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Inclination of a Line:-

An angle θ subtend by a line to the X-axis (Anti Clock wise) is called Inclination of a line. where $0° < \theta < 180°$

Slope of a Line:-

Tangent of the inclination of the line θ is called slope of the line and is denoted by m

$$m = \tan\theta$$

* If $m = 0$ then line is parallel to x-axis.

* If $m = \infty$ then line is parallel to y-axis.

* Let m_1 is the slope of L_1 and m_2 is the slope of L_2 then

L_1 L_2

(i) L_1 and L_2 are parallel if $m_1 = m_2$

(ii) L_1 and L_2 are perpendicular if $m_1 m_2 = -1$

L_1 L_2

$$\text{Slope}(m) = \frac{\text{Rise}}{\text{Horizontal Distance}}$$

Slope of Two Points:-

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the two points then

$$\text{Slope}(m) = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } m = \frac{y_1 - y_2}{x_1 - x_2}$$

which is called Slope Formula.

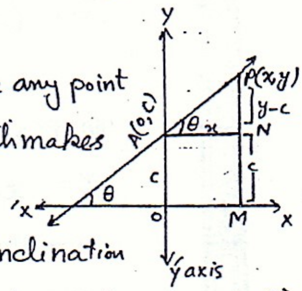
EQUATIONS OF LINE

Slope-intercept Form:-

If m is the slope of a non-verticle line whose y-intercept is "c" then the equation of Line is $y = mx + c$.

Proof:-

Let $P(x, y)$ be any point on the line which makes y intercept "c" also "θ" is the inclination



then draw $\overline{PM}$ perpendicular from P to $\overrightarrow{OX}$

From right triangle APN

$$m = \tan\theta = \frac{y-c}{x-0} = \frac{y-c}{x}$$

$$\Rightarrow m = \frac{y}{x} \Rightarrow mx = y - c$$

$$\boxed{y = mx + c} \quad \text{(Proved)}$$

Deduction:- If Line passes through origion then $c = 0$ and $\boxed{y = mx}$.

Point-Slope Form:-

Let $P(x_1, y_1)$ be any point on a Line L having slope m then equation of Line is

$$y - y_1 = m(x - x_1)$$

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Proof:- Let $P(x_1, y_1)$ be any point on the Line "L" then using Slope intercept form

$$y = mx + c \quad \text{—— (1)}$$

$\because P(x_1, y_1)$ lies on the Line so

$$y_1 = mx_1 + c \quad \text{—— (2)}$$

Subtracting (2) from (1)

$$y - y_1 = (mx + c) - (mx_1 + c)$$

$$\boxed{y - y_1 = m(x - x_1)} \quad (\text{Proved})$$

Two Point Form:-

If $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the two points on a Line "L" then

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Proof:- Using point-slope form

$$y - y_1 = m(x - x_1)$$

also $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$\Rightarrow y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$\boxed{\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}} \quad (\text{Proved})$$

This result can also be written as

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Symmetric Form:-

If θ is the inclination of "L" having a point $P(x_1, y_1)$ then

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

Proof:- We know that for a point $P_1(x_1, y_1)$

$$m = \tan\theta \quad \text{—— (1)}$$

also $m = \dfrac{y - y_1}{x - x_1}$ —— (2)

Comparing (1) and (2), we have

$$\tan\theta = \frac{y - y_1}{x - x_1}$$

$$\frac{y - y_1}{x - x_1} = \frac{\sin\theta}{\cos\theta}$$

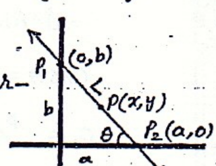
$$\frac{y - y_1}{\sin\theta} = \frac{x - x_1}{\cos\theta} = r \quad (\text{let})$$

$$\boxed{\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r} \quad (\text{Proved})$$

Intercepts Form:-

If "a" and "b" are "x" and "y" intercepts then $\dfrac{x}{a} + \dfrac{y}{b} = 1$ is equation of line.

Proof:- Let "L" line makes "a" and "b" inter-cepts to "x" and "y" axis



then $(a, 0)$ and $(0, b)$ are the two points.

Using Two Point Form of Eq of Line

$$\frac{y - 0}{b - 0} = \frac{x - a}{0 - a} \Rightarrow \frac{y}{b} = \frac{x - a}{-a}$$

$$\Rightarrow -ay = xb - ab$$

$$\Rightarrow ab = xb + ay$$

Dividing throughout by ab

$$\frac{bx}{ab}+\frac{ay}{ab}=\frac{ab}{ab}$$

$$\Rightarrow \boxed{\frac{x}{a}+\frac{y}{b}=1} \quad \text{(Proved)}.$$

Normal Form:-

If θ is the inclination of a line "L" having "p" perpendicular distance from origion then

$$x\cos\theta + y\sin\theta = p$$

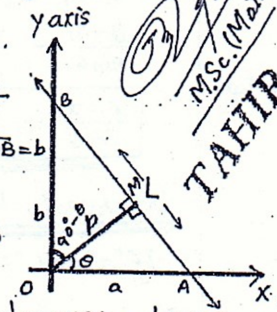
Proof:-

Suppose $\overline{AB}=L$

and $\overline{OA}=a$ and $\overline{OB}=b$

Let $\overline{OM}\perp\overline{AB}$

such that $\overline{OM}=p$ then



from right triangles OAM and OMB

In ΔOAM

$$\cos\theta=\frac{p}{a}$$

$$a=p/\cos\theta$$

In ΔOMB

$$\cos(90^\circ-\theta)=\frac{p}{b}$$

$$\sin\theta=p/b$$

$$b=p/\sin\theta$$

using intercepts form

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{p/\cos\theta}+\frac{y}{p/\sin\theta}=1$$

$$\frac{x\cos\theta}{p}+\frac{y\sin\theta}{p}=1$$

$$\Rightarrow \boxed{x\cos\theta + y\sin\theta = p} \quad \text{(Proved)}.$$

Theorem:- Every linear equation in two variables $ax+by+c=0$ always represents a straight line.

