Chapter. 5 Linear Inequalities and Linear Programming

F.A: F.Sc. Part: II

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INEQUALITIES:-

"The expressions containing conditional symbols are known as inequalities". These conditional symbols are $<, >, \leq, \geq$.

For example: $2x^2+y\leq 0$, $3x-4y>5$, $ax+by<c$ etc.

LINEAR INEQUALITIES:-

"The inequalities in which degree of variable is unit are known as Linear inequalities."

For example: $3x+4\leq 3$, $5x-3y+4z\geq 7$, $2x-6y\leq 2$ etc.

There are two types of Linear inequalities mainly:

(i) Linear inequalities in one variable: e.g. $ax+b\leq c$, $3x+5\geq 7$ etc.

(ii) Linear inequalities in two variables: e.g. $ax+by\leq c$, $3x+7y\geq 3$ etc.

SOLUTION OF INEQUALITIES:-

"The real numbers x, y that satisfy Linear inequality are known as Solution of inequality."

* An inequality may have infinite many solutions.

For example: $3x+4>3$ is true for all values of $x>\frac{-1}{3}$ which is its solution.

CRITERIA USED TO DRAW THE GRAPH OF INEQUALITIES

Following steps should adopt to draw the graph of inequality:

(i) Write down associated/Corresponding equations of inequalities to findout "x" and "y" intercepts by putting "y" and "x" zero respectively.

(ii) Draw the "reference Line" dashed or Solid according to $>, <$ or $\geq, \leq$.

(iii) Choose any test point (which is not on reference Line, usually it is origin) to find region of Solution.

(iv) The graph of inequality of the form $ax+by<d$, $ax+by>c$ is open region.

(v) The graph of inequality of the form $ax+by\leq c$, $ax+by\geq d$ is closed Region.

Solution Of Two or More Simultaneous Inequalities:-

⇒ "The inequalities which have common solution are called Simultaneous inequalities."

⇒ "The set of all those ordered pairs (x, y) which satisfy all inequalitys is called Solution of system of inequalities.

Corner Point or Vertex:-

"The point of a solution region where two of its boundary lines intersect is called Cornerpoint or Vertex.

Exercise 5·1

Q.1 Graph the following linear inequalities in xy-plane.

(i) $2x + y \le 6$

The associated equation is

$2x + y = 6$

To get x, y intercepts

Let $x = 0$ and $y = 0$

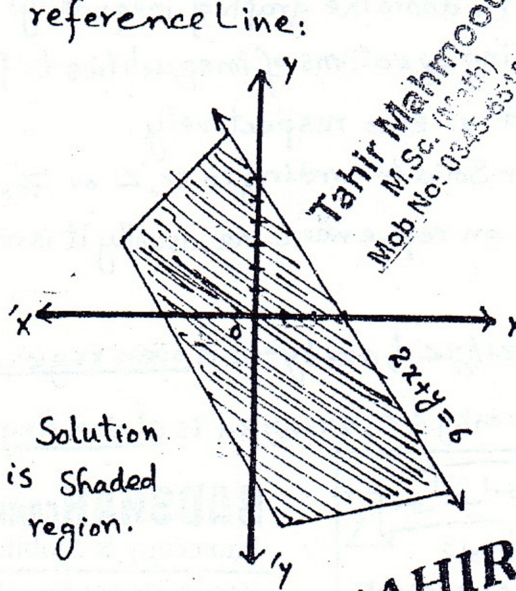
$\Rightarrow y = 6$ $\quad\Rightarrow x = 3$

Pairs are $(0,6), (3,0)$

Let us check whether $(0,0)$ is in Solution region or not:

$\Rightarrow 2(0) + (0) \le 6 \Rightarrow 0 \le 6$ (TRUE)

Thus Solution region is below the reference Line:



Solution is Shaded region.

(ii) $3x + 7y \ge 21$

The associated equation is

$3x + 7y = 21$

To get x, y intercepts

Let $x = 0$ and $y = 0$

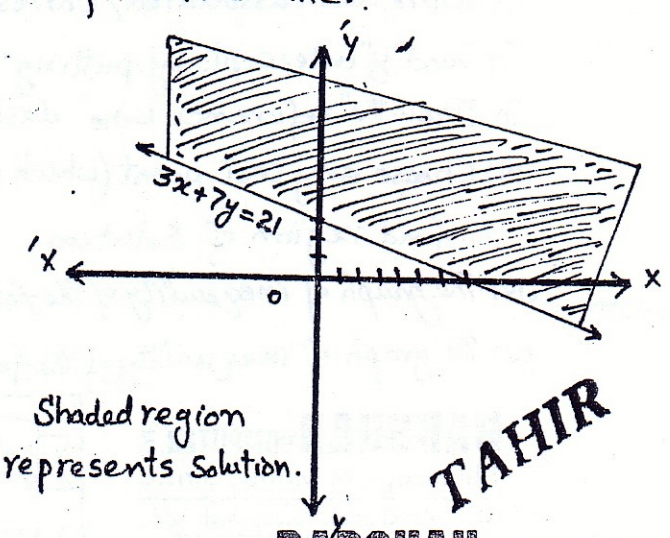
$\Rightarrow y = 3$ $\quad\Rightarrow x = 7$

Pairs are $(0,3), (7,0)$

Let us check whether $(0,0)$ is in Solution region or not:

$\Rightarrow 3(0) + 7(0) \ge 21 \Rightarrow 0 \ge 21$ (false)

Thus Solution region is above the reference Line:



Shaded region represents Solution.

(v) $2x+1 \geq 0$

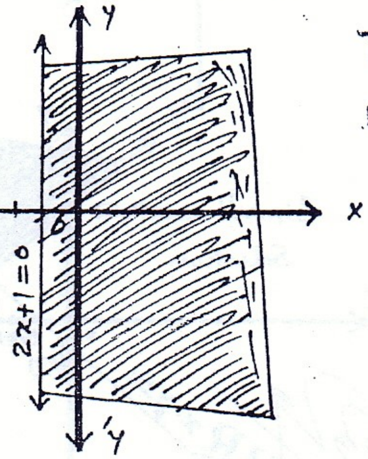
The associated equation is

$2x+1=0$

$\Rightarrow 2x=-1 \Rightarrow x=-\frac{1}{2}$

Let us check whether (0,0) is in the solution region or not:

$2(0)+1 \geq 0 \Rightarrow 1 \geq 0$ (True)



Shaded region represents Solution.

