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Now let $D(c,d)$ divides $\overline{AB}$ in the ratio of 1:1 then "D" is mid of $\overline{AB}$

$$D(c,d) = D\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \text{ Ans.}$$

Now $E(e,f)$ divides $\overline{AB}$ in the ratio 3:1

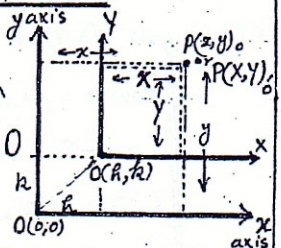
$$E(e,f) = \left[\frac{3(x_2)+1(x_1)}{3+1}, \frac{3(y_2)+1(y_1)}{3+1}\right]$$

$$E(e,f) = E\left(\frac{x_1+3x_2}{4}, \frac{y_1+3y_2}{4}\right) \text{ Ans.}$$

Thus C, D, E are the points which divide $\overline{AB}$ into four equal parts.

Translation Of Axes:-

Let $P(x,y)$ be a point in xy plane w.r.t O and $P(X,Y)$ w.r.t $O(h,k)$ is same point



then $\boxed{x = X+h}$ $\boxed{y = Y+k}$

also $\boxed{X = x-h}$ $\boxed{Y = y-k}$

In the above process, Axes are shifted parallel to old axes upto "h" and "k" distance from O(0,0). These new axes are called translation of old axes.

Rotation Of Axes:-

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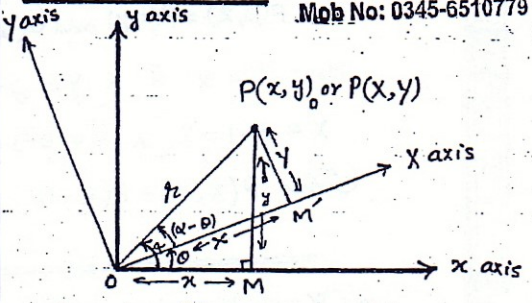


Consider $P(x,y)$ subject to ox, oy then let axes are rotated by an amount of θ angle as shown.

Now from the figure (rt. ΔOMP)

$x = r\cos\alpha$ ∧ $y = r\sin\alpha$

Now $OM' = X = r\cos(\alpha-\theta)$ where $\theta < 90°$

$X = r[\cos\alpha\cos\theta + \sin\alpha\sin\theta]$

$X = (r\cos\alpha)\cos\theta + (r\sin\alpha)\sin\theta$

$$\boxed{X = x\cos\theta + y\sin\theta}$$

$PM' = Y = r\sin(\alpha-\theta)$

$Y = (r\sin\alpha)\cos\theta - (r\cos\alpha)\sin\theta$

$$\boxed{Y = y\cos\theta - x\sin\theta}$$

Thus $P(X,Y)$ is the same point $P(x,y)$ w.r.t. new rotated axes also

$\boxed{x = X\cos\theta - Y\sin\theta}$ and $\boxed{y = X\sin\theta + Y\cos\theta}$

CH#4 (2nd Year)

TAHIR **Exercise: 4.2** TAHIR

Q.1 Find the Point $P(X,Y)$ w.r.t to ox, oy axes from given data.

(i) $P(3,2)=P(x,y)$ and $O(1,3)=O(h,k)$

Now $X=x-h \wedge Y=y-k$

$X=3-1=2 \wedge Y=2-3=-1$

Thus $P(X,Y)=P(2,-1)$ Ans.

(ii) $P(x,y)=P(-2,6)$ and $O(h,k)=O(-3,2)$

Now $X=x-h \wedge Y=y-k$

$X=-2+3=1 \wedge Y=6-2=4$

Thus $P(X,Y)=P(1,4)$ Ans.

(iii) $P(x,y)=P(-6,-8)$ and $O(h,k)=O(-4,-6)$

Now $X=x-h \wedge Y=y-k$

$X=-6+4=-2 \wedge Y=-8+6=-2$

Thus $P(X,Y)=P(-2,-2)$ Ans.

(iv) $P(x,y)=P(\frac{3}{2},\frac{5}{2})$ and $O(h,k)=O(-\frac{1}{2},\frac{7}{2})$

Now $X=x-h \wedge Y=y-k$

$X=\frac{3}{2}+\frac{1}{2}=\frac{4}{2}=2 \wedge Y=\frac{5}{2}-\frac{7}{2}=\frac{5-7}{2}=\frac{-2}{2}$

$X=2 \wedge Y=-1$

Thus $P(X,Y)=P(2,-1)$ Ans.

Q.2 From given data, Find $P(x,y)$ Point w.r.t. xy plane.

(i) $P(X,Y)=P(8,10)$ and $O(h,k)=O(3,4)$

Now $x=X+h \wedge y=Y+k$

$x=8+3=11 \wedge y=10+4=14$

Thus $P(x,y)=P(11,14)$ Ans.

(ii) $P(X,Y)=P(-5,-3)$ and $O(h,k)=O(-2,-6)$

Now $x=X+h \wedge y=Y+k$

$x=-5-2=-7 \wedge y=-3-6=-9$

Thus $P(x,y)=P(-7,-9)$ Ans.

