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CH # 4 (2nd Year)

Chapter: 4 F.A: F.Sc. Part: II

INTRODUCTION TO ANALYTIC GEOMETRY

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Analytic Geometry:-

"The branch of mathematics in which geometry is dealt by means of Co-ordinates and algebric operations is known as Analytic Geometry."

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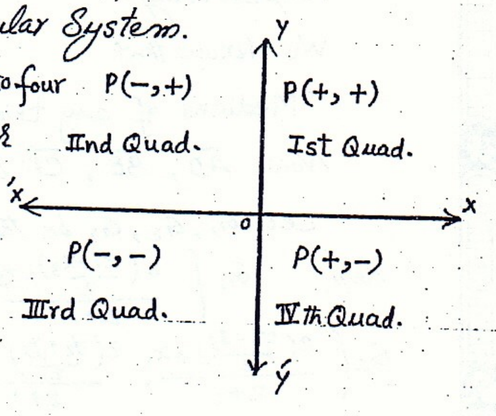
Cartisian Plane:-

The two mutually perpendicular lines which intersect each other at a point form a cartisian Plane or simply XY-plane or Rectangular plane.

* Mutually perpendicular lines are called Co-ordinate Axes and the point of intersection is called Origion.
* Horizontal Co-ordinate axis is called X-axis and Vertical Co-ordinate axis is called Y- axis.
* The point $p(x, y)$ on the plane is called ordered pair where x is "abscissa" or x-coordinate and y is "ordinate" or y-co-ordinate.
* For $(a, b) = (x, y) \Rightarrow x = a \wedge y = b$ but $(x, y) \neq (y, x)$
* The system generated by ordered pairs is called Cartisian Co-ordinate System or Rectangular System.
* Co-ordinate axes divide plane into four equal parts called Quadrents under the mentioned charactristics:



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Distance Formula:-

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the two points then the distance "d" between P and Q is defined as:

$$d = |\overline{PQ}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d = |\overline{PQ}| = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Ratio Formula For Points:-

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the points and C divides $\overline{PQ}$ line in the ratio of $m:n$ then co-ordinates of C are $C\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

NOTE: If $m=n=1$ then C is mid point of $\overline{PQ}$ line and

$$C\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

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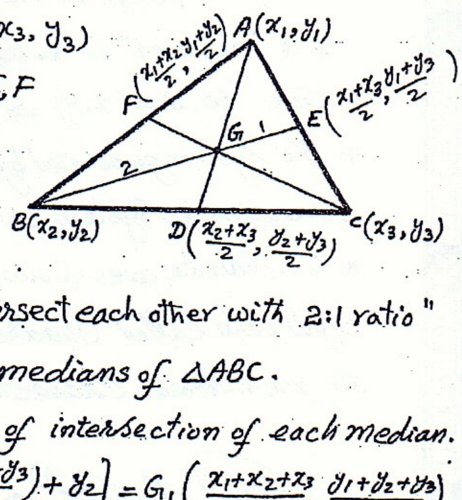
THEOREM:-

Medians of a triangle are Concurrent.

Proof:- Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ be the vertices of ΔABC and D, E, F are the mid points of $\overline{BC}, \overline{AC}, \overline{AB}$ respectively.



We know that

"Medians of any triangle intersect each other with 2:1 ratio"

Now $\overline{AD}$, $\overline{BE}$, $\overline{CF}$ are the medians of ΔABC.

Let G_1, G_2, G_3 be the points of intersection of each median.

then $G_1\left(\frac{2\left(\frac{x_1+x_3}{2}\right)+x_2}{2+1}, \frac{2\left(\frac{y_1+y_3}{2}\right)+y_2}{2+1}\right) = G_1\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$G_2\left(\frac{2\left(\frac{x_2+x_3}{2}\right)+x_1}{2+1}, \frac{2\left(\frac{y_2+y_3}{2}\right)+y_1}{2+1}\right) = G_2\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

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$$G_3\left(\frac{2\left(\frac{x_1+x_2}{2}\right)+x_3}{2+1}, \frac{2\left(\frac{y_1+y_2}{2}\right)+y_3}{2+1}\right) = G_3\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$\Rightarrow$ G_1, G_2, G_3 Concides so $G_1 = G_2 = G_3 = G$ (let)

Thus Median of triangle are Concurrent.