Proof:- Given that $ax+by+c=0$

Case (i) If $a\neq 0$ but $b=0$

then $ax+c=0 \Rightarrow x=\frac{-c}{a}$

which is straight line parallel to y axis.

Case (ii):- If $a=0$ but $b\neq 0$ then

$by+c=0 \Rightarrow y=-c/b$

which is straight line parallel to x axis.

Case (iii) If $a\neq 0, b\neq 0$ then

$$ax+by+c=0$$

$$\Rightarrow by=-ax-c$$

$$y=\left(-\frac{a}{b}\right)x+\left(\frac{-c}{b}\right)$$

which is a st. line of the form

$$y=mx+c.$$

Thus every linear equation in two variables always represents a straight line.

Position of Point w.r.t. Line:-

If $P(x_1,y_1)$ be any point and $L: ax+by+c=0$ be a line then

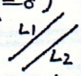
(i) Point is said to be above the line if $ax_1+by_1+c>0$

(ii) Point is below the line if.
$$ax_1+by_1+c<0$$

(iii) Point is on the line if
$$ax_1+by_1+c=0$$

Condition of parallel lines:-

Two lines $\left.\begin{array}{l} L_1: a_1x+b_1y+c_1=0 \\ L_2: a_2x+b_2y+c_2=0 \end{array}\right\}$ are said to be parallel if



$$a_1b_2-a_2b_1=0 \text{ or } a_1b_2=a_2b_1$$

Condition of Perpendicularity

Two lines $\left.\begin{array}{l} L_1: a_1x+b_1y+c_1=0 \\ L_2: a_2x+b_2y+c_2=0 \end{array}\right\}$ are said to be perpendicular if.

$$a_1a_2+b_1b_2=0$$

Condition of Concurrency of 3 Lines:-

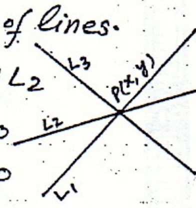
Three non-parallel lines

$L_1: a_1x+b_1y+c_1=0$
$L_2: a_2x+b_2y+c_2=0$
$L_3: a_3x+b_3y+c_3=0$

are concurrent if
$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$

Proof:- Let $P(x,y)$ be the Common point of intersection of lines.

Firstly solving L_1 and L_2



$$a_1x+b_1y+c_1=0$$
$$a_2x+b_2y+c_2=0$$

then
$$x=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1} \quad y=\frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}$$

This point also lies on L_3 so
$$a_3\left[\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}\right]+b_3\left[\frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}\right]+c_3=0$$

Multiplying by $(a_1b_2-a_2b_1)$, we have
$$a_3(b_1c_2-b_2c_1)+b_3(a_2c_1-a_1c_2)+c_3(a_1b_2-a_2b_1)=0$$
$$\Rightarrow \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$

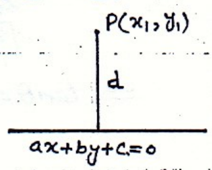
which is the necessary Condition for three lines to be Concurrent.

Distance of a Point from a Line:-

If "d" is the distance of a point $P(x_1,y_1)$ from a line $ax+by+c=0$ then
$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

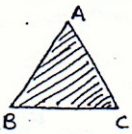
Deductions:-

(i) If $P(x_1,y_1)$ Lies on the Line then
$$d=0$$



Area of triangular Region:-

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangular region then area is defined as



$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where Δ is used to denote area of triangular region and is always positive being area.

Deduction:- If $\Delta = 0$ then points are collinear.

EXERCISE 4.3

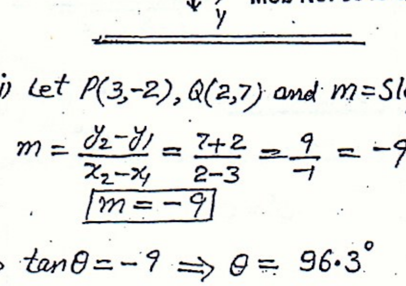
Q.1 Find the slope of the followings:

(i) let $P(-2,4)$, $Q(5,11)$ and $m =$ Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-4}{5+2} = \frac{7}{7} = 1$$

$\boxed{m = 1}$

$\Rightarrow \tan\theta = 1 \Rightarrow \theta = \pi/4 = 45°$

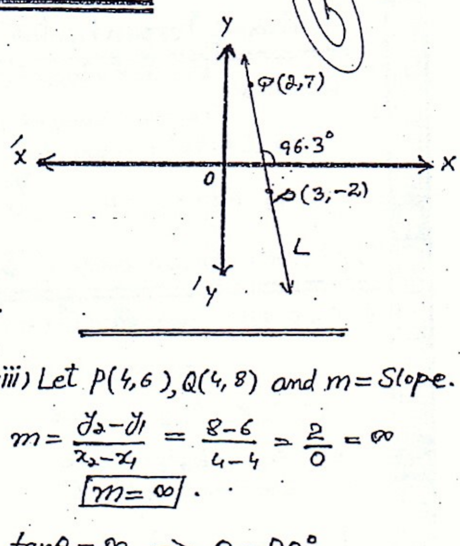


(ii) Let $P(3,-2)$, $Q(2,7)$ and $m =$ Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7+2}{2-3} = \frac{9}{-1} = -9$$

$\boxed{m = -9}$

$\Rightarrow \tan\theta = -9 \Rightarrow \theta = 96.3°$

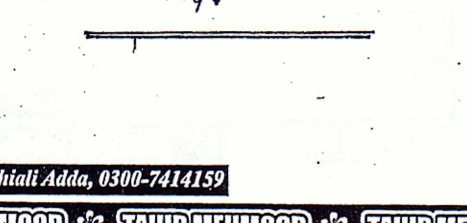


(iii) Let $P(4,6)$, $Q(4,8)$ and $m =$ Slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-6}{4-4} = \frac{2}{0} = \infty$$

$\boxed{m = \infty}$.

