$$y+a^2 = A(y+c^2)(y+d^2)+B(y+b^2)(y+d^2)+C(y+b^2)(y+c^2)$$

Put $y+b^2=0 \Longrightarrow y=-b^2$

$$a^2-b^2 = A(c^2-b^2)(d^2-b^2)+(0)B+(0)C$$

$$\boxed{A = \frac{a^2-b^2}{(c^2-b^2)(d^2-b^2)}}$$

Put $y+c^2=0 \Longrightarrow y=-c^2$

$$a^2-c^2 = (0)A+B(b^2-c^2)(d^2-c^2)+C(0)$$

$$\boxed{B = \frac{a^2-c^2}{(b^2-c^2)(d^2-c^2)}}$$

Put $y+d^2=0 \Longrightarrow y=-d^2$

$$a^2-d^2 = (0)A+(0)B+C(b^2-d^2)(c^2-d^2)$$

$$\boxed{C = \frac{a^2-d^2}{(b^2-d^2)(c^2-d^2)}}$$

Thus Partilization is

$$\frac{y+a^2}{(y+b^2)(y+c^2)(y+d^2)} = \frac{a^2-b^2}{(y+b^2)(c^2-b^2)(d^2-b^2)} + \frac{a^2-c^2}{(y+c^2)(b^2-c^2)(d^2-c^2)} + \frac{a^2-d^2}{(y+d^2)(b^2-d^2)(c^2-d^2)}$$

Now put back $y=x^2$

$$\frac{x^2+a^2}{(x^2+b^2)(x^2+c^2)(x^2+d^2)} = \frac{a^2-b^2}{(x^2+b^2)(c^2-b^2)(d^2-b^2)} + \frac{a^2-c^2}{(x^2+c^2)(b^2-c^2)(d^2-c^2)} + \frac{a^2-d^2}{(x^2+d^2)(b^2-d^2)(c^2-d^2)}$$

Type-II: In Case of repeated linear factor, write upto n powers.

$$\frac{x}{(1+x)^3} = \frac{A}{1+x}+\frac{B}{(1+x)^2}+\frac{C}{(1+x)^3}$$

Exercise 5.2

Q.1 $\dfrac{2x^2-3x+4}{(x-1)^3}$

$$\frac{2x^2-3x+4}{(x-1)^3} = \frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}$$

Multiplying both Sides by $(x-1)^3$

$$2x^2-3x+4 = A(x-1)^2+B(x-1)+C$$

Put $x-1=0 \Longrightarrow x=1$

$$2-3+4 = (0)A+0(B)+C$$

$$\boxed{C=3}$$

Comparing the Coefficients of

$x^2 \Longrightarrow \boxed{2=A}$

$x \Longrightarrow -3=-2A+B$

$B=-3+2A$

$B=-3+2(2)=-3+4$

$$\boxed{B=1}$$

Thus Partilization is

$$\frac{2x^2-3x+4}{(x-1)^3} = \frac{2}{x-1}+\frac{1}{(x-1)^2}+\frac{3}{(x-1)^3}$$

Q.2 $\dfrac{5x^2-2x+3}{(x+2)^2}$

$$= \frac{5x^2-2x+3}{x^2+4x+4}$$

$$\begin{array}{r|l} & \quad 5 \\ x^2+4x+4 & \overline{5x^2-2x+3} \\ & \underline{5x^2+20x+20} \\ & -22x-17 \end{array}$$

$$= 5-\frac{22x+17}{x^2+4x+4}$$

$$= 5-\frac{22x+17}{(x+2)^2}$$

Now Partilization is

$$\frac{22x+17}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$$

Multiplying both Sides by $(x+2)^2$

$$22x+17 = A(x+2) + B$$

Put $x+2=0 \Rightarrow x=-2$

$$22(-2)+17 = (0)A + B$$

$$\boxed{B = -27}$$

Comparing the Coefficients of

$x \Rightarrow \boxed{22 = A}$

Thus the Partilization is

$$\frac{5x^2-2x+3}{(x+2)^2} = 5 - \left\{\frac{22}{x+2} + \frac{-27}{(x+2)^2}\right\}$$