TAHIR **Exercise 4.1** TAHIR

Q.1 Describe the location of the point $P(x, y)$ for which:

(i) $x > 0$

The set of all points lying in Ist and 4th Quadrent.

(ii) $x > 0$ and $y > 0$

The set of all points which lie in Ist Quadrent.

(iii) $x = 0$

The set of all those points which lie only on y-axis.

(iv) $y = 0$

The set of all those points which lie only on x-axis.

(v) $x < 0$ and $y \geq 0$

The set of all the points lying in IInd Quadrent and on the negative side of x-axis.

(vi) $x = y$

The set of all points inwhich x and y Co-ordinates are equal (i.e. line bisecting Ist and IIIrd Quad.)

(vii) $|x| = -|y|$

The only point $O(0,0)$ is represented by $|x| = -|y|$

(viii) $|x| \geq 3$

$\Rightarrow$ $-3 \leq x$ and $x \geq 3$

The set of all the points of abscissa greater or equal to 3 and less or equal to -3.

(ix) $x > 2$ and $y = 2$

The set of all the points $(x, 2)$ for which $x > 2$.

(x) x and y have opposite Signs.

The set of all the points lying in IInd and IVth Quadrent.

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Q.2: Find (i) mid point (ii) Distance

(a) $A(3,1)$, $B(-2,-4)$

(i) $|AB| = \sqrt{(3+2)^2+(1+4)^2}$

$|AB| = \sqrt{5^2+5^2}$

$|AB| = \sqrt{25+25} = 5\sqrt{2}$ Ans.

(ii) Mid point of $\overline{AB} = \left(\frac{3-2}{2}, \frac{1-4}{2}\right)$

$= \left(\frac{1}{2}, \frac{-3}{2}\right)$ Ans.

(b) $A(-8,3)$, $B(2,-1)$

(i) $|AB| = \sqrt{(-8-2)^2+(3+1)^2}$

$|AB| = \sqrt{(-10)^2+(4)^2} = \sqrt{100+16}$

$|AB| = \sqrt{116} = 2\sqrt{29}$ Ans.

(ii) Mid point of $\overline{AB} = \left(\frac{-8+2}{2}, \frac{3-1}{2}\right)$

$= \left(\frac{-6}{2}, \frac{2}{2}\right) = (-3,1)$ Ans.

(c) $A\left(-\sqrt{5}, \frac{-1}{3}\right)$; $B\left(-3\sqrt{5}, 5\right)$

(i) $|AB| = \sqrt{(-\sqrt{5}+3\sqrt{5})^2+\left(\frac{-1}{3}-5\right)^2}$

$|AB| = \sqrt{(2\sqrt{5})^2+\left(-\frac{16}{3}\right)^2}$

$= \sqrt{20+\frac{256}{9}} = \sqrt{\frac{180+256}{9}}$

$= \sqrt{\frac{436}{9}} = \frac{2\sqrt{109}}{3}$ Ans.

(ii) Mid point of $\overline{AB} = \left(\frac{-\sqrt{5}-3\sqrt{5}}{2}, \frac{-\frac{1}{3}+5}{2}\right)$

$= \left(-\frac{4\sqrt{5}}{2}, \frac{14}{6}\right) = \left(-2\sqrt{5}, \frac{7}{3}\right)$

$= \left(-2\sqrt{5}, \frac{7}{3}\right)$ Ans.

Q.3 Which of the following points are 15 unit distant from origion?

