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0345-6510779

(iii) System of three Lines can be represented as

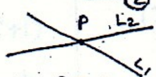
$$\left.\begin{aligned} L_1 &: a_1x+b_1y+c_1=0 \\ L_2 &: a_2x+b_2y+c_2=0 \\ L_3 &: a_3x+b_3y+c_3=0 \end{aligned}\right\} \Rightarrow \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

EXERCISE 4.4

Q.1 Find points of Intersection.

(i) $x-2y+1=0$ —① and $2x-y+2=0$ —②

$x = 2y-1$ from ①



Putting in ② $2(2y-1)-y+2=0$

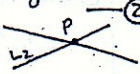
$\Rightarrow 4y-2-y+2=0 \Rightarrow 3y=0$

$\boxed{y=0} \Rightarrow \boxed{x=2(0)-1=-1}$

Thus $P(-1,0)$ is pt of Intersection.

(ii) $3x+y+12=0$ —① $x+2y-1=0$ —②

① $\Rightarrow y=-3x-12$



Putting in ② $x+2(-3x-12)-1=0$

$x-6x-24-1=0 \Rightarrow -5x=25$

$\boxed{x=-5}$ and $y=-3(-5)-12$

$\boxed{y=15-12=3}$

Thus $P(-5,3)$ is pt of Intersection.

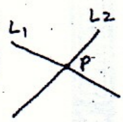
(iii) $x+4y-12=0$ —① $x-3y+3=0$ —②

① $\Rightarrow x=12-4y$

Putting in ② $\Rightarrow (12-4y)-3y+3=0$

$12-4y-3y+3=0 \Rightarrow 7y=15$

$\boxed{y=\frac{15}{7}}$ and $x=12-4\left(\frac{15}{7}\right)$



$\boxed{x=\frac{84-60}{7}=\frac{24}{7}}$

Thus $P\left(\frac{24}{7},\frac{15}{7}\right)$ is pt of intersection.

Q.2 Equations of Lines:-

(i) $P(2,-9)$

Now $2x+5y-8=0$ —① $3x-4y-6=0$ —②

$2\times② \Rightarrow 6x-8y=12$

$3\times① \Rightarrow \underline{6x+15y=24}$

$-23y=-12 \Rightarrow \boxed{y=\frac{12}{23}}$

$2x=8-5\left(\frac{12}{23}\right) \Rightarrow x=4-\frac{30}{23}$

$\boxed{x=\frac{92-30}{23}=\frac{62}{23}}$

Pt of intersection $=Q\left(\frac{62}{23},\frac{12}{23}\right)$

Equation of Line:

$$\frac{y+9}{\frac{12}{23}+9}=\frac{x-2}{\frac{62}{23}-2} \Rightarrow \frac{(y+9)23}{12+207}=\frac{(x-2)23}{62-46}$$

$$\Rightarrow \frac{y+9}{219}=\frac{x-2}{16} \Rightarrow 16y+144=219x-438$$

$\boxed{219x-16y-582=0}$ Ans.

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(ii). $x-y-4=0$ ① and $7x+y+20=0$ ②

① $\Rightarrow x=y+4 \Rightarrow 7(y+4)+y+20=0$

$7y+28+y+20=0$

$8y=-48 \Rightarrow \boxed{y=-6}$

$\boxed{x=-6+4=-2}$

Pt of intersection $= P(-2,-6)$

Slope of Given line $(m_0) = -6$

$(\because y=-6x+14)$

(i) Eq $\parallel$ to $6x-y-14=0$ is

$y+6=-6(x+2)$

$y+6=-6x-12$

$\Rightarrow \boxed{6x+y+18=0}$

(ii) Eq $\perp$ to $6x-y-14=0$

$y+6=\frac{1}{6}(x+2) \quad \because m=\frac{+1}{6}$

$6y+36=x+2$

$\boxed{x-6y-34=0}$

(iii) If L_1 and L_2 are the two lines then the line "L" through intersection of L_1 & L_2 is written as