$$\frac{5x^2-2x+3}{(x+2)^2} = 5 - \frac{22}{x+2} + \frac{27}{(x+2)^2}$$

Q.3 $\dfrac{4x}{(x+1)^2(x-1)}$

The Partilization is

$$\frac{4x}{(x+1)^2(x-1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

Multiplying both Sides by $(x+1)^2(x-1)$

$$4x = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

Put $x-1=0 \Rightarrow x=1$

$$4 = A(1+1)^2 + (0)B + C(0)$$

$$4 = 4A \Rightarrow \boxed{A=1}$$

Put $x+1=0 \Rightarrow x=-1$

$$-4 = (0)A + (0)B + C(-2)$$

$$\boxed{C = 2}$$

Comparing the Coefficients of

$x^2 \Rightarrow 0 = A + B$

$$A = -B \Rightarrow B = -A$$

$$\boxed{B = -1}$$

Thus Partialization is

$$\frac{4x}{(x+1)^2(x-1)} = \frac{1}{x-1} + \frac{-1}{x+1} + \frac{2}{(x+1)^2}$$

$$\frac{4x}{(x+1)^2(x-1)} = \frac{1}{(x-1)} - \frac{1}{(x+1)} + \frac{2}{(x+1)^2}$$

Q.4 $\dfrac{9}{(x+2)^2(x-1)}$

The partilization is

$$\frac{9}{(x+2)^2(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{x-1}$$

Multiplying both Sides by $(x+2)^2(x-1)$

$$9 = A(x+2)(x-1) + B(x-1) + C(x+2)^2$$

Put $x+2=0 \Rightarrow x=-2$

$$9 = (0)A + B(-3) + (0)C \Rightarrow B = \frac{-9}{3}$$

$$\boxed{B = -3}$$

Put $x-1=0 \Rightarrow x=1$

$$9 = (0)A + (0)B + C(3)^2$$

$$9 = 9C \Rightarrow \boxed{C=1}$$

Comparing Coefficients of

$x^2 \Rightarrow 0 = A + C \Rightarrow A = -C$

$$\boxed{A = -1}$$

Thus the Partialization is

$$\frac{9}{(x+2)^2(x-1)} = \frac{-1}{(x+2)} + \frac{-3}{(x+2)^2} + \frac{1}{(x-1)}$$

$$\frac{9}{(x+2)^2(x-1)} = \frac{1}{(x-1)} - \frac{1}{(x+2)} - \frac{3}{(x+2)^2}$$

Q.5 $\dfrac{1}{(x-3)^2(x+1)}$

$$\frac{1}{(x-3)^2(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x-3)^2} + \frac{C}{x+1}$$

Multiplying both Sides by $(x-3)^2(x+1)$

$$1 = A(x-3)(x+1) + B(x+1) + C(x-3)^2$$

Put $x-3=0 \Rightarrow x=3$

$$1 = (0)A + B(4) + (0)C$$

$$\boxed{B = 1/4}$$

Put $x+1=0 \Rightarrow x=-1$

$$1 = (0)A + (0)B + C(-4)^2$$

$$16C = 1 \Longrightarrow \boxed{C = \frac{1}{16}}$$

Comparing the Coefficients of

$$x^2 \Rightarrow 0 = A + C$$

$$A = -C \Longrightarrow \boxed{A = \frac{-1}{16}}$$

Thus Partialization is

$$\frac{1}{(x-3)^2(x+1)} = \frac{-1/16}{x-3} + \frac{1/4}{(x-3)^2} + \frac{1/16}{(x+1)}$$

$$\frac{1}{(x-3)^2(x+1)} = \frac{1}{4(x-3)^2} - \frac{1}{16(x-3)} + \frac{1}{16(x+1)}$$

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Q.6 $\dfrac{x^2}{(x-2)(x-1)^2}$

$$\frac{x^2}{(x-2)(x-1)^2} = \frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