(a) Let $A(\sqrt{176}, 7)$

$|OA| = \sqrt{(\sqrt{176}-0)^2+(7-0)^2}$

$|OA| = \sqrt{176+49} = \sqrt{225}$

$|OA| = 15$ unit.

(b) Let $B(10,-10)$

$|OB| = \sqrt{(10-0)^2+(-10-0)^2}$

$|OB| = \sqrt{100+100} = \sqrt{200}$

$|OB| = 10\sqrt{2}$ units

(c) let $C(1,15)$

$|OC| = \sqrt{(1-0)^2+(15-0)^2}$

$|OC| = \sqrt{1+225} = \sqrt{226}$ units

(d) let $D\left(\frac{15}{2}, \frac{15}{2}\right)$

$|OD| = \sqrt{\left(\frac{15}{2}-0\right)^2+\left(\frac{15}{2}-0\right)^2}$

$|OD| = \sqrt{\frac{225}{4}+\frac{225}{4}} = \sqrt{\frac{2\times 225}{4}}$

$|OD| = \frac{15\sqrt{2}}{2}$ units.

$|OD| = \frac{15}{\sqrt{2}}$

Thus only $A(\sqrt{176}, 7)$ is 15 unit distant from origion.

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Q.4 Show that: TAHIR

(i) $A(0,2), B(\sqrt{3},-1), C(0,-2)$ are the vertices of a right triangle.

Soln:- $|AB| = \sqrt{(\sqrt{3}-0)^2+(-1-2)^2}$

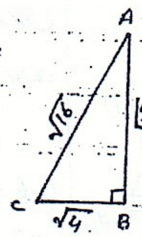
$|AB| = \sqrt{3+9} = \sqrt{12}$

$|BC| = \sqrt{(\sqrt{3}-0)^2+(-1+2)^2}$

$|BC| = \sqrt{3+1} = \sqrt{4}$

$|AC| = \sqrt{(0-0)^2+(2+2)^2}$

$|AC| = \sqrt{0+16} = \sqrt{16}$



Now $|AC|^2 = |AB|^2+|BC|^2$

$(\sqrt{16})^2 = (\sqrt{12})^2+(\sqrt{4})^2 \Rightarrow 16=16$

Pythagorean theorem is verified so A, B, C are the vertices of right triangle.

(ii) $A(3,1), B(-2,-3), C(2,2)$ are the vertices of isosceles triangle.

Soln:- $|AB| = \sqrt{(3+2)^2+(1+3)^2}$

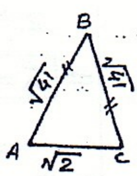
$|AB| = \sqrt{25+16} = \sqrt{41}$

$|AC| = \sqrt{(3-2)^2+(1-2)^2}$

$|AC| = \sqrt{1+1} = \sqrt{2}$

$|BC| = \sqrt{(2+2)^2+(2+3)^2}$

$|BC| = \sqrt{16+25} = \sqrt{41}$



Clearly $|AB| = |BC|$ so A, B, C are the vertices of isosceles triangle.

(iii) $A(5,2), B(-2,3), C(-3,-4), D(4,-5)$ are the vertices of a parallelogram. Is this parallelogram a square?

Soln:- $|AB| = \sqrt{(5+2)^2+(2-3)^2}$

$|AB| = \sqrt{49+1} = \sqrt{50}$

$|BC| = \sqrt{(-2+3)^2+(3+4)^2}$

$|BC| = \sqrt{1+49} = \sqrt{50}$

$|CD| = \sqrt{(-3-4)^2+(-4+5)^2}$

$|CD| = \sqrt{49+1} = \sqrt{50}$

$|AD| = \sqrt{(5-4)^2+(2+5)^2}$

$|AD| = \sqrt{1+49} = \sqrt{50}$

Clearly $|AB| = |CD|$ & $|BC| = |AD|$

So A, B, C, D forms a parallelogram.