$\tan\theta = \infty \Rightarrow \theta = 90°$



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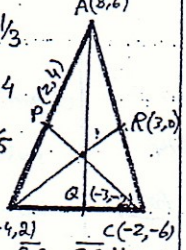
Q.2 $A(8,6), B(-4,2), C(-2,-6)$

(i) Slope of $\overline{AB} = \frac{6-2}{8+4} = \frac{4}{12} = 1/3$

Slope of $\overline{BC} = \frac{2+6}{-4+2} = \frac{8}{-2} = -4$

Slope of $\overline{AC} = \frac{6+6}{8+2} = \frac{12}{10} = 6/5$



(ii) Now Let P, Q, R be the mid points of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ then

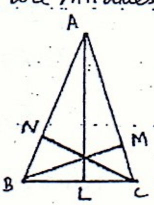
$P(2,4)$, $Q(-3,-2)$, $R(3,0)$

Slope of $\overline{AQ}$ median $= \frac{6+2}{8+3} = 8/11$

Slope of $\overline{BR}$ median $= \frac{2-0}{-4-3} = -\frac{2}{7}$

Slope of $\overline{PC}$ median $= \frac{4+6}{2+2} = \frac{10}{4} = 5/2$

(iii) Now let AL, BM, CN are Altitudes then slope are



Slope of $\overline{AL} = \frac{-1}{BC\ Slope} = \frac{-1}{-4} = 1/4$

Slope of $\overline{BM} = \frac{-1}{Slope\ AC} = \frac{-1}{6/5} = -5/6$

Slope of $\overline{CN} = \frac{-1}{Slope\ \overline{AB}} = \frac{-1}{1/3} = -3$ Ans.

Q.3 Show points are collinear.

(i) Let $A(-1,-3)$, $B(1,5)$, $C(2,9)$

Slope of $\overline{AB} = \frac{5+3}{1+1} = \frac{8}{2} = 4$

Slope of $\overline{BC} = \frac{9-5}{2-1} = \frac{4}{1} = 4$

$\because$ Slope of $\overline{AB}$ = Slope of BC

Thus A, B, C points are Collinear.

(ii) let $A(4,-5)$, $B(7,5)$, $C(10,15)$

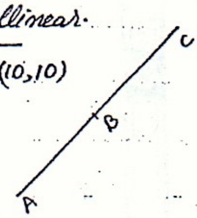
Slope of $\overline{AB} = \frac{5+5}{7-4} = 10/3$

Slope of $\overline{BC} = \frac{15-5}{10-7} = 10/3$

$\because$ Slope of $\overline{AB}$ = Slope of $\overline{BC}$

Thus A, B, C points are Collinear.

(iii) Let $A(-4,6)$, $B(3,8)$, $C(10,10)$



Slope of $\overline{AB} = \frac{8-6}{3+4} = 2/7$

Slope of $\overline{BC} = \frac{10-8}{10-3} = \frac{2}{7}$

$\because$ Slope of $\overline{AB}$ = Slope of $\overline{BC}$

Thus A, B, C are Collinear points.

(iv) Let $A(a, 2b)$, $B(c, a+b)$, $C(2c-a, 2a)$

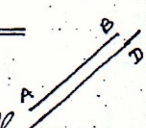
Slope of $\overline{AB} = \frac{a+b-2b}{c-a} = \frac{a-b}{c-a}$

Slope of $\overline{BC} = \frac{2a-a-b}{2c-a-c} = \frac{a-b}{c-a}$

$\because$ Slope of $\overline{AB}$ = Slope of $\overline{BC}$

Thus A, B, C are Collinear points.

Q.4 Find K when



(i) $\overline{AB}$ and $\overline{CD}$ are parall

$\Rightarrow$ Slope of $\overline{AB}$ = Slope of $\overline{CD}$

$\frac{3+6}{7-K} = \frac{4-5}{-6+4} \Rightarrow \frac{9}{7-K} = \frac{-1}{-2}$

$\Rightarrow 9 \times 2 = (7-K) \times 1$

$\Rightarrow 18 = 7-K \Rightarrow K = 7-18$

$\boxed{K = -11}$ Ans.

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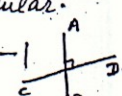
(ii) $\overline{AB}$ and $\overline{CD}$ are perpendicular.



$\Rightarrow$ (Slope of $\overline{AB}$)(Slope of $\overline{CD}$) $= -1$

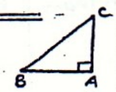
$\left(\frac{3+6}{7-K}\right)\left(\frac{4-5}{-6+4}\right) = -1$

$\frac{9}{7-k} \times \frac{-1}{-2} = -1 \Rightarrow \frac{9}{14-2k} = -1$

$9 = 2k - 14 \Rightarrow 2k = 9 + 14$

$2k = 23 \Rightarrow \boxed{k = 23/2}$ Ans.

Q.5 $A(6,1), B(2,7), C(-6,-7)$



Slope of $\overline{AB} = \frac{7-1}{2-6} = \frac{6}{-4} = -3/2$

Slope of $\overline{BC} = \frac{7+7}{2+6} = \frac{14}{8} = \frac{7}{4}$

Slope of $\overline{AC} = \frac{1+7}{6+6} = \frac{8}{12} = \frac{2}{3}$

Now (Slope of $\overline{AB}$)(Slope of $\overline{AC}$) $= -3/2 \times 2/3 = -1$

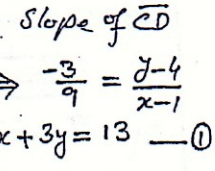
Thus $\overline{AB} \perp \overline{AC}$

Thus ABC is right triangle.

