CH#4 (2nd Year)

EXERCISE 4.5

Q.1 $10x^2 - 23xy - 5y^2 = 0$

$10x^2 - 25xy + 2xy - 5y^2 = 0$

$5x(2x-5y) + y(2x-5y) = 0$

$(2x-5y)(5x+y) = 0$

$\Rightarrow 2x-5y=0$ and $5x+y=0$

Now $a=10$ $h=-\frac{23}{2}$ $b=-5$

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b} = \frac{2\sqrt{(\frac{-23}{2})^2-(10)(-5)}}{10-5}$$

$$= \frac{2\sqrt{\frac{529}{4}+50}}{5} = \frac{2\sqrt{\frac{529+200}{2}}}{5} = \frac{\sqrt{729}}{5}$$

$\tan\theta = \frac{27}{5} \Rightarrow \boxed{\theta = \tan^{-1}(\frac{27}{5})}$

Q.2 $3x^2 + 7xy + 2y^2 = 0$

$3x^2 + 6xy + xy + 2y^2 = 0$

$3x(x+2y) + y(x+2y) = 0$

$(3x+y)(x+2y) = 0$

$\Rightarrow x+2y=0$ and $3x+y=0$

$a=3$ $h=\frac{7}{2}$ $b=2$

$$\tan\theta = \frac{2\sqrt{(\frac{7}{2})^2-(3)(2)}}{3+2} = \frac{2\sqrt{\frac{49}{4}-6}}{5} = \frac{2\sqrt{\frac{25}{4}}}{5}$$

$\tan\theta = \frac{5}{5} = 1$

$\tan\theta = 1 \Rightarrow \theta = \tan^{-1}(1)$

$\boxed{\theta = 45^\circ}$

Q.3 $9x^2 + 24xy + 16y^2 = 0$

$9x^2 + 12xy + 12xy + 16y^2 = 0$

$3x(3x+4y) + 4y(3x+4y) = 0$

$(3x+4y)(3x+4y) = 0$

$\Rightarrow 3x+4y=0$ and $3x+4y=0$

Now $a=9$ $h=12$ $b=16$

$$\tan\theta = \frac{2\sqrt{(12)^2-(9)(16)}}{9+16} = \frac{2\sqrt{144-144}}{25}$$

$\tan\theta = 0 \Rightarrow \boxed{\theta = 0^\circ}$

Q.4 $2x^2 + 3xy - 5y^2 = 0$

$2x^2 + 5xy - 2xy - 5y^2 = 0$

$x(2x+5y) - y(2x+5y) = 0$

$(2x+5y)(x-y) = 0$

$\Rightarrow 2x+5y=0$ and $x-y=0$

Now $a=2$ $b=-5$ $h=3/2$

$$\tan\theta = \frac{2\sqrt{(3/2)^2-2(-5)}}{2+(-5)} = \frac{2\sqrt{9/4+10}}{-3} = \frac{2\sqrt{49/4}}{-3}$$

$\tan\theta = \frac{2\times 7}{-3\times 2} = \frac{-7}{3}$

$\boxed{\theta = \tan^{-1}(-7/3)}$

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Q.5 $6x^2-19xy+15y^2=0$

$6x^2-10xy-9xy+15y^2=0$

$2x(3x-5y)-3y(3x-5y)=0$

$(3x-5y)(2x-3y)=0$

$\Rightarrow 3x-5y=0$ and $2x-3y=0$

Now $a=6 \quad h=-19/2 \quad b=15$

$$\tan\theta=\frac{2\sqrt{(-19/2)^2-(6)(15)}}{6+15}$$

$$=\frac{2\sqrt{\frac{361}{4}-90}}{21}=\frac{2\sqrt{\frac{361-360}{4}}}{21}$$

$$=\frac{2\times\frac{1}{2}}{21}=\frac{1}{21}$$

$\boxed{\theta=\tan^{-1}(1/21)}$ Ans.

Q.6 $x^2+2xy\sec\alpha+y^2=0$

$$\frac{y^2}{x^2}+2\frac{y}{x}\sec\alpha+1=0$$

which is quadratic in y/x

$$\frac{y}{x}=\frac{-2\sec\alpha\pm\sqrt{4\sec^2\alpha-4}}{2}$$

$$\frac{y}{x}=\frac{-2\sec\alpha\pm2\sqrt{\tan^2\alpha}}{2}$$

$$\frac{y}{x}=-\sec\alpha\pm\tan\alpha$$

$y=(-\sec\alpha\pm\tan\alpha)x$

$y=(-\sec\alpha+\tan\alpha)x$ ...