(Remaining Parts do yourself Similarly)

(vi) $3y-4 \leq 0$

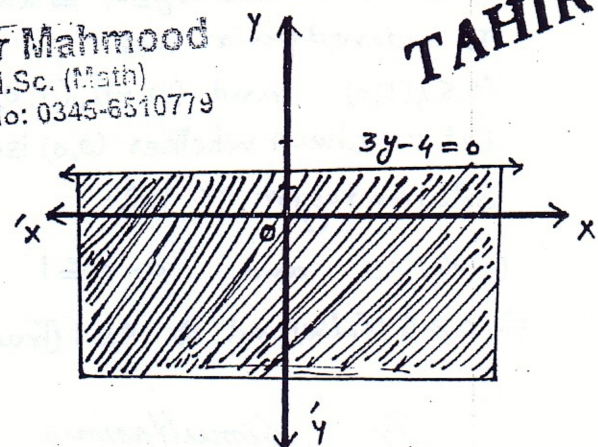
The associated equation is

$3y-4=0$

$\Rightarrow 3y=4 \Rightarrow y=4/3$

Let us check whether (0,0) is in the Solution region or not:

$3(0)-4 \leq 0 \Rightarrow -4 \leq 0$ (TRUE)



Shaded region represents solution.

Q.2: Graph the Solution of following Systems of linear inequalities:

(i) $2x-3y \leq 6$ and $2x+3y \leq 12$

The associated Equations are

$2x-3y=6$ and $2x+3y=12$

To get x, y intercepts let $x=0, y=0$

$x=0 \quad y=0$ and $x=0 \quad y=0$

$\Rightarrow y=-2 \Rightarrow x=3$ and $\Rightarrow y=4 \Rightarrow x=6$

Thus ordered pairs are

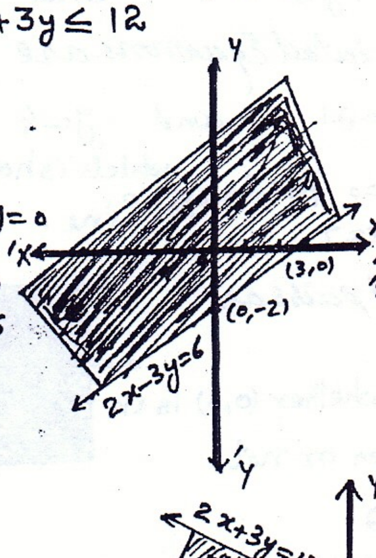
$(0,-2),(3,0)$ and $(0,4),(6,0)$

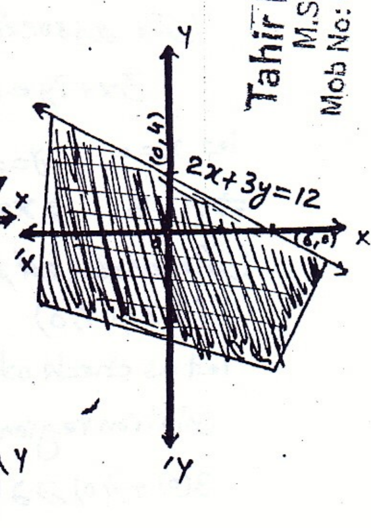
let us check whether (0,0) is in the Solution region or not

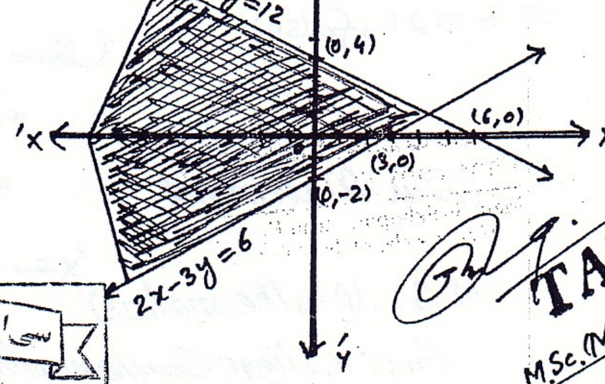
$2(0)-3(0) \leq 6$ and $2(0)+3(0) \leq 12$

$\Rightarrow 0 \leq 6$ (True) $\Rightarrow 0 \leq 12$ (True)

The Simultaneous Solution is shaded.







(ii) $x+y \geq 5$ and $x-y \leq 1$ (4)

The associated equations are

$x+y=5$ and $x-y=1$

To get x, y intercepts

$x=0$ $y=0$ and $x=0$ $y=0$

$\Rightarrow y=5 \Rightarrow x=5$ and $\Rightarrow y=-1 \Rightarrow x=1$

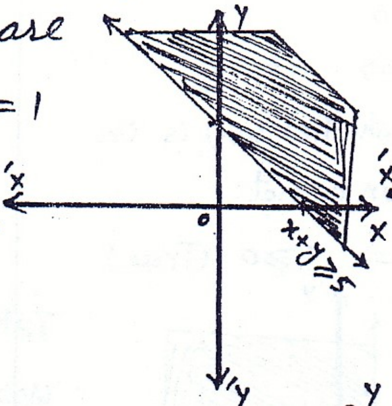
The ordered pairs are

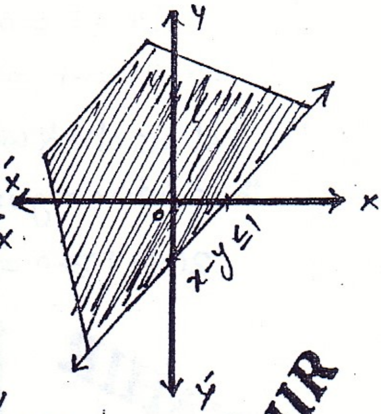
$(0,5),(5,0)$ and $(0,-1),(1,0)$

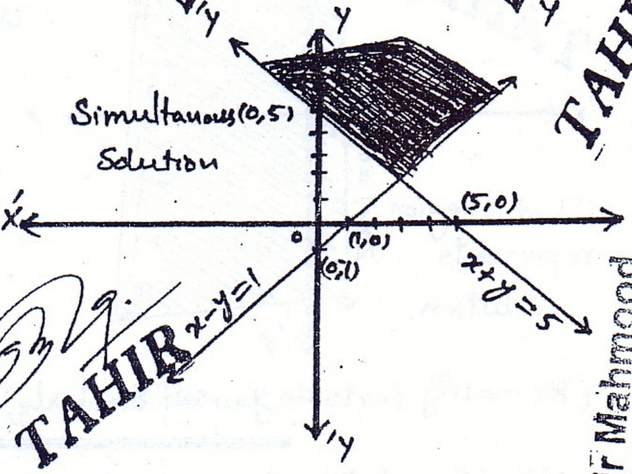
Let us check whether $(0,0)$ is in Solution region or not:

$(0)+(0) \geq 5$ and $0-0 \leq 1$

$\Rightarrow 0 \geq 5$ (false) and $\Rightarrow 0 \leq 1$ (True)







The Simultanous solution is shaded.

(V) $3x+7y \geq 21$ and $y \leq 4$

The associated Equations are

$3x+7y=21$ and $y=4$

let $x=0$ $y=0$ for intercepts which is horizontal line.