Q.6 Let $D(x,y)$ be the Fourth Vertex

so $\overline{AB} \parallel \overline{CD}$

and $\overline{AD} \parallel \overline{BC}$



Firstly $\overline{AB} \parallel \overline{CD}$

Slope of $\overline{AB}$ = Slope of $\overline{CD}$

$\frac{2+1}{-2-7} = \frac{y-4}{x-1} \Rightarrow \frac{-3}{9} = \frac{y-4}{x-1}$

$-\frac{1}{3} = \frac{y-4}{x-1} \Rightarrow x + 3y = 13$ —①

Now $\overline{AD} \parallel \overline{BC}$

Slope of $\overline{AD}$ = Slope of $\overline{CB}$

$\frac{y+1}{x-7} = \frac{4-2}{1+2} \Rightarrow \frac{y+1}{x-7} = \frac{2}{3}$

$3y + 3 = 2x - 14 \Rightarrow 2x - 3y = 17$ —②

From ① $x = 13 - 3y$

Putting in ② $2(13-3y) - 3y = 17$

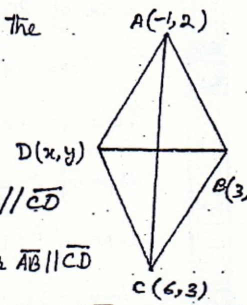
$26 - 6y - 3y = 17 \Rightarrow -9y = -9$

$\Rightarrow \boxed{y = 1}$ $x = 13 - 3(1)$

$\boxed{x = 10}$

Thus $D(x,y) = D(10,1)$ Ans.

Q.7 Let $D(x,y)$ be the fourth Vertex.



In Rhumbos,

$\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{CD}$

For $\overline{AD} \parallel \overline{BC}$

Slope of $\overline{AD}$ = Slope of $\overline{BC}$

$\frac{y-2}{x+1} = \frac{3+1}{6-3}$

$\frac{y-2}{x+1} = \frac{4}{3}$

$3y - 6 = 4x + 4$

$\Rightarrow 4x - 3y = -10$ —①

For $\overline{AB} \parallel \overline{CD}$

Slope of $\overline{AB}$ = Slope of $\overline{CD}$

$\frac{2+1}{-1-3} = \frac{y-3}{x-6}$

$\frac{-3}{4} = \frac{y-3}{x-6}$

$-3x + 18 = 4y - 12$

$3x + 4y = 30$ —②

$3 \times ② \Rightarrow 9x + \cancel{12y} = 90$

$4 \times ① \Rightarrow 16x - \cancel{12y} = -40$

$25x = 50 \Rightarrow \boxed{x = 2}$

$3x + 4y = 30 \Rightarrow 3(2) + 4y = 30$

$4y = 30 - 6 = 24 \Rightarrow \boxed{y = 6}$

Thus $D(x,y) = D(2,6)$

Slope of diagonal $\overline{AC} = \frac{3-2}{6+1} = 1/7$

Slope of diagonal $\overline{BD} = \frac{6+1}{2-3} = -7$

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Now (Slope of $\overline{AC}$) × (Slope of $\overline{BD}$) $= \frac{1}{7} \times -7 = -1$

Thus $\overline{AC} \perp \overline{BD}$

Thus diagonals of rhombus are $\perp$

Q.8 (a) Let $A(1,-2)$, $B(2,4)$ & $C(4,1)$, $D(-8,2)$

Slope of $\overline{AB} = \frac{4+2}{2-1} = 6$

Slope of $\overline{CD} = \frac{2-1}{-8-4} = \frac{-1}{12}$

$\because$ Slope of $\overline{AB} \neq$ Slope of $\overline{CD}$

also (Slope of $\overline{AB}$)(Slope of $\overline{CD}$) $\neq -1$

Thus neither $\parallel$ nor $\perp$.

(b) Let $A(-3,4)$, $B(6,2)$ & $C(4,5)$, $D(-2,-7)$

(m_1) Slope of $\overline{AB} = \frac{4-2}{-3-6} = \frac{-2}{9}$

(m_2) Slope of $\overline{CD} = \frac{5+7}{4+2} = \frac{12}{6} = 2$

$\because m_1 \neq m_2$ and $m_1 m_2 \neq -1$

Thus neither $\parallel$ nor $\perp$.

Q.9 (a) Horizontal Line through $P(7,-9)$

$\because$ Slope (m) of Horizontal Line $= 0$ so

$y - y_1 = m(x - x_1)$

$y + 9 = 0(x-7) \Rightarrow \boxed{y+9=0}$

(b) Vertical Line through $P(-5,3)$

$\because$ Slope (m) of Vertical line $= \infty$

so $y - y_1 = m(x - x_1)$

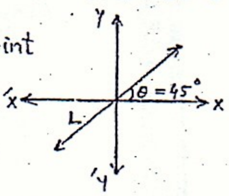
$\frac{1}{m}(y - y_1) = x - x_1$

$\frac{1}{\infty}(y-3) = x+5$

$\Rightarrow 0(y-3) = x+5 \Rightarrow \boxed{x+5=0}$

(c) Line bisecting Ist and IIIrd Quadrent

Let $O(0,0)$ be the point



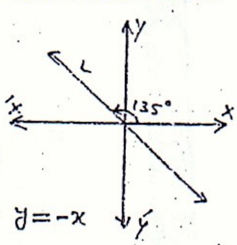
$m = \tan 45° = 1$

$y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 0) \Rightarrow \boxed{y = x}$

(d) Line bisecting IInd and IVth Quadrent.

Let $O(0,0)$ be the point



$m = \tan(135°) = -1$

$y - y_1 = m(x - x_1)$

$y - 0 = -1(x - 0) \Rightarrow y = -x$

$\boxed{x + y = 0}$ Required Line.