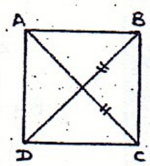
Now diagonals are of length

$|AC| = \sqrt{(5+3)^2+(2+4)^2}$

$|AC| = \sqrt{64+36} = \sqrt{100} = 10$

$|BD| = \sqrt{(-2-4)^2+(3+5)^2}$

$|BD| = \sqrt{36+64} = \sqrt{100} = 10$



Now diagonals are equal so this parallelogram is square.

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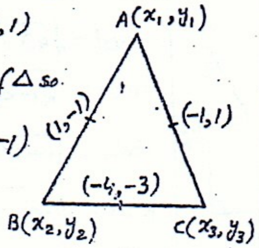
Q.5: Find the vertiees of triangle.

Soln:- Let $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ are the vertices of triangle so

Now $(1,-1),(-4,-3),(-1,1)$ are mid points of Δ so.



$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=(1,-1)$

$\Rightarrow \frac{x_1+x_2}{2}=1 \wedge \frac{y_1+y_2}{2}=-1$

$x_1+x_2=2$ ——① $\quad y_1+y_2=-2$ ——②

Now $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)=(-4,-3)$

$\Rightarrow \frac{x_2+x_3}{2}=-4 \wedge \frac{y_2+y_3}{2}=-3$

$x_2+x_3=-8$ ——③ $\quad y_2+y_3=-6$ ——④

Again $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)=(-1,1)$

$\Rightarrow \frac{x_1+x_3}{2}=-1 \wedge \frac{y_1+y_3}{2}=1$

$x_1+x_3=-2$ ——⑤ $\quad y_1+y_3=2$ ——⑥

By ①−③ and ②−④

$x_1+\cancel{x_2}=2$
$\underline{-x_3 \mp \cancel{x_2}=-8}$
$x_1-x_3=10$ ——⑦

$y_1+\cancel{y_2}=-2$
$\underline{-y_3 \mp \cancel{y_2}=-6}$
$y_1-y_3=4$ ——⑧

Now ⑤+⑦ and ⑥+⑧

$2x_1=8 \quad\quad 2y_1=6$

$\boxed{x_1=\frac{8}{2}=4} \quad\quad \boxed{y_1=3}$

Now ⑤ ⇒ $x_3=-2-x_1$ $\quad$ ⑥ ⇒ $y_3=2-y_1$

$x_3=-2-4 \quad\quad y_3=2-3$

$\boxed{x_3=-6} \quad\quad \boxed{y_3=-1}$

① ⇒ $x_2=2-x_1$ $\quad$ ② ⇒ $y_2=-2-y_1$

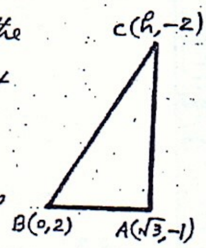
$x_2=2-4=-2 \quad\quad y_2=-2-3=-5$

$\boxed{x_2=-2} \quad\quad \boxed{y_2=-5}$

Thus the vertices of triangle are $(4,3),(-2,-5),(-6,-1)$

Q.6 Find the Value of h.

Soln:- ∵ A, B, C are the vertices of right triangle so by Pythagorean theorem



$|BC|^2=|AB|^2+|AC|^2$

$\{(h-0)^2+(-2-2)^2\}=\{(\sqrt{3}-0)^2+(-1-2)^2\}+\{(\sqrt{3}-h)^2+(-1+2)^2\}$

$\{h^2+16\}=\{3+9\}+\{3+h^2-2\sqrt{3}h+1\}$

$h^2+16=12+4+h^2-2\sqrt{3}h$

$2\sqrt{3}h=h^2+16-h^2-16$

$2\sqrt{3}h=0$

$\Rightarrow \boxed{h=0}$ Ans $\quad ∵ 2\sqrt{3}\neq 0$

* Three points $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ are said to be collinear if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=0$$

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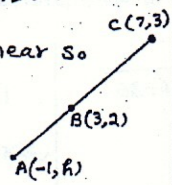
Q.7 Find the value of "h".