and $y=(-\sec\alpha-\tan\alpha)x$

Now $a=1 \quad b=1 \quad h=\sec\alpha$

$$\tan\theta=\frac{2\sqrt{\sec^2\alpha-1}}{2}=\frac{2\sqrt{\tan^2\alpha}}{2}$$

$\tan\theta=\tan\alpha \Rightarrow \boxed{\theta=\alpha}$

Q.7 $x^2-2xy\tan\alpha-y^2=0$

$\Rightarrow y^2+2xy\tan\alpha-x^2=0$

$$\frac{y^2}{x^2}+2\frac{y}{x}\tan\alpha-1=0$$

which is quadratic in y/x so

$$\frac{y}{x}=\frac{-2\tan\alpha\pm\sqrt{4\tan^2\alpha+4}}{2}$$

$$\frac{y}{x}=\frac{-2\tan\alpha\pm2\sqrt{\sec^2\alpha}}{2}$$

$$\frac{y}{x}=\frac{-2\tan\alpha\pm2\sec\alpha}{2}=-\tan\alpha\pm\sec\alpha$$

$\frac{y}{x}=-\tan\alpha+\sec\alpha$ and $\frac{y}{x}=-\tan\alpha-\sec\alpha$

$y=(\sec\alpha-\tan\alpha)x$ and $y=(-\tan\alpha-\sec\alpha)x$

Lines perpendicular to above are:

$$y=\frac{-x}{(\sec\alpha-\tan\alpha)} \text{ and } y=\frac{-x}{-(\tan\alpha+\sec\alpha)}$$

$$y=\frac{x}{(\tan\alpha-\sec\alpha)} \text{ and } y=\frac{x}{(\tan\alpha+\sec\alpha)}$$

$y(\tan\alpha-\sec\alpha)-x=0$ and $y(\tan\alpha+\sec\alpha)-x=0$

Joint Eq is

$[y(\tan\alpha-\sec\alpha)-x][y(\tan\alpha+\sec\alpha)-x]=0$

$y^2(\tan^2\alpha-\sec^2\alpha)+xy(\tan\alpha+\sec\alpha)$

$-xy(\tan\alpha-\sec\alpha)+x^2=0$

$y^2(-1)-2xy\tan\alpha+x^2=0$

$\boxed{x^2-2xy\tan\alpha-y^2=0}$ Ans.

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779
TAHIR

$ax^2+2hxy+by^2=0$

Q.8 Let $y=m_1x$ and $y=m_2x$ are two lines then

$m_1+m_2=-\frac{2h}{b}$ $\quad m_1m_2=\frac{a}{b}$

Eqs $\perp$ to above are

$y=\frac{-1}{m_1}x$ $\quad y=\frac{-1}{m_2}x$

$m_1y+x=0$ $\quad m_2y+x=0$

The Joint Eq is

$(m_1y+x)(m_2y+x)=0$

$(m_1m_2)y^2+(m_1+m_2)xy+x^2=0$

$\frac{a}{b}y^2-\frac{2h}{b}xy+x^2=0$

$$\boxed{ay^2-2hxy+bx^2=0}$$

Required Joint Equation.

Q.9 Find area of region:

$x+y+1=0$ —①

$10x^2-xy-21y^2=0$

$10x^2-15xy+14xy-21y^2=0$

$5x(2x-3y)+7y(2x-3y)=0$

$(2x-3y)(5x+7y)=0$

$2x-3y=0$ —② $\quad 5x+7y=0$ —③

Sloving ①, ②, ③, we get

$A(0,0)$, $B(-7/2, 5/2)$, $C(\frac{-3}{5}, \frac{-2}{5})$

Now Area of $\triangle ABC$

$$\Delta=\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -7/2 & 5/2 & 1 \\ -3/5 & -2/5 & 1 \end{vmatrix}$$

$$\Delta=\frac{1}{2}\left\{0-0+1\left(\frac{7}{5}+\frac{3}{2}\right)\right\}$$

$$=\frac{1}{2}\left\{\frac{14+15}{10}\right\}=\frac{29}{20}$$

$$\boxed{\triangle ABC=\frac{29}{20}\ sq-units}$$

For $ax^2+2hxy+by^2=0$

let $y=m_1x$ and $y=m_2x$ are two lines then

$(y-m_1x)(y-m_2x)=0$

$y^2-(m_1+m_2)xy+(m_1m_2)x^2=0$ —①

$y^2+\frac{2h}{b}xy+\frac{a}{b}x^2=0$ —②

$\Rightarrow m_1+m_2=-\frac{2h}{b}$ and $m_1m_2=\frac{a}{b}$