Q.10 Find Eq of Line:

(a) $A(-6,5)$ $m = 7$

$y - y_1 = m_1(x - x_1)$

$y - 5 = 7(x+6) \Rightarrow \boxed{7x - y + 47 = 0}$

(b) Let $A(8,-3)$ $m = 0$

$y - y_1 = m(x - x_1)$

$y + 3 = 0(x - 8) \Rightarrow \boxed{y+3=0}$

(C) Let $A(-8,5)$ $m = \infty$

$y - y_1 = m(x - x_1) \Rightarrow \frac{1}{m}(y - y_1) = x - x_1$

$\frac{1}{\infty}(y-5) = x+8 \Rightarrow 0(y-5) = x+8$

$\boxed{x+8=0}$

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(d) let $A(-5,-3)$ $B(9,-1)$

Using $\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$

$\dfrac{y+3}{-1+3} = \dfrac{x+5}{9+5} \Rightarrow \dfrac{y+3}{2} = \dfrac{x+5}{14}$

$7(y+3) = x+5 \Rightarrow x+5 = 7y+21$

$\boxed{x-7y=16}$

(e) y-intercept $=(C)=-7$ $m=-5$

$\because$ $y = mx+c$

$y = -5(x)+(-7) \Rightarrow \boxed{5x+y+7=0}$

(f) x-intercept$(a)=-3$ y-intercept $(b)=4$

$\because$ $\dfrac{x}{a}+\dfrac{y}{b}=1$

$\Rightarrow \dfrac{x}{-3}+\dfrac{y}{4}=1 \Rightarrow 4x-3y=-12$

$\boxed{4x-3y+12=0}$ Ans.

(g) x-intercept$(c)=-9$ $m=-4$

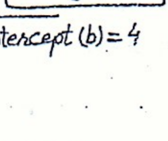
$\Rightarrow Pt(-9,0)$ $m=-4$

$\because$ $y-y_1 = m(x-x_1)$

$y-0 = -4(x+9) \Rightarrow y=-4x-36$

$\boxed{4x+y+36=0}$ Ans.

Q.11 Let $A(3,5)$, $B(9,8)$



Let P be the mid point of $\overline{AB}$

$P\left(\dfrac{3+9}{2}, \dfrac{5+8}{2}\right) = P\left(6, \dfrac{13}{2}\right)$

Slope of $\overline{AB} = \dfrac{8-5}{9-3} = \dfrac{3}{6} = 1/2$

Slope of $\perp$ bisector$(L) = \dfrac{-1}{1/2} = -2$

Eq of L: $y-y_1 = m(x-x_1)$

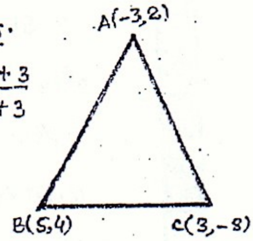
$y-\dfrac{13}{2} = -2(x-6)$

$2y-13 = -4x+24$

$\boxed{4x+2y=37}$

Q.12 $A(-3,2)$, $B(5,4)$, $C(3,-8)$

Equations of Sides.



Eq of $\overline{AB}$: $\dfrac{y-2}{4-2} = \dfrac{x+3}{5+3}$

$\dfrac{y-2}{2} = \dfrac{x+3}{8}$

$4y-8 = x+3$

$\boxed{x-4y+11=0}$

Eq of $\overline{BC}$: $\dfrac{y+8}{4+8} = \dfrac{x-3}{5-3}$

$\Rightarrow \dfrac{y+8}{12} = \dfrac{x-3}{2} \Rightarrow y+8 = 6x-18$

$\Rightarrow \boxed{6x-y-26=0}$

Eq of $\overline{AC}$: $\dfrac{y+8}{2+8} = \dfrac{x-3}{-3-3}$

$\dfrac{y+8}{10} = \dfrac{x-3}{-6} \Rightarrow \dfrac{y+8}{5} = \dfrac{3-x}{3}$

$3y+24 = 15-5x \Rightarrow \boxed{5x+3y+9=0}$

Slope of $\overline{AB} = (m_1) = \dfrac{4-2}{5+3} = \dfrac{2}{8} = 1/4$

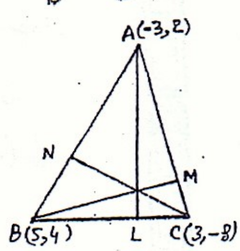
Slope of $\overline{BC} = (m_2) = \dfrac{4+8}{5-3} = \dfrac{12}{2} = 6$

Slope of $\overline{AC} = (m_3) = \dfrac{2+8}{-3-3} = \dfrac{10}{-6} = -5/3$

Slope of $\overline{AL} = \dfrac{-1}{m_2} = \dfrac{-1}{6}$

Slope of $\overline{BM} = \dfrac{-1}{m_3} = 3/5$

Slope of $\overline{CN} = \dfrac{-1}{m_1} = -4$



Equations of Altitudes:-

$\overline{AL}$: $y-2 = \dfrac{-1}{6}(x+3) \Rightarrow 6y-12 = -x-3$

$\boxed{x+6y-9=0}$

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Eq of $\overline{BM}$: $y-4=\frac{3}{5}(x-5)$

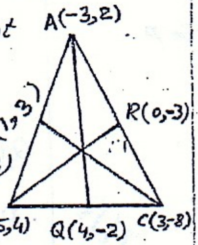
$5y-20=3x-15 \Rightarrow \boxed{3x-5y+5=0}$