Soln:- $\because$ A, B, C are collinear so



$$\begin{vmatrix} -1 & h & 1 \\ 3 & 2 & 1 \\ 7 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow -1(2-3) - h(3-7) + 1(9-14) = 0$

$\Rightarrow 1 + 4h - 5 = 0$

$\Rightarrow 4h - 4 = 0 \Rightarrow 4h = 4$

$\Rightarrow \boxed{h = 1}$ Ans.

Q.8 A(-5,-2), B(5,-4) are the ends of the diameter of Circle so

Centre of Circle = mid of A and B

Centre $= \left(\frac{-5+5}{2}, \frac{-2-4}{2}\right)$

$\boxed{\text{Centre} = (0, -3)}$ Ans.

Radius of Circle $= \frac{1}{2}|AB|$

Radius $= \frac{1}{2}\sqrt{(-5-5)^2 + (-2+4)^2}$

Radius $= \frac{1}{2}\sqrt{100+4} = \frac{1}{2}\sqrt{104}$

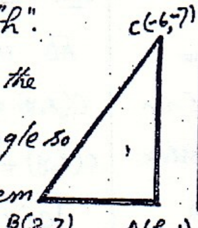
Radius $= \frac{1}{2}(2\sqrt{26}) = \sqrt{26}$

$\boxed{\text{Radius} = \sqrt{26}}$ Ans.

Q.9 Find the value of "h".

Soln:- $\because$ A, B, C are the vertices of right triangle so

By Pythagorean theorem



$|BC|^2 = |AB|^2 + |AC|^2$

$\{(2+6)^2 + (7+7)^2\} = \{(h-2)^2 + (1-7)^2\} + \{(h+6)^2 + (1+7)^2\}$

$64 + 196 = h^2 + 4 - 4h + 36 + h^2 + 36 + 12h + 64$

$2h^2 + 8h + 140 - 64 - 196 = 0$

$\Rightarrow 2h^2 + 8h - 120 = 0$

$\Rightarrow h^2 + 4h - 60 = 0 \quad (\because 2 \neq 0)$

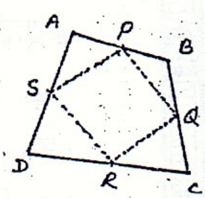
$\Rightarrow (h+10)(h-6) = 0$

$\Rightarrow h + 10 = 0 \quad \vee \quad h - 6 = 0$

$\Rightarrow \boxed{h = -10}$ or $\boxed{h = 6}$

Q.10 A(9,3), B(-7,7), C(-3,-7), D(5,-5) are the vertices of Quadrilateral.

Soln:- Let P, Q, R, S be the mid points of each side.



$P\left(\frac{9-7}{2}, \frac{3+7}{2}\right)$

$P(1, 5)$

$Q\left(\frac{-7-3}{2}, \frac{7-7}{2}\right) = Q(-5, 0)$

$R\left(\frac{-3+5}{2}, \frac{-7-5}{2}\right) = R(1, -6)$

$S\left(\frac{9+5}{2}, \frac{3-5}{2}\right) = S(7, -1)$

Now $|PQ| = \sqrt{(1+5)^2 + (5-0)^2} = \sqrt{36+25}$

$|PQ| = \sqrt{61}$

$|QR| = \sqrt{(-5-1)^2 + (0+6)^2} = \sqrt{36+36}$

$|QR| = \sqrt{72}$

$|RS| = \sqrt{(1-7)^2 + (-6+1)^2} = \sqrt{36+25}$

$|RS| = \sqrt{61}$

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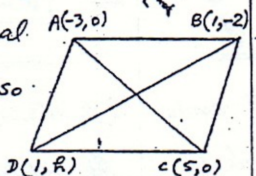
$|PS| = \sqrt{(7-1)^2+(-1-5)^2} = \sqrt{36+36}$

$|PS| = \sqrt{72}$

Clearly $|PQ| = |RS|$ and $|QR| = |PS|$
so opposite sides are equal
So figure formed is parallelogram.