Multiplying both Sides by $(x-2)(x-1)^2$

$$x^2 = A(x-1)^2 + B(x-2)(x-1) + C(x-2)$$

Put $x-2=0 \Longrightarrow x=2$

$$4 = A(2-1)^2 + (0)B + C(0)$$

$$\boxed{A = 4}$$

Put $x-1=0 \Longrightarrow x=1$

$$1 = (0)A + (0)B + C(-1)$$

$$\boxed{C = -1}$$

Comparing the Coefficients of

$$x^2 \Longrightarrow 1 = A + B$$

$$B = 1 - A = 1 - 4 = -3$$

$$\boxed{B = -3}$$

Thus Partialization is

$$\frac{x^2}{(x-2)(x-1)^2} = \frac{4}{(x-2)} - \frac{3}{(x-1)} - \frac{1}{(x-1)^2}$$

Q.7 $\dfrac{1}{(x-1)^2(x+1)}$

$$\frac{1}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$$

Multiplying both Sides by $(x-1)^2(x+1)$

$$1 = A(x+1)(x-1) + B(x+1) + C(x-1)^2$$

Put $x-1=0 \Longrightarrow x=1$

$$1 = (0)A + B[2] + 0\,C \Longrightarrow 2B = 1$$

$$\boxed{B = \frac{1}{2}}$$

Put $x+1=0 \Rightarrow x=-1$

$1 = (0)A + (0)B + C[-2]^2$

$1 = 4C \Rightarrow \boxed{C = \frac{1}{4}}$

Comparing the Coefficients of

$x^2 \Rightarrow 0 = A + C$

$A = -C \Rightarrow \boxed{A = \frac{-1}{4}}$

Thus the Partialization is

$$\frac{1}{(x-1)^2(x+1)} = \frac{-1/4}{(x-1)} + \frac{1/2}{(x-1)^2} + \frac{1/4}{(x+1)}$$

$$\frac{1}{(x-1)^2(x+1)} = \frac{1}{4(x+1)} - \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2}$$

Q.8 $\dfrac{x^2}{(x-1)^3\,(x+1)}$

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{x+1}$$

Multiplying both Sides by $(x-1)^3(x+1)$

$x^2 = A(x+1)(x-1)^2 + B(x-1)(x+1) + C(x+1) + D(x-1)^3$

$x^2 = A(x^3 - x^2 - x + 1) + B(x^2 - 1) + C(x+1) + D(x^3 - 1 - 3x^2 + 3x)$

Put $x-1=0 \Rightarrow x=1$

$1 = (0)A + (0)B + C[2] + (0)D$

$\boxed{C = \frac{1}{2}}$

Put $x+1=0 \Rightarrow x=-1$

$1 = (0)A + (0)B + (0)C + D(-2)^3$

$-8D = 1 \Rightarrow \boxed{D = \frac{-1}{8}}$

Comparing the Coefficients of

$x^3 \Rightarrow 0 = A + D \Rightarrow A = -D$ $\quad \boxed{A = \frac{1}{8}}$

$x^2 \Rightarrow 1 = -A + B - 3D$

$A + 3D = B$

$1 + A + 3D = B \Rightarrow B = 1 + \frac{1}{8} - \frac{3}{8}$

$B = \frac{8+1-3}{8} = \frac{6}{8} = 3/4$

$\boxed{B = 3/4}$

Thus Partialization is

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{1/8}{x-1} + \frac{3/4}{(x-1)^2} + \frac{1/2}{(x-1)^3} + \frac{-1/8}{x+1}$$

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{1}{8(x-1)} + \frac{3}{4(x-1)^2} + \frac{1}{2(x-1)^3} - \frac{1}{8(x+1)}$$

Q.9 $\dfrac{x-1}{(x-2)\,(x+1)^3}$ **TAHIR**

$$\frac{x-1}{(x-2)(x+1)^3} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$$