$\Rightarrow y=3$ $\Rightarrow x=7$

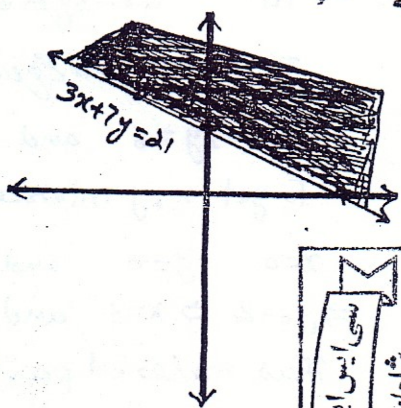
The ordered pairs are

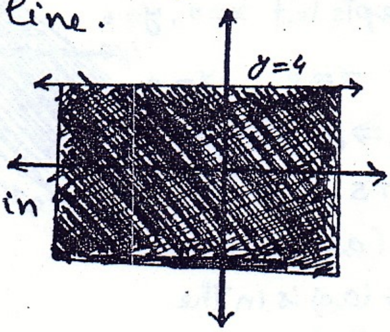
$(0,3),(7,0)$

let us check whether $(0,0)$ is in Solution region or not:

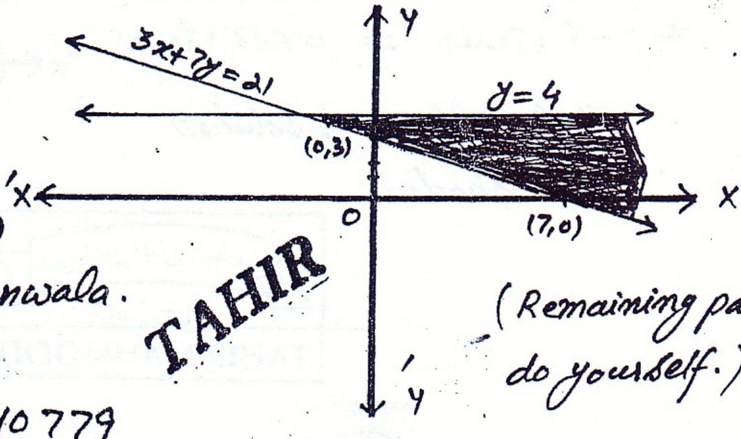
$3(0)+7(0) \geq 21$

$\Rightarrow 0 \geq 21$ (false)





Thus Simultaneous Solution is shaded



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(Remaining parts do yourself.)

Q.3 Find the graphical Solutions of the following Systems:

(i) $2x-3y\le 6$ $\quad$ $2x+3y\le 12$ $\quad$ $y\ge 0$

The associated Equations are

$2x-3y=6$ $\quad$ $2x+3y=12$ $\quad$ $y=0$

To get x, y intercepts

which is upper +ve Portion of xy plane along y-axis.

$x=0$ $\quad$ $y=0$ $\quad$ and $\quad$ $x=0$ $\quad$ $y=0$

$\Rightarrow y=-2$ $\quad$ $\Rightarrow x=3$ $\quad$ $\Rightarrow y=4$ $\quad$ $\Rightarrow x=6$

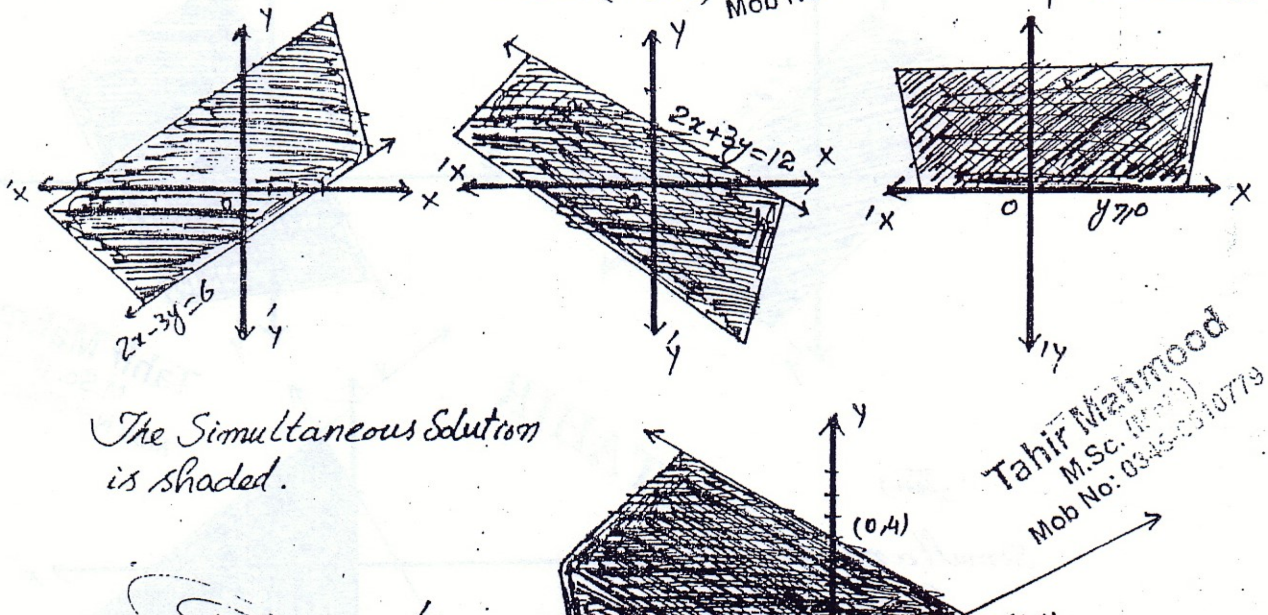
The ordered pairs are

$(0,-2),(3,0)$ and $(0,4),(6,0)$

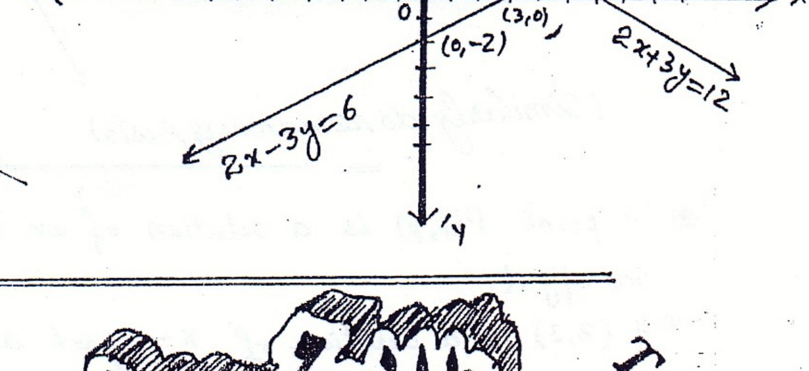
Let us check whether $(0,0)$ is in solution region or not:

$2(0)-3(0)\le 6$ $\quad$ and $\quad$ $2(0)+3(0)\le 12$

$\Rightarrow 0\le 6$ (True) $\quad$ and $\quad$ $0\le 12$. (True)



The Simultaneous Solution is shaded.