$L = L_1 + kL_2 \quad k\in\mathbb{R}$

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(iii) $x+2y+3=0$ ① $\quad 3x+4y+7=0$ ②

The line through intersection of ① & ②

$(x+2y+3)+k(3x+4y+7)=0$

$(1+3k)x+(2+4k)y+(3+7k)=0$

Let $y=0$ to get x-intercept

$(1+3k)x=-(3+7k) \Rightarrow x=\frac{-(3+7k)}{(1+3k)}$

Let $x=0$ to get y-intercept

$y(2+4k)=-(3+7k) \Rightarrow y=\frac{-(3+7k)}{2+4k}$

$\because$ x and y intercepts are equal

So $\frac{-(3+7k)}{2+4k} = \frac{-(3+7k)}{1+3k} \Rightarrow 2+4k=1+3k$

$\boxed{k=-1}$

Thus $(x+2y+3)-1(3x+4y+7)=0$

$-2x-2y-4=0$

$\boxed{x+y+2=0}$ required line

Q.3 $16x-10y-33=0$ ① $\quad 12x+14y+29=0$ ②

①×3 $\Rightarrow \cancel{48x}-30y-99=0$

②×4 $\Rightarrow \underline{\cancel{48x} \pm 56y \pm 116=0}$

$-86y-215=0 \Rightarrow y=\frac{-215}{86}=\frac{-5}{2}$

$\boxed{y=\frac{-5}{2}}$ $\quad 16x-10\left(\frac{-5}{2}\right)-33=0$

$16x+25-33=0 \Rightarrow 16x=8$

$\boxed{x=\frac{1}{2}}$

Pt of intersection $= P\left(\frac{1}{2}, \frac{-5}{2}\right)$

Eq of line through intersection of

$x-y+4=0$ & $x-7y+2=0$

$(x-y+4)+k(x-7y+2)=0$

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Now $P(\frac{1}{2}, \frac{-5}{2})$ lies on line so

$$\left(\frac{1}{2}+\frac{5}{2}+4\right)+K\left(\frac{1}{2}+\frac{35}{2}+2\right)=0$$

$$7+20K=0 \Rightarrow K=\frac{-7}{20}$$

$$(x-y+4)-\frac{7}{20}(x-7y+2)=0$$

$$20x-20y+80-7x+49y-14=0$$

$$\boxed{13x+29y+66=0}$$

Q.4 Condition of Concurrency

$y=m_1x+c_1$ —① $\quad y=m_2x+c_2$ —② $\quad y=m_3x+c_3$ —③

Comparing ① & ②

$$m_1x+c_1=m_2x+c_2$$

$$m_2x-m_1x=c_1-c_2$$

$$\boxed{x=\frac{c_1-c_2}{m_2-m_1}}$$

$$y=m_1\left(\frac{c_1-c_2}{m_2-m_1}\right)+c_1$$

$$y=\frac{\cancel{m_1c_1}-m_1c_2+m_2c_1-\cancel{m_1c_1}}{m_2-m_1}$$

$$\boxed{y=\frac{m_2c_1-m_1c_2}{m_2-m_1}}$$

Putting in (3)

$$m_3x-y+c_3=0$$

$$m_3\left[\frac{c_1-c_2}{m_2-m_1}\right]-\left[\frac{m_2c_1-m_1c_2}{m_2-m_1}\right]+c_3=0$$

Multiplying by (m_2-m_1), we have

$$m_3c_1-m_3c_2-m_2c_1+m_1c_2+m_2c_3-m_1c_3=0$$

$$m_1(c_2-c_3)+m_2(c_3-c_1)+m_3(c_1-c_2)=0$$

This can also written as

$$\begin{vmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ m_3 & -1 & c_3 \end{vmatrix}=0$$

Q.5 $P=?$ Lines are Concurrent so

$$\begin{vmatrix} 2 & -3 & -1 \\ 3 & -1 & -5 \\ 3 & P & 8 \end{vmatrix}=0$$

$$2(-8+5P)+3(24+15)-1(3P+3)=0$$

$$-16+10P+117-3P-3=0$$

$$7P+98=0 \Rightarrow P=\frac{-98}{7}=-14$$

$$\boxed{P=-14}$$

Q.6 Now condition of Concurrency is to verify

$$\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}$$

$$=4(8-6)+3(-6+6)-8(-3+4)$$

$$=8+0-8=0$$

Thus Lines are Concurrent.