Eq of $\overline{CN}$: $y+8=-4(x-3)$

$y+8=-4x+12 \Rightarrow \boxed{4x+y-4=0}$

Let P, Q, R be mid point of each sides

then $P(1,3)$, $Q(4,-2)$, $R(0,-3)$



Equation of Medians

Eq of $\overline{AQ}$: $\frac{y+2}{2+2}=\frac{x-4}{-3-4}$

$\frac{y+2}{4}=\frac{x-4}{-7} \Rightarrow -7y-14=4x-16$

$\boxed{4x+7y-2=0}$

Eq of $\overline{BR}$: $\frac{y+3}{4+3}=\frac{x-0}{5-0}$

$5y+15=7x \Rightarrow \boxed{7x-5y-15=0}$

Eq of $\overline{PC}$ $=\frac{y+8}{3+8}=\frac{x-3}{1-3}$

$\frac{y+8}{11}=\frac{x-3}{-2} \Rightarrow -2y-16=11x-33$

$\boxed{11x+2y-17=0}$

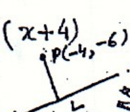
Q.13 Slope of the given line $(m)=-3/2$

Slope of required line $=\frac{-1}{m}=2/3$

Eq of Line: $y+6=\frac{2}{3}(x+4)$

$3y+18=2x+8$

$\Rightarrow \boxed{2x-3y-10=0}$



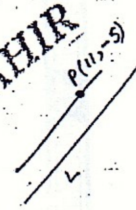
Q.14 Slope of required line = Slope of given line

Slope of required line $(m)=-24$

Eq of line: $y+5=-24(x-11)$

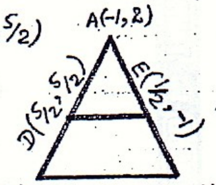
$y+5=-24x+264$

$\Rightarrow \boxed{24x+y-259=0}$ Ans.



Q.15 Co-ordinates of $D\left(\frac{5}{2},\frac{5}{2}\right)$

Co-ordinates of $E=\left(\frac{1}{2},-1\right)$



Slope of $\overline{BC}\,(m_1)=\frac{3+4}{6-2}=7/4$

Slope of $\overline{DE}\,(m_2)=\frac{5/2+1}{5/2-1}$

$m_2=\frac{7/2}{2}=\frac{7}{4}$

$\because m_1=m_2 \Rightarrow \overline{DE} \parallel \overline{BC}$ (Proved).

$|BC|=\sqrt{(6-2)^2+(3+4)^2}=\sqrt{4^2+7^2}$

$|BC|=\sqrt{16+49}=\sqrt{65}$

$|DE|=\sqrt{\left(\frac{5}{2}-\frac{1}{2}\right)^2+\left(\frac{5}{2}+1\right)^2}=\sqrt{(2)^2+(7/2)^2}$

$|DE|=\sqrt{4+\frac{49}{4}}=\sqrt{\frac{65}{4}}=\frac{\sqrt{65}}{2}$

$|DE|=\frac{1}{2}|BC|$ (Proved)

Q.16 Let "l" denotes milk in litres and "P" denotes Prices in rupees.

then $(l_1, P_1)=(560, 12.50)$

$(l_2, P_2)=(700, 12.00)$

Slope $(m)=\frac{12.50-12.00}{560-700}=\frac{0.50}{-160}$

$m=\frac{-\cancel{50}}{2\cancel{100}\times 160}=\frac{-1}{280}$

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(24)

Eq of line $P-P_1 = m(l-l_1)$

$P-12.50 = \frac{-1}{280}(l-560)$

$280P - 12.50\times280 = -l+560$

$l = 560+3500 - 280p$

$\boxed{l = -280p+4060}$

If $P = 12.25$ Rs. then

$l = -280(12.25)+4060$

$l = -3430+4060$

$\boxed{l = 630 \text{ litres}}$

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Q.17 Let P denotes population and t denotes time after 1961 then

$(t_1, P_1) = (1961, 60)$

$(t_2, P_2) = (1981, 95)$

$\text{Slope}(m) = \frac{95-60}{1981-1961} = \frac{35}{20} = 7/4$

Eq of Line

$P-P_1 = m(t-t_1)$

$P-60 = \frac{7}{4}(t-1961)$

$P = \frac{7}{4}t - \frac{13727}{4}+60$

$\boxed{P = 7/4 t + \frac{(-13487)}{4}}$

(a) If $t = 1947$

$P = \frac{7}{4}(1947) - \frac{13487}{4}$

$P = \frac{13629-13487}{4} = \frac{142}{4}$

$\boxed{P = 35.5 \text{ million}}$

(b) If $t = 1997$

$P = \frac{7}{4}(1997) - \frac{13487}{4}$

$P = \frac{13979-13487}{4} = \frac{492}{4}$

$\boxed{P = 123 \text{ million}}$

Q.18 Let P denotes the purchased price and t denotes the time after 1980.

$(P_1, t_1) = (1, 1980)$

$(P_2, t_2) = (4, 1996)$

$\text{Slope}(m) = \frac{1996-1980}{4-1} = \frac{16}{3}$

Eq of line: $t-t_1 = m(P-P_1)$

$t-1980 = \frac{16}{3}(P-1)$

$\frac{3}{16}(t-1980) = P-1$

$P = \frac{3}{16}t+1-\frac{1485}{4}$

$\boxed{P = \frac{3}{16}t - \frac{1481}{4}}$

Now $t = 1990$

$P = \frac{3}{16}\times1990 - \frac{1481}{4}$

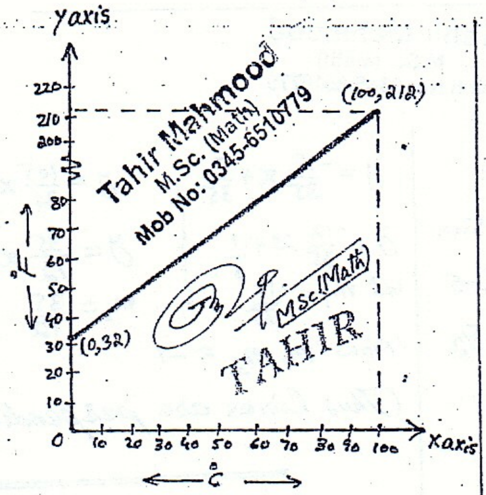
$P = \frac{5970-5924}{16} = \frac{46}{16}$

