ —— (3)

$\text{Constt} \Rightarrow 3 = B + D$ —— (4)

From (2) and (3)

$\cancel{A} + C - \cancel{B} + D = -2$

$+ \quad -\cancel{A} + C + \cancel{B} + D = 1$

$C + D = -\frac{1}{2}$ —— (5)

Eq (2) − Eq (4)

$-A + C + \cancel{B} + \cancel{D} = 1$

$\cancel{B} + \cancel{D} = 3$

$-A + C = -2$ —— (6)

Adding (1) and (6)

$\cancel{A} + C = 0$

$-\cancel{A} + C = -2$

$2C = -2$

$\boxed{C = -1}$

$A + C = 0 \qquad A = -C$

$\boxed{A = 1}$

$\therefore C + D = -\frac{1}{2}$

$D = -\frac{1}{2} - C \Rightarrow -\frac{1}{2} + 1$

$\boxed{D = \frac{1}{2}}$

$\therefore B + D = 3 \Rightarrow B = 3 - D$

$B = 3 - \frac{1}{2} = \frac{5}{2}$

$\boxed{B = \frac{5}{2}}$