Now $L_1 \Rightarrow 4x-3y-8=0 \Rightarrow y=\frac{4}{3}x-\frac{8}{3}$

$L_2 \Rightarrow 3x-4y-6=0 \Rightarrow y=\frac{3}{4}x-\frac{3}{2}$

$L_3 \Rightarrow x-y-2=0 \Rightarrow y=x-2$

Slope of $L_1=(m_1)=4/3$
Slope of $L_2=(m_2)=3/4$
Slope of L_3 $(m_3)=1$

Let α and β are the angles between L_1, L_3 and L_2, L_3 then

$$\tan\alpha = \frac{m_3 - m_1}{1 + m_3 m_1} \qquad \tan\beta = \frac{m_3 - m_2}{1 + m_3 m_2}$$

$$\tan\alpha = \frac{1 - 4/3}{1 + 1\cdot 4/3} \qquad \tan\beta = \frac{1 - 3/4}{1 + 3/4}$$

$$\tan\alpha = \left|\frac{3-4}{3+4}\right| \qquad \tan\beta = \left|\frac{4-3}{4+3}\right|$$

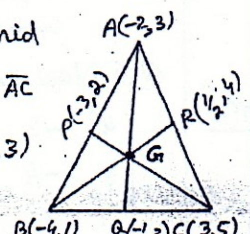
$$\angle\alpha = \tan^{-1}(1/7) \qquad \angle\beta = \tan^{-1}(1/7)$$

$$\because m\angle\alpha = m\angle\beta$$

Thus L_3 bisect the angle between L_1 and L_2.

Q.7 $A(-2,3), B(-4,1), C(3,5)$

(i) Let P, Q, R be the mid points of $\overline{AB}, \overline{BC}, \overline{AC}$ then $P(-3, 2), Q(-\frac{1}{2}, 3)$, $R(\frac{1}{2}, 4)$



Eq of $\overline{AQ}$ median:

$$\frac{y-3}{3-3} = \frac{x+2}{-\frac{1}{2}+2} \Rightarrow \frac{y-3}{0} = \frac{(x+2)2}{3}$$

$$y - 3 = 0\left[\frac{2x+4}{3}\right]$$

$$y - 3 = 0 \Rightarrow \boxed{y = 3}$$

Now Eq of $\overline{CP}$ Median

$$\frac{y-2}{5-2} = \frac{x+3}{3+3}$$

$$\frac{y-2}{3} = \frac{x+3}{6} \Rightarrow 2y - 4 = x + 3$$

$$\boxed{x - 2y + 7 = 0}$$

Point of Intersection of $\overline{AQ}$ and $\overline{CP}$

$$x - 2(3) + 7 = 0 \Rightarrow x = -1$$

Thus Centriod $= (-1, 3)$

(ii) **Orthocentre:-**

"The point of intersection of the altitudes of triangle is called orthocentre."

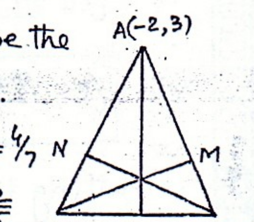
Centroid:-

"The point of intersection of the medians of a triangle is called centriod."

Circum Centre:-

"The point of intersection of the perpendicular bisectors is called Circum Centre."

Let $\overline{AL}, \overline{BM}, \overline{CN}$ be the altitudes of ΔABC.