(ii) $x+y \leq 5$ $\quad$ $y-2x \leq 2$ $\quad$ $x \geq 0$

The associated Equations are

$x+y=5$ $\quad$ $-2x+y=2$ $\quad$ $x=0$

which is the right half of the xy plane

To get x, y intercepts

$x=0$ $\quad$ $y=0$ $\quad$ and $\quad$ $x=0$ $\quad$ $y=0$

$\Rightarrow y=5$ $\quad$ $\Rightarrow x=5$ $\quad$ and $\Rightarrow y=2$ $\quad$ $\Rightarrow x=-1$

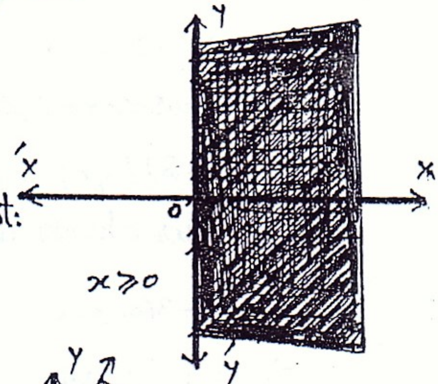
The ordered pairs are

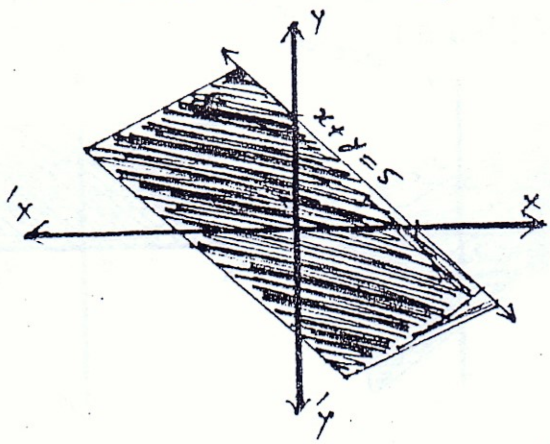
$(0,5),(5,0)$ and $(0,2),(-1,0)$

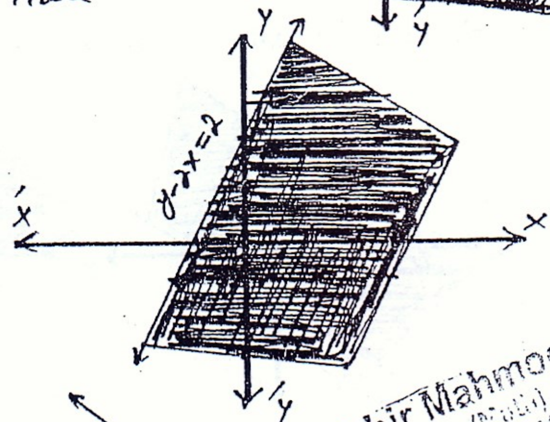
Let us check whether $(0,0)$ is in Solution region or not:

$(0)+(0) \leq 5$ $\quad$ and $\quad$ $(0)-2(0) \leq 2$

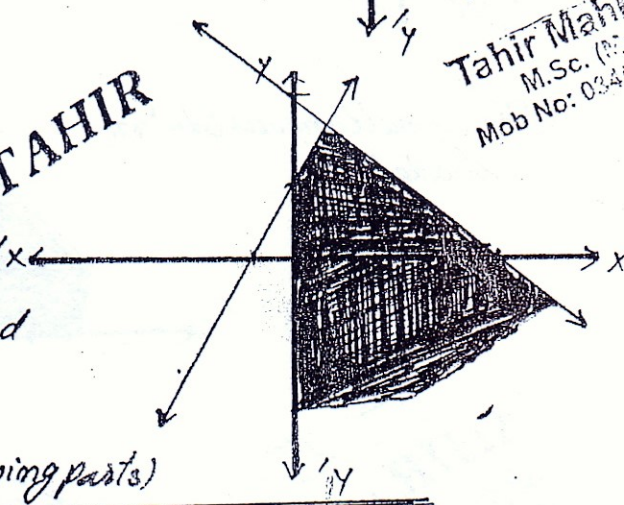
$\Rightarrow 0 \leq 5$ (true) $\quad$ $\Rightarrow 0 \leq 2$ True







Thus Simultaneous Solution is shaded



(Similarly do remaining parts)

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* A point $P(p,q)$ is a solution of an inequality $ax+by \leq c$ if it satisfy it.

e.g $(2,3)$ is a Solution of $x-3y \leq 6$ as $(2)-3(3) \leq 6 \Rightarrow 2-9 \leq 6$
$\Rightarrow -7 \leq 6$ (True)

* Solution of an inequality of one variable lies on real Line.
* Solution of an inequality of two variables lie on plane.
* $x \geq 0$ represents right half plane. * $x \leq 0$ represents left half plane.
* $y \geq 0$ represents upper half plane. * $y \leq 0$ represents lower half plane.

Q.4 Graph the solution and find Corner Points of the followings: (7)

(i) $2x-3y\le 6$ $\quad$ $2x+3y\le 12$ $\quad$ $x\ge 0$

The associated Equations are $\quad$ which is right half of the xy plane.

$2x-3y=6$ ——① $\quad$ $3y+2x=12$ ——②

To get the x, y intercepts

$x=0$ $\quad$ $y=0$ $\quad$ and $\quad$ $x=0$ $\quad$ $y=0$

$\Rightarrow y=-2 \Rightarrow x=3$ $\quad$ and $\Rightarrow y=4$ $\Rightarrow x=6$