$\boxed{P = 2.8 \text{ millions}}$

Q.19 Freezing point of water $= (0°c, 32°F)$

Boiling point of water $= (100°c, 212°F)$

$m(\text{Slope}) = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$



Eq of Line: $F - F_1 = m(C - C_1)$

$F - 32 = \frac{9}{5}(C - 0)$

$\boxed{F^\circ = \frac{9}{5}C^\circ + 32}$

Q.20:- Let "S" be the average entry test scores and "t" be used to denote the years

$(S_1, t_1) = (592, 1998)$

$(S_2, t_2) = (564, 2002)$

Slope $(m) = \frac{2002 - 1998}{564 - 592} = \frac{4}{-28}$

$m = -1/7$

Equation: $t - t_1 = m(S - S_1)$

$t - 1998 = \frac{-1}{7}(S - 592)$

$-7(t - 1998) = S - 592$

$S = -7t + 13986 + 592$

$\boxed{S = -7t + 14578}$

Now $t = 2006$

$S = -7(2006) + 14578$

$S = -14042 + 14578$

$\boxed{S = 536}$ Ans.

Q.21 (a) $2x - 4y + 11 = 0$

(i) Slope intercept form

$4y = 2x + 11$

$y = \frac{2}{4}x + \frac{11}{4}$

$y = \frac{1}{2}x + 11/4$ where $m = 1/2$, $c = 11/4$

(ii) Two intercepts form

$2x - 4y = -11$

(Dividing by -11)

$\frac{2x}{-11} - \frac{4y}{-11} = 1$

$\frac{x}{(-11/2)} + \frac{y}{(11/4)} = 1$

for $a = \frac{-11}{2}$ $b = \frac{11}{4}$

(iii) Normal Form:-

$2x - 4y + 11 = 0$

$-2x + 4y = 11$

$\sqrt{a^2 + b^2} = \sqrt{(-2)^2 + (4)^2} = \sqrt{4 + 16} = \sqrt{20}$

$x\left(\frac{-2}{\sqrt{20}}\right) + y\left(\frac{4}{\sqrt{20}}\right) = \frac{11}{\sqrt{20}}$

For $\cos\theta = \frac{-2}{\sqrt{20}}$, $\sin\theta = \frac{4}{\sqrt{20}}$, $p = \frac{11}{\sqrt{20}}$

where $p = \frac{11}{\sqrt{20}}$ is length of perpendicular from origion.

(Remainings do your self.)

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Q.22 Which are $\parallel$ or $\perp$ lines:

(a) $2x+y-3=0$ | $4x+2y+5=0$

$y=-2x+3$ | $2y=-4x-5$

Let $m_1=-2$ | $y=-2x-5/2$, $m_2=-2$

$\because m_1=m_2$

So lines are parallel.

(b) $3y=2x+5$ | $3x+2y-8=0$

$y=\frac{2}{3}x+5/3$ | $2y=-3x+8$

let $m_1=\frac{2}{3}$ | $y=\frac{-3}{2}x+\frac{8}{2}$, $m_2=-3/2$

Now $m_1\cdot m_2=-1$

Thus lines are perpendicular.

(c) $4y+2x-1=0$ | $x-2y-7=0$

$4y=-2x+1$ | $2y=x-7$

$y=\frac{-1}{2}x+1/4$ | $y=\frac{1}{2}x-7/2$

let $m_1=-1/2$ | $m_2=1/2$

$\because m_1\neq m_2$ and $m_1\cdot m_2\neq -1$

So lines are neither $\parallel$ nor $\perp$.

(d) $4x-y+2=0$ | $12x-3y+1=0$

$y=4x+2$ | $3y=12x+1$

let $m_1=4$ | $y=4x+1/3$, $m_2=4$

$\because m_1=m_2$

So lines are parallel.

(e) $12x+35y-7=0$ | $105x-36y+11=0$

$35y=-12x+7$ | $36y=105x+11$

$y=\frac{-12}{35}x+\frac{7}{35}$ | $y=\frac{105}{36}x+\frac{11}{36}$

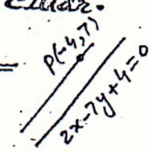
$y=\frac{-12}{35}x+\frac{1}{5}$ | $y=\frac{35}{12}x+11/36$

let $m_1=\frac{-12}{35}$ | $m_2=\frac{35}{12}$

Now $m_1 m_2=-1$

Thus lines are perpendicular.

Q.24 Pt $(-4,7)$



Req. line $\parallel$ to $2x-7y+4=0$

Slope of given line(m_1) $\Rightarrow 7y=2x+4$

$y=\frac{2}{7}x+4/7$

$m_1=2/7$

Now Slope of Req line = Slope of Given line.

$m=m_1=2/7$

Eq of line $y-y_1=m(x-x_1)$

$y-7=\frac{2}{7}(x+4)$

$7y-49=2x+8 \Rightarrow 2x-7y+8+49=0$

$\boxed{2x-7y+57=0}$

Q.25 Pt$(5,-8)$

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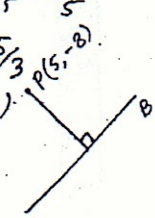
Req. line $\perp$ to $\overline{AB}$

Slope of $\overline{AB}$ $(m_1)=\frac{7+8}{10+15}=\frac{15}{25}=\frac{3}{5}$

Slope of req. line $(m)=\frac{-1}{m}=-5/3$

Eq of line: $y+8=-5/3(x-5)$



$3y+24=-5x+25$

$\boxed{5x+3y-1=0}$

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$2x-y=-3 \Rightarrow y=2x+3$

Q.26 Slope of Given lines(m') = 2

Slope of perpendicular line $(m) = -\frac{1}{2}$

Eq of lines $y = -\frac{1}{2}x + c$

$2y + x = c$

Now x-intercept let $y=0$

$x = c$ ___(i)

Now y-intercept let $x=0$

$y = \frac{c}{2}$ ___(ii)

$(x\text{-intercept})(y\text{-intercept}) = 3$

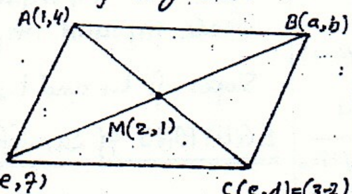
$\frac{c}{2}\cdot c = 3 \Rightarrow c^2 = 6 \Rightarrow c = \pm\sqrt{6}$

Thus lines are

$x + 2y = \pm\sqrt{6}$

$x+2y+\sqrt{6}=0$ & $x+2y-\sqrt{6}=0$

Q.27 Let $A(1,4), B(a,b), C(c,d), D(e,f)$ are the vertices of ‖gram.