Slope of $\overline{BC}\ (m_1) = \frac{5-1}{3+4} = 4/7$

Slope of $\overline{AC}\ (m_2) = \frac{5-3}{3+2} = \frac{2}{5}$

Slope of $\overline{AL} = \frac{-1}{m_1} = -7/4$

Eq of $\overline{AL} \Rightarrow y - 3 = -7/4(x+2)$

$$4y - 12 = -7x - 14$$

$$\boxed{7x + 4y + 2 = 0} \quad (1)$$

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Mob No: 0345-6510779

Slope of $\overline{BM} = \frac{-1}{m_2} = -5/2$

Eq of $\overline{BM} \Rightarrow y-1 = -\frac{5}{2}(x+4)$

$2y-2 = -5x-20$

$\boxed{5x+2y+18=0}$ —②

Point of intersection of $\overline{AL}$, $\overline{BM}$ is

①×1 ⇒ $7x+4y+2=0$

②×2 ⇒ $\underline{10x \pm 4y \pm 36=0}$

$-3x-34=0 \Rightarrow \boxed{x=\frac{-34}{3}}$

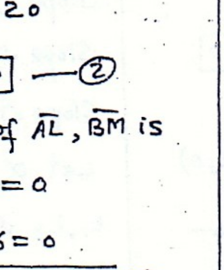
$2y = -18-5\left(\frac{-34}{3}\right)$

$y = -9-5\left(\frac{-17}{3}\right) = \frac{-27+85}{3}$

$\boxed{y=\frac{58}{3}}$

Thus Orthocentre $= \left(\frac{-34}{3}, \frac{58}{3}\right)$

(iii) Let D, E, F are the mid points of $\overline{BC}$, $\overline{AC}$, $\overline{AB}$ then



$D(-1/2, 3)$

$E\left(\frac{1}{2}, 4\right), F(-3, 2)$

Slope of $\overline{BC} = (m_1) = 4/7$

Slope of $\overline{AC} = (m_2) = 2/5$

Slope of $\overline{ID} = \frac{-7}{4}$

Eq of $\overline{ID} \Rightarrow y-3 = \frac{-7}{4}\left(x+\frac{1}{2}\right)$

$8y-24 = -14x-7$

$\boxed{14x+8y-17=0}$ —③

Slope of $\overline{IE} = \frac{-1}{m_2} = \frac{-5}{2}$

Eq of $\overline{IE} \Rightarrow y-4 = -\frac{5}{2}\left(x-\frac{1}{2}\right)$

$4y-16 = -10x+5$

$\boxed{10x+4y-21=0}$ —④

Point of intersection of $\overline{IE}$, $\overline{ID}$

③×1 ⇒ $14x+8y-17=0$

④×2 ⇒ $\underline{20x \pm 8y \mp 42=0}$

$-6x+25 \Rightarrow \boxed{x=\frac{25}{6}}$

$4y = 21-10\left(\frac{25}{6}\right) = \frac{126-250}{6}$

$y = \frac{-124}{24} = \frac{-31}{6} \Rightarrow \boxed{y=\frac{-31}{6}}$

Thus Circum Centre $= \left(\frac{25}{6}, \frac{-31}{6}\right)$

Now To show them collinear take

$$\begin{vmatrix} -1 & 3 & 1 \\ \frac{-34}{3} & 58/3 & 1 \\ \frac{25}{6} & \frac{-31}{6} & 1 \end{vmatrix}$$

$= -1\left(\frac{58}{3}+\frac{31}{6}\right) - 3\left(\frac{-34}{3}-\frac{25}{6}\right) + 1\left(\frac{1054}{18}-\frac{1450}{18}\right)$

$= -\frac{147}{6} + \frac{279}{6} - \frac{396}{18} = \frac{-441+837-396}{18}$

$= \frac{837-837}{18} = 0$

Thus Pts. are Collinear.

Q.8 $\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}$

$= 4(8-6)+3(-6+6)-8(-3+4)$

$= 8+0-8 = 0$ Lines are Concurrent.