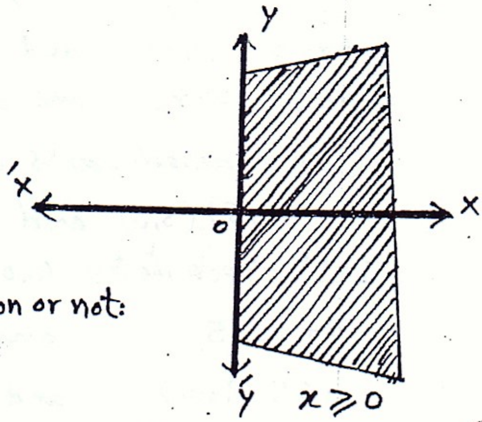
The ordered pairs are

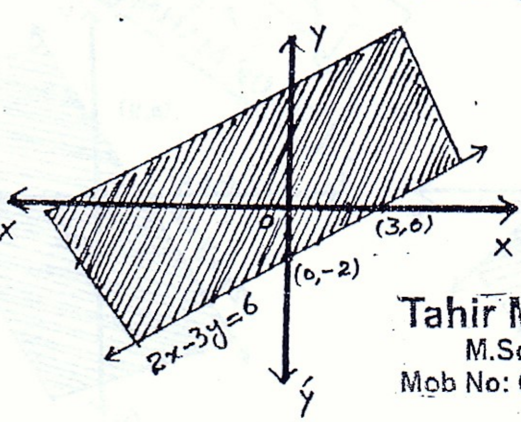
$(0,-2),(3,0)$ and $(0,4),(6,0)$

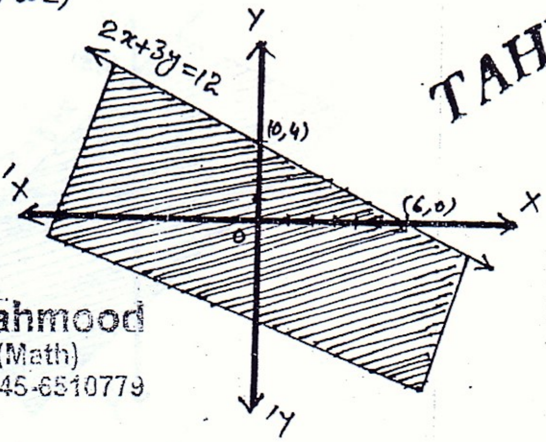
Let us check whether $(0,0)$ is in Solution region or not:

$2(0)-3(0)\le 6$ $\quad$ and $\quad$ $2(0)+3(0)\le 12$

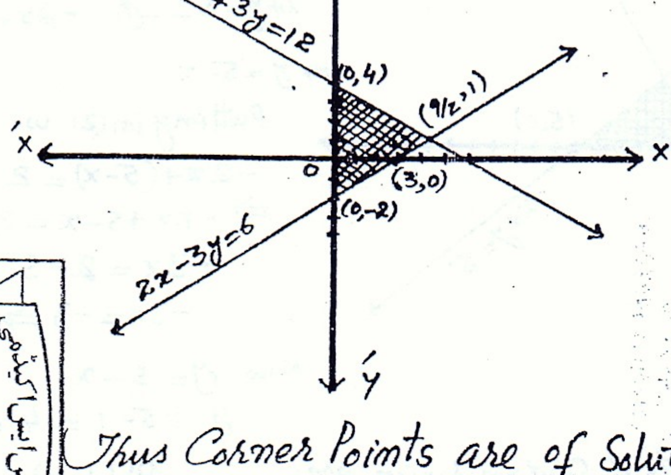
$0\le 6$ (True) and $\quad$ $0\le 12$ (True)







The simultaneous solution is shaded



Solving ① and ②

$2x+3y=12$ ——② $\quad$ $2x-3y=6$ ——①

Adding ① and ②

$4x=18 \Rightarrow x=\frac{9}{2}$

Putting in ② $\quad 3y=12-2\left(\frac{9}{2}\right)$

$3y=12-9=3$

$3y=3 \Rightarrow y=1$

Pt$\left(\frac{9}{2},1\right)$

Thus Corner Points are of Solution region:

$(0,4), (0,-2), (3,0), \left(\frac{9}{2},1\right)$

(ii) $x+y \leq 5$ $\quad -2x+y \leq 2$ $\quad y \geq 0$

The associated equations are:

$x+y=5$ —— ① $\quad -2x+y=2$ —— ②

To get x, y intercepts

let $x=0$ $\quad y=0$ $\quad$ and $\quad x=0$ $\quad y=0$

$\Rightarrow y=5$ $\quad \Rightarrow x=5$ $\quad$ and $\quad \Rightarrow y=2$ $\quad \Rightarrow x=-1$

The ordered pairs are:

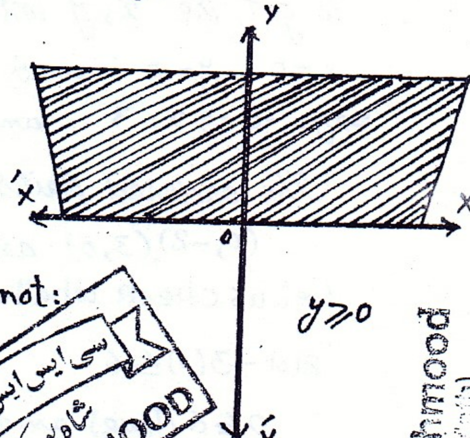
$(0,5),(5,0)$ and $(0,2),(-1,0)$

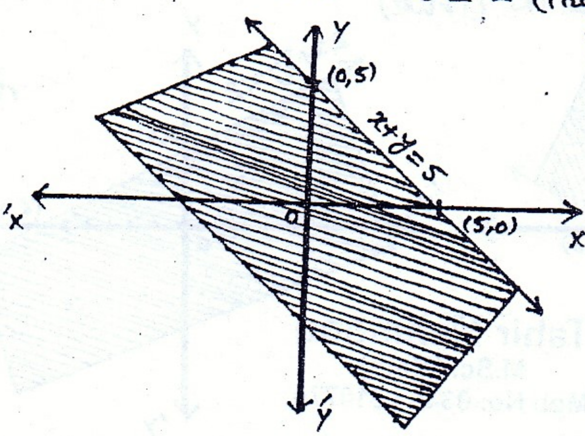
Let's check whether $(0,0)$ is in Solution region or not:

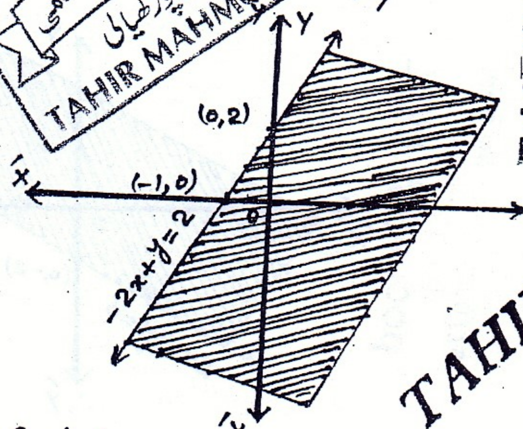
$0+0 \leq 5$ $\quad$ and $\quad -2(0)+0 \leq 2$

$0 \leq 5$ (True) $\quad$ and $\quad 0 \leq 2$ (True)