Multiplying both Sides by $(x-2)(x+1)^3$

$x-1 = A(x+1)^3 + B(x-2)(x+1)^2 + C(x-2)(x+1) + D(x-2)$

$x-1 = A(x^3+3x^2+3x+1) + B(x-2)(x^2+2x+1) + C(x^2-x-2)$
$+ D(x-2)$

$x-1 = A(x^3+3x^2+3x+1) + B(x^3-3x-2) + C(x^2-x-2)$
$+ D(x-2)$

Put $x-2=0 \Rightarrow x=2$

$1 = A[3]^3 + (0)B + (0)C + (0)D$

$\boxed{A = \frac{1}{27}}$

Put $x+1=0 \Rightarrow x=-1$

$-2 = -3D + (0)A + (0)B + (0)C \Rightarrow \boxed{D = \frac{2}{3}}$

Comparing the Coefficients of

$x^3 \Rightarrow 0 = A + B \Rightarrow B = -A$

$\boxed{B = \frac{-1}{27}}$

$x^2 \Rightarrow 0 = 3A + C$

$C = -3A \Rightarrow C = -\cancel{3}\left(\frac{1}{\cancel{27}_{9}}\right)$

$\boxed{C = \frac{-1}{9}}$

Thus Partialization is

$$\frac{x-1}{(x-2)(x+1)^3} = \frac{1}{27(x-2)} - \frac{1}{27(x+1)} - \frac{1}{9(x+1)^2} + \frac{2}{3(x+1)^3}$$

Q.10 $\dfrac{4x^3}{(x^2-1)(x+1)^2}$

$$= \frac{4x^3}{(x-1)(x+1)(x+1)^2} = \frac{4x^3}{(x-1)(x+1)^3}$$

$$\frac{4x^3}{(x-1)(x+1)^3} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$$

Multiplying both sides by $(x-1)(x+1)^3$.

$4x^3 = A(x+1)^3 + B(x-1)(x+1)^2 + C(x-1)(x+1) + D(x-1)$

$4x^3 = A(x^3+3x^2+3x+1) + B(x^3+x^2-x-1) + C(x^2-1) + D(x-1)$

Put $x-1=0 \Rightarrow x=1$

$4 = A(2)^3 + (0)B + (0)C + (0)D$

$A = \frac{4}{8} = \frac{1}{2} \Rightarrow \boxed{A = \frac{1}{2}}$

Put $x+1=0 \Rightarrow x=-1$

$-4 = (0)A + (0)B + (0)C + D(-2)$

$4 = 2D \Rightarrow \boxed{D = 2}$

Comparing the Coefficients of

$x^3 \Rightarrow 4 = A + B$

$B = 4 - A = 4 - \frac{1}{2} = \frac{7}{2} \Rightarrow \boxed{B = \frac{7}{2}}$

$x^2 \Rightarrow 0 = 3A + B + C$

$C = -3A - B \Rightarrow C = -\frac{7}{2} - \frac{3}{2}$

$C = \frac{-10}{2} \Rightarrow \boxed{C = -5}$

Thus the Partialization is

$$\frac{4x^3}{(x^2-1)(x+1)^2} = \frac{1}{2(x-1)} + \frac{7}{2(x+1)} - \frac{5}{(x+1)^2} + \frac{2}{(x+1)^3}$$