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Mob No: 0345-6510779

$x = y+2$ From ③

$① \Rightarrow \quad 4(y+2) - 3y = 8$

$4y - 3y + 8 = 8 \Rightarrow \boxed{y=0}$

$x = 0+2 = 2 \Rightarrow \boxed{x=2}$

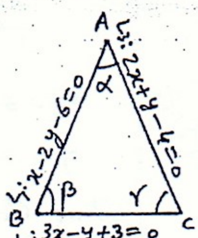
Thus Pt. of intersection $= (2,0)$

Q.9 Let.

$L_1: x - 2y - 6 = 0$

$L_2: 3x - y + 3 = 0$

$L_3: 2x + y - 4 = 0$



$L_2 \Rightarrow y = 3x+3$ & $L_3 \Rightarrow y = 4-2x$

Now $3x+3 = 4-2x \Rightarrow 5x = 1$

$x = \frac{1}{5} \qquad y = 3(\frac{1}{5}) + 3$

$y = \frac{18}{5}$

$\boxed{C\left(\frac{1}{5}, \frac{18}{5}\right)}$

Now $L_1 \Rightarrow x - 2(3x+3) = 6$

$x - 6x - 6 = 6 \Rightarrow -5x = 12$

$x = \frac{-12}{5} \Rightarrow y = 3\left(\frac{-12}{5}\right) + 3$

$y = \frac{-36+15}{5} = \frac{-21}{5}$

$\boxed{B\left(\frac{-12}{5}, \frac{-21}{5}\right)}$

Now $y = 4-2x$ from L_3

$L_1 \Rightarrow x - 2(4-2x) = 6$

$x - 8 + 4x = 6 \Rightarrow 5x = 14$

$x = \frac{14}{5}$ and $y = 4 - 2\left(\frac{14}{5}\right)$

$y = \frac{+20-28}{5}$

$y = -\frac{8}{5}$

Thus $\boxed{A\left(\frac{14}{5}, \frac{-8}{5}\right)}$

Slope of L_1 $(m_1) = \frac{1}{2}$

Slope of L_2 $(m_2) = 3$

Slope of L_3 $(m_3) = -2$

Let α, β, γ be the angles between L_1, L_3 & L_1, L_2 and L_2, L_3 resp.

$\tan\alpha = \left|\frac{m_1 - m_3}{1 + m_1 m_3}\right|$

$\tan\alpha = \left|\frac{\frac{1}{2}+2}{1 - 2 \cdot \frac{1}{2}}\right| = \left|\frac{5/2}{0}\right|$

$\tan\alpha = |\infty| \Rightarrow \alpha = \tan^{-1}(\infty)$

$\boxed{\alpha = \frac{\pi}{2}}$

Now $\tan\beta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\beta = \tan^{-1}\left|\frac{\frac{1}{2} - 3}{1 + \frac{3}{2}}\right| = \tan^{-1}\left(\frac{5/2}{5/2}\right)$

$\boxed{\beta = \tan^{-1}(1) = \frac{\pi}{4}}$

Now $\tan\gamma = \left|\frac{m_2 - m_3}{1 + m_2 m_3}\right|$

$\gamma = \tan^{-1}\left|\frac{3+2}{1-6}\right| = \tan^{-1}\left(\frac{5}{5}\right)$

$\boxed{\gamma = \frac{\pi}{4}}$

Thus angles and Vertices of triangle are given.

Q.10 Find the angle between ℓ_1 and ℓ_2

(i) Slope of $(\ell_1)(m_1) = \frac{10-7}{7-2} = 3/5$

Slope of $\ell_2\,(m_2) = \frac{3-1}{-5-1} = \frac{-2}{6} = -1/3$

Let θ be the required angle then

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1+m_1 m_2}\right|$$

$$\theta = \tan^{-1}\left|\frac{3/5 + 1/3}{1 - 3/5 \cdot 1/3}\right|$$

$$\theta = \tan^{-1}\left(\frac{14/15}{12/15}\right) = \tan^{-1}\left(\frac{7}{6}\right)$$

$$\boxed{\theta = \tan^{-1}(7/6)}$$ Ans.