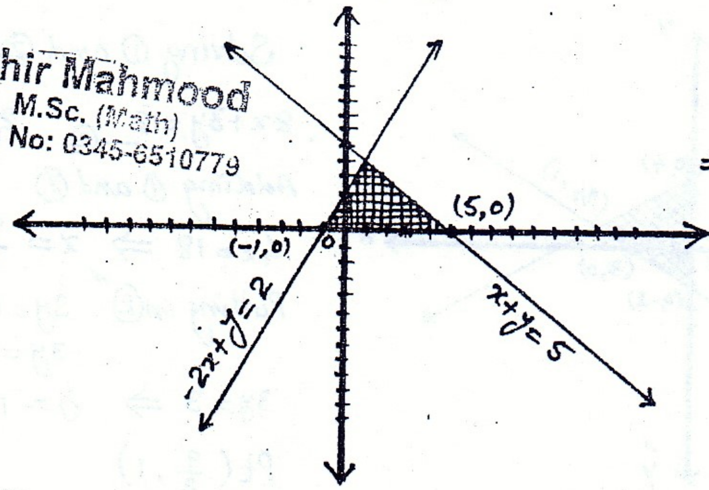
$y \geq 0$ which is the upper half of the xy-plane







The Simultaneous Solution is Shaded



Solving ① and ②

$x+y=5$ —— ① $\quad -2x+y=2$ —— ②

$\Rightarrow y=5-x$

Putting in (2) we have

$-2x+(5-x)=2$

$\Rightarrow -2x+5-x=2$

$-3x=2-5$

$-3x=-3 \Rightarrow \boxed{x=1}$

Now $y=5-x$

$y=5-1=4 \Rightarrow \boxed{y=4}$

Pt$(1,4)$

Thus Corner Points of Solution region are:

$(5,0),(-1,0)$ and $(1,4)$

(iv) $3x+2y \geq 6$ $\qquad$ $x+3y \leq 6$ $\qquad$ $y \geq 0$

The associated Equations are

$3x+2y=6$ and $x+3y=6$

To get x, y intercepts

let $x=0$ $\quad$ $y=0$ $\quad$ and $\quad$ $x=0$ $\quad$ $y=0$

$\Rightarrow y=3$ $\quad \Rightarrow x=2$ $\quad$ and $\Rightarrow y=2$ $\quad \Rightarrow x=6$

Thus ordered pairs are

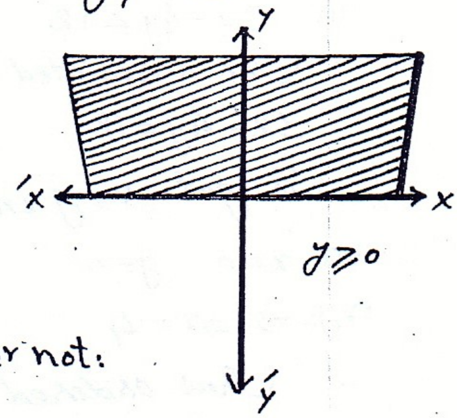
$(0,3),(2,0)$ and $(0,2),(6,0)$

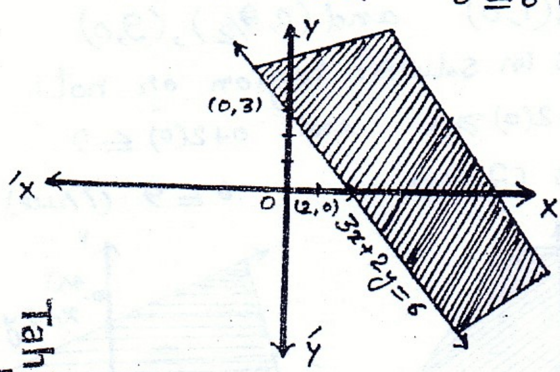
Let's check whether $(0,0)$ is in solution region or not:

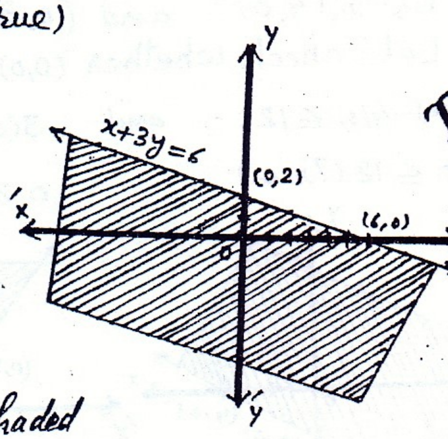
$3(0)+2(0) \geq 6$ $\quad$ and $\quad$ $0+3(0) \leq 6$

$\Rightarrow 0 \geq 6$ (False) $\quad$ and $\quad$ $0 \leq 6$ (True)

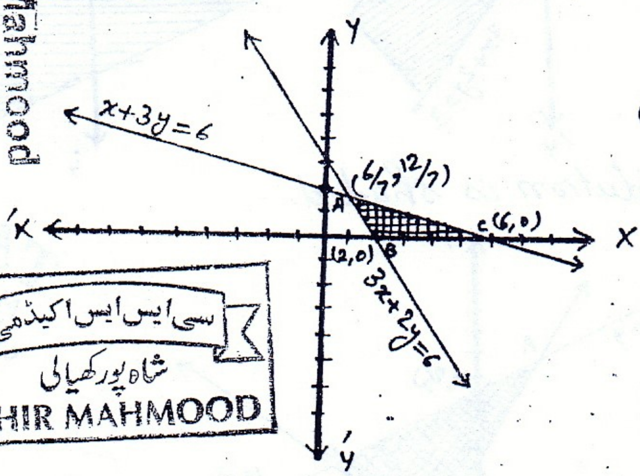
$y \geq 0$ which is upper half of the xy plane.







The Simultaneous solution is shaded



let $3x+2y=6$ — ① $\quad$ $x+3y=6$ — ②

② $\Rightarrow x=6-3y$

Putting in (1), we have

$3(6-3y)+2y=6$

$\Rightarrow 18-9y+2y=6$

$\Rightarrow -7y+18=6 \Rightarrow -7y=6-18$

$\Rightarrow -7y=-12 \Rightarrow \boxed{y=\frac{12}{7}}$

$\because x=6-3y \Rightarrow x=6-3\left(\frac{12}{7}\right)$

$\Rightarrow x=\frac{42-36}{7} \Rightarrow \boxed{x=6/7}$

Pt $\left(\frac{6}{7}, \frac{12}{7}\right)$

Thus Common Points of the Solution Region are:

$(2,0),(6,0)$ and $\left(\frac{6}{7}, \frac{12}{7}\right)$

(Similarly do remaining Parts yourself)

Q.5 Graph the following system's solution regions:

(ii) $3x-4y \leq 12$ $\quad$ $3x+2y \geq 3$ $\quad$ $x+2y \leq 9$

The associated Equations are:

$3x-4y=12$ $\quad$ $3x+2y=3$ $\quad$ $x+2y=9$

To get x, y intercepts

let $x=0$ $\quad$ $y=0$ $\quad$ let $x=0$ $\quad$ $y=0$ $\quad$ let $x=0$ $\quad$ $y=0$