Q.11 $\dfrac{2x+1}{(x+3)(x-1)(x+2)^2}$

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{x+2} + \frac{D}{(x+2)^2}$$

Multiplying both sides by $(x+3)(x-1)(x+2)^2$

$2x+1 = A(x-1)(x+2)^2 + B(x+3)(x+2)^2 + C(x+3)(x-1)(x+2) + D(x+3)(x-1)$

$2x+1 = A(x-1)(x^2+4x+4) + B(x+3)(x^2+4x+4) + C(x-1)(x^2+5x+6) + D(x^2+2x-3)$

$2x+1 = A(x^3+3x^2-4) + B(x^3+7x^2+16x+12) + C(x^3+4x^2+x-6) + D(x^2+2x-3)$

Put $x+3=0 \Rightarrow x=-3$

$-6+1 = A(-4)(-1)^2 + (0)B + (0)C + (0)D$

$-5 = -4A \Rightarrow \boxed{A = \frac{5}{4}}$

Put $x-1=0 \Rightarrow x=1$

$3 = (0)A + B(4)(3)^2 + (0)C + (0)D$

$3 = 36B \Rightarrow \boxed{B = \frac{1}{12}}$

Put $x+2=0 \Rightarrow x=-2$

$-4+1 = (0)A + (0)B + (0)C + D(1)(-3)$

$-3D = -3 \Rightarrow \boxed{D = 1}$

Comparing the Coefficients of

$x^3 \Rightarrow 0 = A + B + C$

$C = -A - B = -\frac{5}{4} - \frac{1}{12} = \frac{-15-1}{12} = \frac{-\cancel{16}^{\,4}}{\cancel{12}_{\,3}}$

$\boxed{C = -4/3}$

Thus the Partialization is

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{5}{4(x+3)} + \frac{1}{12(x-1)} - \frac{4}{3(x+2)} + \frac{1}{(x+2)^2}$$

Q.12 $\dfrac{2x^4}{(x-3)(x+2)^2}$

$$= \frac{2x^4}{(x-3)(x^2+4x+4)}$$

$$= \frac{2x^4}{x^3+x^2-8x-12}$$

$$\begin{array}{r|l} & 2x-2 \\ \hline x^3+x^2-8x-12 & 2x^4 \\ & \underline{\pm 2x^4 \pm 2x^3 \mp 16x^2 \mp 24x} \\ & -2x^3+16x^2+24x \\ & \underline{\mp 2x^3 \mp 2x^2 \pm 16x \pm 24} \\ & 18x^2+8x-24 \end{array}$$

$$= 2x-2 + \frac{18x^2+8x-24}{(x-3)(x+2)^2}$$

Now

$$\frac{18x^2+8x-24}{(x-3)(x+2)^2} = \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

Multiplying both Sides by $(x-3)(x+2)^2$

$$18x^2+8x-24 = A(x+2)^2 + B(x-3)(x+2) + C(x-3)$$

Put $x-3=0 \Rightarrow x=3$

$$18(9)+8(3)-24 = A(5)^2+(0)B+(0)C$$

$$162+\cancel{24}-\cancel{24} = 25A \Rightarrow \boxed{A = \frac{162}{25}}$$

Put $x+2=0 \Rightarrow x=-2$

$$18(4)-16-24 = (0)A+(0)B+C(-5)$$

$$72-16-24 = -5C \Rightarrow \boxed{\frac{-32}{5} = C}$$

Comparing the Coefficients of

$$x^2 \Rightarrow 18 = A+B$$

$$\therefore B = 18-A = 18-\frac{162}{25}$$

$$B = \frac{450-162}{25} = \frac{288}{25}$$

$$\boxed{B = \frac{288}{25}}$$

Thus the Partialization is

$$\frac{2x^4}{(x-3)(x+2)^2} = 2x-2+\frac{162}{25(x-3)}+\frac{288}{25(x+2)}-\frac{32}{5(x+2)^2}$$

Type III In Case of non-repeated Quadratic (2nd degree) factors, write in the form of:

$$\frac{x}{(x^2+3)(x^2+5)} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+5} \text{ etc:}$$

Note: Quadratic factors here are those factors which are not reducible to Linear factors.

Exercise 5.3

Q.1 $\dfrac{9x-7}{(x^2+1)(x+3)}$

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$$

Multiplying both Sides by $(x+3)(x^2+1)$

$$9x-7 = A(x^2+1)+B(x^2+3x)+C(x+3)$$

Put $x+3=0 \Rightarrow x=-3$

$$-27-7 = A(10)+0(B)+(0)C \Rightarrow A = \frac{-34}{10}$$

$$\boxed{A = \frac{-17}{5}}$$