(iii) $P(X,Y)=P(-\frac{3}{4},-\frac{7}{6}) \wedge O(h,k)=O(\frac{1}{4},-\frac{1}{6})$

Now $x=X+h$ and $y=Y+k$

$x=-\frac{3}{4}+\frac{1}{4}=\frac{-2}{4}=-\frac{1}{2}$ and $y=\frac{-7}{6}-\frac{1}{6}=\frac{-8}{6}=-\frac{4}{3}$

Thus $P(x,y)=P(-\frac{1}{2},-\frac{4}{3})$

(iv) $P(X,Y)=P(4,-3) \wedge O(h,k)=O(-2,3)$

Now $x=X+h \wedge y=Y+k$

$x=4-2=2 \wedge y=-3+3=0$

Thus $P(x,y)=P(2,0)$

Q.3 (i) $P(x,y)=P(5,3)$ $\theta=45^\circ$ Find $P(X,Y)=?$

$X=x\cos\theta+y\sin\theta \Rightarrow X=5\cos45^\circ+3\sin45^\circ=\frac{5}{\sqrt{2}}+\frac{3}{\sqrt{2}}=\frac{8}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=4\sqrt{2}$

$Y=y\cos\theta-x\sin\theta \Rightarrow Y=3\cos45^\circ-5\sin45^\circ=\frac{3}{\sqrt{2}}-\frac{5}{\sqrt{2}}=\frac{-2}{\sqrt{2}}=-\sqrt{2}$

Thus $P(X,Y)=P(4\sqrt{2},-\sqrt{2})$

Q.3 (ii) $P(x,y) = P(3,-7)$ $\theta = 30°$

Now $X = x\cos\theta + y\sin\theta$

$X = 3\cos 30° - 7\sin 30°$

$X = \frac{3\sqrt{3}}{2} - \frac{7}{2} = \frac{3\sqrt{3}-7}{2}$

Now $Y = y\cos\theta - x\sin\theta$

$Y = -7\cos 30° - 3\sin 30°$

$Y = \frac{-7\sqrt{3}}{2} - \frac{3}{2} = \frac{-7\sqrt{3}-3}{2}$

Thus $P(X,Y) = \left(\frac{3\sqrt{3}-7}{2}, \frac{-7\sqrt{3}-3}{2}\right)$

(iii) $P(x,y) = P(11,-15)$ $\theta = 60°$

$X = x\cos\theta + y\sin\theta$

$X = 11\cos 60° - 15\sin 60°$

$X = \frac{11}{2} - \frac{15\sqrt{3}}{2} = \frac{11-15\sqrt{3}}{2}$

Now $Y = y\cos\theta - x\sin\theta$

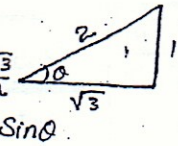
$Y = -15\cos 60° - 11\sin 60°$

$Y = \frac{-15}{2} - \frac{11\sqrt{3}}{2} = \frac{-15-11\sqrt{3}}{2}$

Thus $P(X,Y) = P\left(\frac{11-15\sqrt{3}}{2}, \frac{-15-11\sqrt{3}}{2}\right)$

(iv) $P(x,y) = P(15,10)$ $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$



$\sin\theta = \frac{1}{2}$ $\cos\theta = \frac{\sqrt{3}}{2}$

$X = x\cos\theta + y\sin\theta$

$X = 15\cdot\frac{\sqrt{3}}{2} + 10\cdot\frac{1}{2} = \frac{15\sqrt{3}+10}{2}$

Now $Y = y\cos\theta - x\sin\theta$

$Y = 10\frac{\sqrt{3}}{2} - 15\times\frac{1}{2} = \frac{10\sqrt{3}-15}{2}$

Thus $P(X,Y) = P\left(\frac{15\sqrt{3}+10}{2}, \frac{10\sqrt{3}-15}{2}\right)$

Q.4 (i) $P(X,Y) = P(-5,3)$ $\theta = 30°$

$x = X\cos\theta - Y\sin\theta$

$x = -5\cos 30° - 3\sin 30° = -\frac{5\sqrt{3}}{2} - \frac{3}{2}$

$x = \frac{-5\sqrt{3}-3}{2}$

$y = X\sin\theta + Y\cos\theta$

$y = -5\sin 30° + 3\cos 30° = -\frac{5}{2} + \frac{3\sqrt{3}}{2}$

$y = \frac{-5+3\sqrt{3}}{2}$

Thus $P(x,y) = P\left(\frac{-5\sqrt{3}-3}{2}, \frac{-5+3\sqrt{3}}{2}\right)$

(ii) $P(X,Y) = P(-7\sqrt{2}, 5\sqrt{2})$ $\theta = 45°$

$x = X\cos\theta - Y\sin\theta$

$x = -7\sqrt{2}\cos 45° - 5\sqrt{2}\sin 45°$

$x = -7\sqrt{2}\cdot\frac{1}{\sqrt{2}} - 5\sqrt{2}\cdot\frac{1}{\sqrt{2}} = -12$

$y = X\sin\theta + Y\cos\theta$

$y = -7\sqrt{2}\sin 45° + 5\sqrt{2}\cos 45°$

$y = -7\sqrt{2}\cdot\frac{1}{\sqrt{2}} + 5\sqrt{2}\cdot\frac{1}{\sqrt{2}} = -2$

Thus $P(x,y) = P(-12,-2)$ Ans.