$\Rightarrow y=-3$ $\quad$ $\Rightarrow x=4$ $\quad$ $\Rightarrow y=3/2$ $\quad$ $\Rightarrow x=1$ $\quad$ $\Rightarrow y=9/2$ $\quad$ $\Rightarrow x=9$

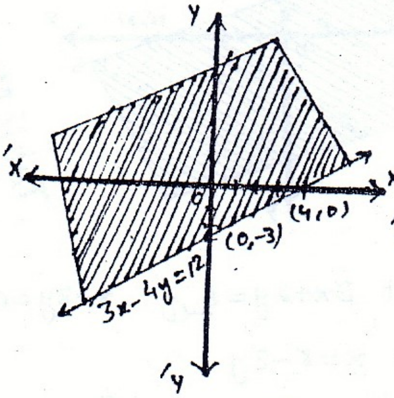
Thus ordered pairs are

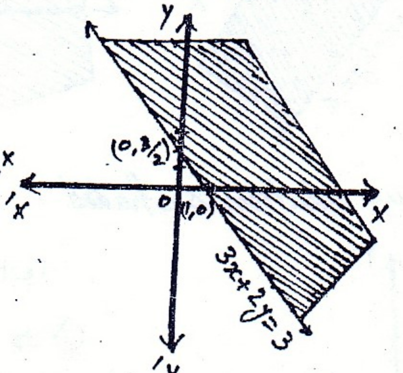
$(0,-3),(4,0)$ and $(0,3/2),(1,0)$ and $(0,9/2),(9,0)$

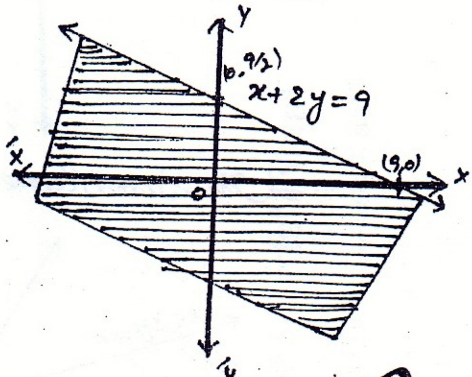
Let's check whether $(0,0)$ is in solution region or not:

$3(0)-4(0) \leq 12$ $\quad$ and $\quad$ $3(0)+2(0) \geq 3$ $\quad$ and $\quad$ $0+2(0) \leq 9$

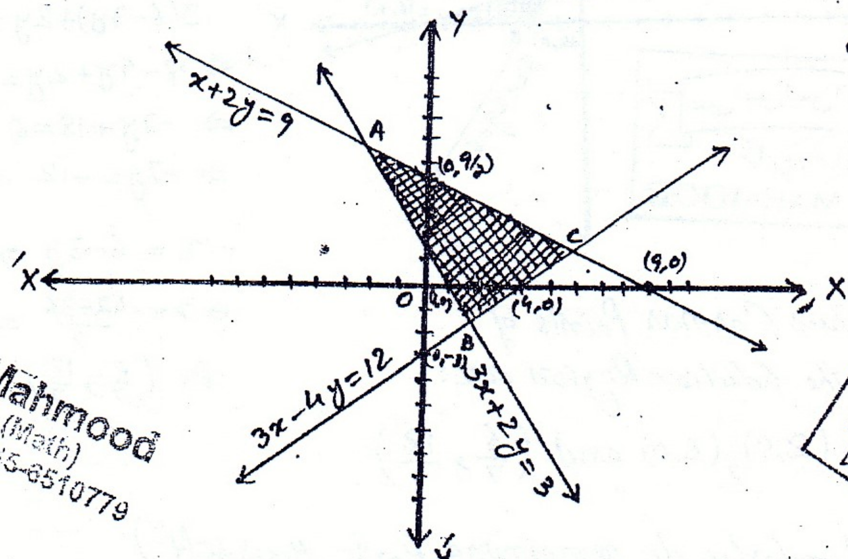
$0 \leq 12$ (True) $\quad$ $0 \geq 3$ (False) $\quad$ $0 \leq 9$ (True)







The Simultaneous Solution is shaded.



Shaded Region Represents Simultaneous Solution.

(iii) $3x-4y\le 12$ $\qquad x+2y\le 6$ $\qquad x+y\ge 1$

The associated Equations are

$3x-4y=12$ $\qquad x+2y=6$ $\qquad x+y=1$

To get x, y intercepts

let $x=0$ $\quad y=0$ $\qquad$ Let $x=0$ $\quad y=0$ $\qquad$ Let $x=0$ $\quad y=0$

$\Rightarrow y=-3$ $\quad \Rightarrow x=4$ $\qquad \Rightarrow y=3$ $\quad \Rightarrow x=6$ $\qquad \Rightarrow y=1$ $\quad \Rightarrow x=1$

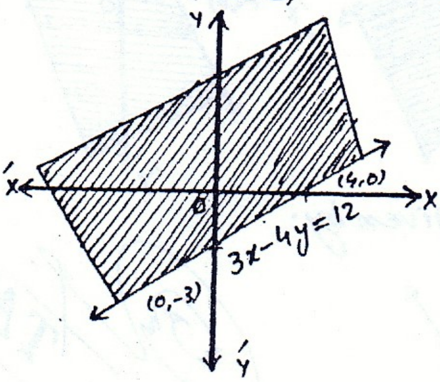
Thus ordered pairs are

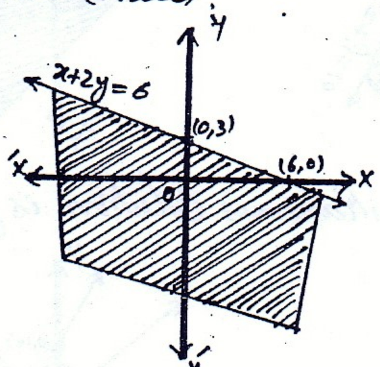
$(0,-3),(4,0)$ and $(0,3),(6,0)$ and $(0,1),(1,0)$

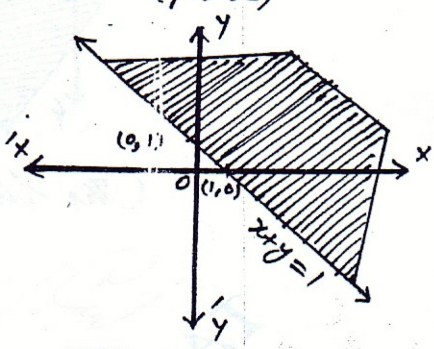
Let's check whether $(0,0)$ is in Solution region or not:

$3(0)-4(0)\le 12$ $\qquad (0)+2(0)\le 6$ $\qquad 0+0\ge 1$

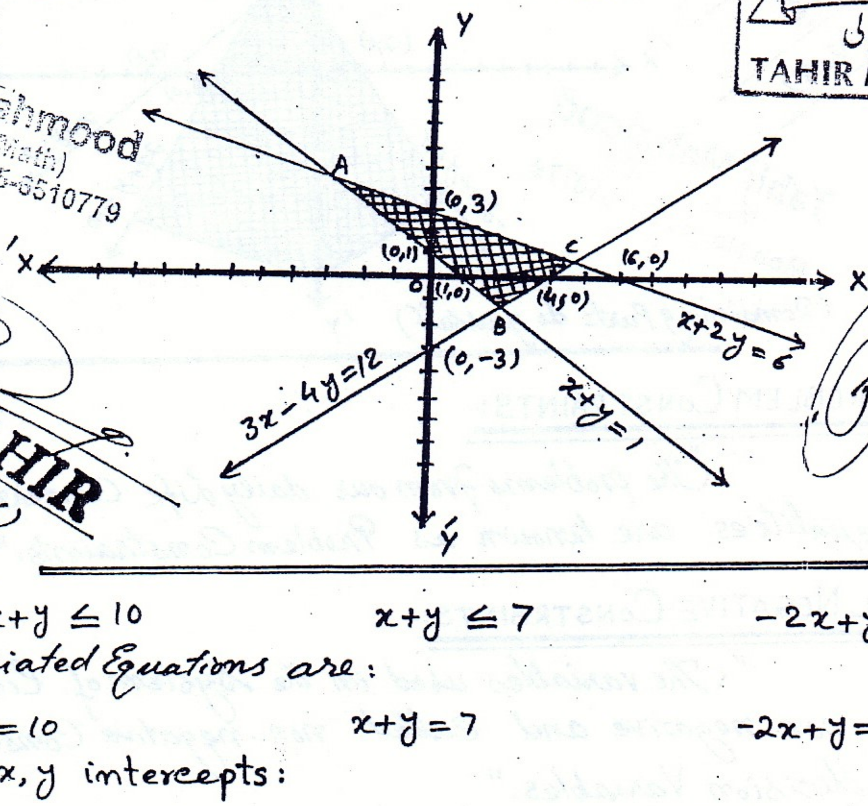
$0\le 12$ (True) $\qquad 0\le 6$ (True) $\qquad 0\ngeq 1$ (False)







The Simultaneous Solution is shaded.



(iv) $2x+y\le 10$ $\qquad x+y\le 7$ $\qquad -2x+y\le 4$

The associated Equations are:

$2x+y=10$ $\qquad x+y=7$ $\qquad -2x+y=4$

To get x, y intercepts:

Let $x=0$ $\quad y=0$ $\qquad$ let $x=0$ $\quad y=0$ $\qquad$ let $x=0$ $\quad y=0$

$\Rightarrow y=10$ $\quad \Rightarrow x=5$ $\qquad \Rightarrow y=7$ $\quad \Rightarrow x=7$ $\qquad \Rightarrow y=4$ $\quad \Rightarrow x=-2$

The ordered pairs are:

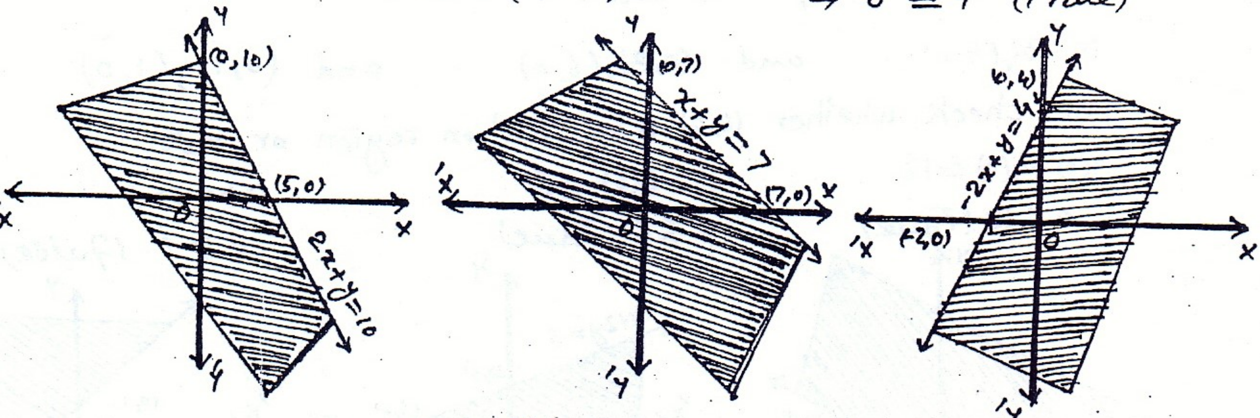
$(0,10),(5,0)$ and $(0,7),(7,0)$ and $(0,4),(-2,0)$

Now let's check whether $(0,0)$ is in solution region or not:

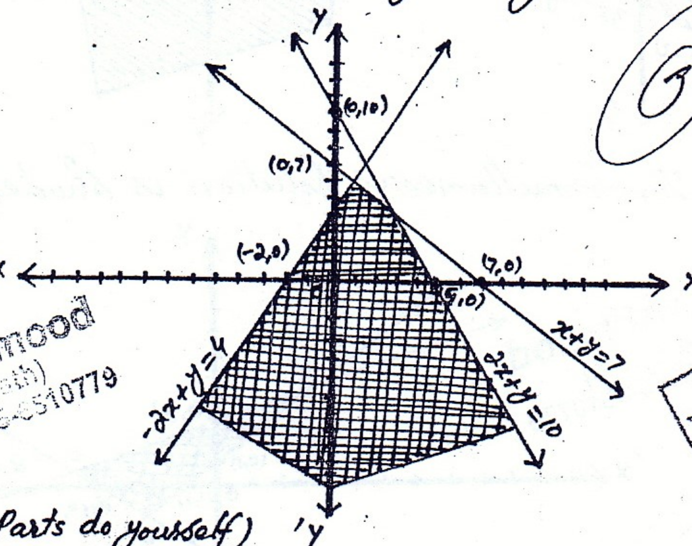
$2(0)+(0) \leq 10$ $\Rightarrow 0 \leq 10$ (True)

$0+0 \leq 7$ $\Rightarrow 0 \leq 7$ (True)

$-2(0)+0 \leq 4$ $\Rightarrow 0 \leq 4$ (True)



The Simultaneous Solution is given by:



(Remaining Parts do yourself)

PROBLEM CONSTRAINTS:-

"The problems from our daily life concerning to linear inequalities are known as Problem Constraints."

NON-NEGATIVE CONSTRAINTS:-

"The variables used in the system of linear inequalities are non-negative and called non-negative constraints or decision variables."

FEASIBLE REGION:-

"The solution region of an inequality restricted to the first quadrant is called Feasible region."