(ii) Slope of $\ell_1\,(m_1) = \frac{7+1}{5-3} = \frac{8}{2} = 4$

Slope of $\ell_2\,(m_2) = \frac{4-2}{2+8} = \frac{2}{10} = 1/5$

Let θ be the required angle then

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1+m_1 m_2}\right|$$

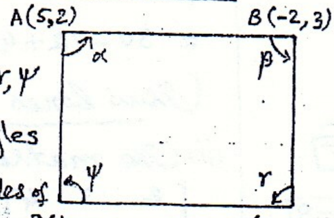
$$\theta = \tan^{-1}\left(\frac{4-1/5}{1+4/5}\right) = \tan^{-1}\left(\frac{19/5}{9/5}\right)$$

$$\boxed{\theta = \tan^{-1}\left(\frac{19}{9}\right)}$$ Ans.

(iii), (iv) do your self.

Q.11 Similar to ⑨ do yourself.

Q.12



Let $\alpha, \beta, \gamma, \psi$ be the angles between sides of ABCD quadrilateral.

Slope of $\overline{AB}\,(m_1) = \frac{3-2}{-2-5} = -1/7$

Slope of $\overline{BC}\,(m_2) = \frac{3+4}{-2+3} = 7$

Slope of $\overline{CD}\,(m_3) = \frac{-4+5}{-3-4} = -1/7$

Slope of $\overline{AD}\,(m_4) = \frac{2+5}{5-4} = 7$

Now $\tan\alpha = \left|\frac{m_1 - m_4}{1+m_1 m_4}\right|$

$$\alpha = \tan^{-1}\left(\frac{-\frac{1}{7} - 7}{1 - 1/7 \cdot 7}\right)$$

$$\boxed{\alpha = \tan^{-1}(\infty) = \pi/2}$$

Now $\tan\beta = \left|\frac{m_1 - m_2}{1+m_1 m_2}\right|$

$$\beta = \tan^{-1}\left|\frac{-1/7 - 7}{1 - 7/7}\right|$$

$$\boxed{\beta = \tan^{-1}(\infty) = \pi/2}$$

Now $\tan\gamma = \left|\frac{m_2 - m_3}{1+m_2 m_3}\right|$

$$\tan\gamma = \left|\frac{7+1/7}{1-7\cdot 1/7}\right|$$

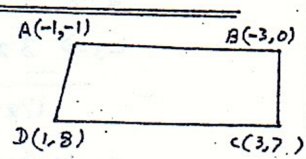
$$\boxed{\gamma = \tan^{-1}(\infty) = \pi/2}$$

Now $\tan\psi = \left|\frac{m_1 - m_4}{1+m_1 m_4}\right|$

$$\tan\psi = \left|\frac{-1/7 - 7}{1 - 7\cdot 1/7}\right|$$

$$\boxed{\psi = \tan^{-1}(\infty) = \pi/2}$$

Q.13



Trapezium:-

"A quadrilateral in which two sides are parallel and other two may or may not be parallel Called Trapezium."

Slope of $\overline{AB}\ (m_1) = \frac{0+1}{-3+1} = -\frac{1}{2}$

Slope of $\overline{BC}\ (m_2) = \frac{7-0}{3+3} = \frac{7}{6}$

Slope of $\overline{CD}\ (m_3) = \frac{8-7}{1-3} = \frac{-1}{2}$

Slope of $\overline{AD}\ (m_4) = \frac{8+1}{1+1} = \frac{9}{2}$

$\because m_1 = m_3 \Rightarrow \overline{AB} \parallel \overline{CD}$

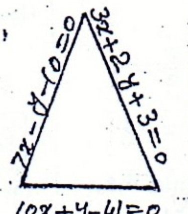
Thus ABCD is trapazium.