Diagonal of ‖gram bisect eachother

$2 = \frac{1+c}{2}$ ∧ $1 = \frac{4+d}{2}$

$c = 4-1$ $d = 2-4$

$\boxed{c=3}$ $\boxed{d=-2}$

Let Slope of $\overline{AD}$ = Slope of $\overline{BC}$ = 1

$\frac{4-f}{1-e} = \frac{b+2}{a-3} = 1$

$4-f = 1-e$ and $b+2 = a-3$

$f - e = 3$ ___① $a - b = 5$ ___②

Also Slope of $\overline{AB}$ = Slope of $\overline{CD} = -\frac{1}{7}$

$\frac{b-4}{a-1} = \frac{f+2}{e-3} = -\frac{1}{7}$

$\frac{b-4}{a-1} = -\frac{1}{7}$ and $\frac{f+2}{e-3} = -\frac{1}{7}$

$7b-28 = -a+1$ ∧ $7f+14 = -e+3$

$a+7b = 29$ ___③ $e + 7f = -11$ ___④

By ③ − ②

$\cancel{a}+7b = 29$
$-\cancel{a} \mp b = -5$

$8b = 24$

$\boxed{b=3}$

$a = 5+b$

$a = 5+3 = 8$

$\boxed{a=8}$

By ① + ④

$7f + \cancel{e} = -11$
$f - \cancel{e} = 3$

$8f = -8$

$\boxed{f=-1}$

① ⇒ $e = f-3$

$e = -1-3 = -4$

$\boxed{e=-4}$

Thus vertices are

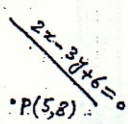
$A(1,4), B(8,3), C(3,-2), D(-4,-1)$

Q.28 Locate the position of point:

(a) $(5,8)$: $2x-3y+6=0$

$2(5)-3(8)+6 = 10-24+6 = -8 < 0$

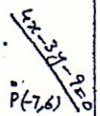
Thus $(5,8)$ lies below the line $2x-3y+6=0$.



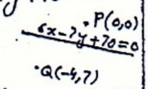
(b) $(-7,6)$: $4x-3y-9=0$

$4(-7)-3(6)-9=-28-18-9<0$

Thus $(-7,6)$ lies below the line.



Q.29:- (a) $(0,0)$ and $(-4,7)$: $6x-7y+70=0$



$(0,0) \Rightarrow 6(0)-7(0)+70=70>0$

$(-4,7) \Rightarrow 6(-4)-7(7)+70=-24-49+70=-3<0$

The points lie in opposite sides.

(b) $(2,3)$ and $(-2,3)$: $3x-5y+8=0$

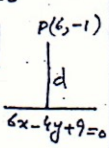
$(2,3) \Rightarrow 3(2)-5(3)+8=6-15+8=-1<0$

$(-2,3) \Rightarrow 3(-2)-5(3)+8=-6-15+8=-13<0$

The points lie in same sides.

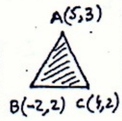
Q.30:- Let "d" be the distance of $P(6,-1)$ from line $6x-4y+9=0$

then $d=\dfrac{|6(6)-4(-1)+9|}{\sqrt{(6)^2+(-4)^2}}$



$d=\dfrac{|36+4+9|}{\sqrt{36+16}}=\dfrac{49}{\sqrt{52}}$ Ans.

Q.31 Find Area of ΔABC.



$$\Delta=\frac{1}{2}\begin{vmatrix} 5 & 3 & 1 \\ -2 & 2 & 1 \\ 4 & 2 & 1 \end{vmatrix}$$

$\Delta=\frac{1}{2}\{5(2-2)-3(-2-4)+1(-4-8)\}$

$\Delta=\frac{1}{2}\{0+18-12\}=\frac{6}{2}=3$ Sq. units Ans.

Q.32:- $A(2,3)$, $B(-1,1)$, $C(4,-5)$

$$\Delta=\frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & 1 & 1 \\ 4 & -5 & 1 \end{vmatrix}$$

$\Delta=\frac{1}{2}\{2(1+5)-3(-1-4)+1(5-4)\}$

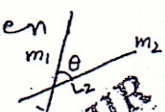
$\Delta=\frac{1}{2}\{12+15+1\}=\frac{28}{2}$

$\Delta=14\neq 0$

Thus points are non-Collinear.

ANGLE Between two LINES:-

Let θ is the angle between L_1 and L_2 Lines then



$$\tan\theta=\frac{m_2-m_1}{1+m_1m_2}$$

or

$$\tan\theta=\frac{m_1-m_2}{1+m_1m_2}$$

where m_1 and m_2 are the Slopes of L_1 and L_2 respectively.

EQUTIONS of Straight Lines in Matrix form:-

(i) Any Line $ax+by+c=0$ Can be represented as

$$ax+by+c=0 \Rightarrow \begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -c \end{bmatrix}$$

(ii) For two lines (System of two lines)

$L_1 : a_1x+b_1y+c_1=0$

$L_2 : a_2x+b_2y+c_2=0$

$$\Rightarrow \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -c_1 \\ -c_2 \end{bmatrix}$$