Q.14 Take

$L_1: 7x - y - 10 = 0$

$L_2: 10x + y - 41 = 0$

$L_3: 3x + 2y + 3 = 0$



Solving L_1 and L_2

$$\begin{aligned} 7x - y - 10 &= 0 \\ 10x + y - 41 &= 0 \\ \hline 17x - 51 = 0 &\Rightarrow \boxed{x=3} \end{aligned}$$

$y = 41 - 10(3) = 41 - 30 = 11 \Rightarrow \boxed{y=11}$

Let $A(3,11)$ ——— (1)

Solving L_2, L_3

$$\begin{aligned} 2\times L_2 \Rightarrow\ 20x + 2y - 82 &= 0 \\ L_3 \Rightarrow\ 3x + 2y + 3 &= 0 \\ \hline 17x - 85 = 0 &\Rightarrow \boxed{x=5} \end{aligned}$$

$y = 41 - 10(5) = -9 \Rightarrow \boxed{y=-9}$

Let $B(5,-9)$ ——— (2)

Solving L_1 and L_3

$$\begin{aligned} 2\times L_1 \Rightarrow\ 14x - 2y - 20 &= 0 \\ 3x + 2y + 3 &= 0 \\ \hline 17x - 17 = 0 &\Rightarrow \boxed{x=1} \end{aligned}$$

$y = 7x - 10 = 7(1) - 10 = -3 \Rightarrow \boxed{y=-3}$

Let $C(1,-3)$ ——— (3)

Now area of ΔABC

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & 11 & 1 \\ 5 & -9 & 1 \\ 1 & -3 & 1 \end{vmatrix}$$

$\Delta = \frac{1}{2}\{3(-9+3) - 11(5-1) + 1(-15+9)\}$

$\Delta = \frac{1}{2}\{-18 - 44 - 6\} = \frac{1}{2}(-68)$

$\Delta = |-34| = 34$ sq units.

Q.15 Similar to Q.7

Q.16 Express in matrix form

(i) The matrix form is

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now $\begin{vmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{vmatrix}$

$= 1(-14+44) - 3(28-4) - 2(-22+1)$

$= 30 - 72 + 42 = 72 - 72 = 0$

Thus lines are Concurrent.

(ii) The matrix form is

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now $\begin{vmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{vmatrix}$

$= 2(16+3) - 3(-8+9) + 4(1+6) = 38 - 3 - 28$
$= 38 - 31 = 7 \neq 0$ Non-Concurrent

TAHIR MEHMOOD
M.Sc Math
0345-6510779

(iii) The matrix form is

$$\begin{bmatrix} 3 & -4 & -2 \\ 1 & 2 & -4 \\ 3 & -2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now $\begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & -4 \\ 3 & -2 & 5 \end{vmatrix}$

$= 3(10-8)+4(5+12)-2(-2-6)$
$= 6+68+16 = 90 \neq 0$

Thus lines are non-Concurrent.

Q.17 Express as system of equations.

(a) $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{cases} x-1=0 \\ 2x+1=0 \\ -y+2=0 \end{cases}$ (required)

Now $\begin{vmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{vmatrix}$

$= 1(0+1)-0-1(-2-0) = 1+2 = 3 \neq 0$

Lines are nonConcurrent

(b) $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{cases} x+y+2=0 \\ 2x+4y-3=0 \\ 3x+6y-5=0 \end{cases}$ (required)

Now $\begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{vmatrix}$

$= 1(-20+18)-1(-10+9)+2(12-12)$
$= -2+1+0 = -1 \neq 0$

Lines are non-Concurrent.

Second degree Homogenous Equation

An equation.

$$ax^2+2hxy+by^2=0 \quad \text{—— (1)}$$

is called 2nd degree homogenous equation such that a, b, h are not simultaneously zero.

* Equation (1) represents a pair of straight lines through origion.
* Lines are real and distinct if $h^2-ab>0$
* Lines are Coincident if $h^2-ab=0$
* lines are imaginary if $h^2-ab<0$

Angle between Pair of Lines:-

Let θ is the angle between pair of lines $ax^2+2hxy+by^2=0$

then $\tan\theta = \dfrac{2\sqrt{h^2-ab}}{a+b}$

* Lines are Coincident if $h^2-ab=0$
$\Rightarrow \theta = 0^\circ$ (lines are parallel)
* Lines are perpendicular if $\theta = 90^\circ$ and $a+b=0$