Q.11 Find the value of "h".

Soln:- $\because$ quadrilateral is parallelogram so



$|AB| = |CD|$

$\Rightarrow \sqrt{(1+3)^2+(-2-0)^2} = \sqrt{(5-1)^2+(0-h)^2}$

$\Rightarrow \sqrt{16+4} = \sqrt{16+h^2} \Rightarrow 16+4 = 16+h^2$

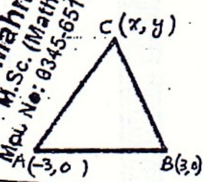
$\Rightarrow h^2 = 4 \Rightarrow \boxed{h = \pm 2}$ Ans.

For $h = 2$	For $h = -2$
$|AC| = \sqrt{(-3-5)^2+(0-0)^2}$	$|AC| = \sqrt{(-3-5)^2+(0-0)^2}$
$|AC| = \sqrt{8^2+0} = 8$	$|AC| = \sqrt{8^2+0} = 8$
$|BD| = \sqrt{(1-1)^2+(2+2)^2}$	$|BD| = \sqrt{(1-1)^2+(-2+2)^2}$
$|BD| = \sqrt{0+4^2} = 4$	$|BD| = \sqrt{0+0} = 0$
$\because |AC| \neq |BD|$	$\because |AC| \neq |BD|$
So it is not Square.	So it is not Square.

Q.12 $A(-3,0)$, $B(3,0)$ are vertices of an equilatral triangle. Find Third Vertex.

Soln:- Let $C(x,y)$ be the 3rd vertex of equilateral triangle so

$|AB| = |BC| = |AC|$



Firstly, let us take

$|AB| = |BC|$

$\sqrt{(3+3)^2+(0-0)^2} = \sqrt{(x-3)^2+(y-0)^2}$

$\Rightarrow 36 = x^2+9-6x+y^2$

$\Rightarrow x^2+y^2-6x = 27$ ——— (1)

Now Consider $|AC| = |BC|$

$\sqrt{(x+3)^2+(y-0)^2} = \sqrt{(x-3)^2+(y-0)^2}$

$\Rightarrow x^2+6x+9+y^2 = x^2-6x+9+y^2$

$\Rightarrow 6x = -6x \Rightarrow 12x = 0$

$\boxed{x=0}$ $(\because 12 \neq 0)$

Putting in (1), we have

$(0)^2+y^2-6(0) = 27 \Rightarrow y^2 = 27$

$\boxed{y = \pm 3\sqrt{3}}$

Thus 3rd Vertex of the triangle is

$C(x,y) = C(0, \pm 3\sqrt{3})$

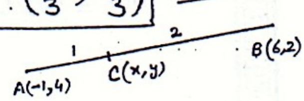
Q.13 Find the point trisecting $A(-1,4)$, $B(6,2)$.

Soln:- Let $C(x,y)$ be the point that divides $\overline{AB}$ in the ratio $1:2$ so

$C(x,y) = C\left(\frac{1(6)+2(-1)}{1+2}, \frac{1(2)+2(4)}{1+2}\right)$

$C(x,y) = C\left(\frac{6-2}{3}, \frac{2+8}{3}\right)$

$\boxed{C(x,y) = C\left(\frac{4}{3}, \frac{10}{3}\right)}$ Ans.

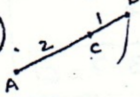


Now if $C(x,y)$ divides AB in the ratio of 2:1 then $C(x,y) = C\left(\frac{2(6)+1(-1)}{2+1}, \frac{2(2)+1(4)}{2+1}\right)$

$C(x,y) = C\left(\frac{12-1}{3}, \frac{4+4}{3}\right)$

$\boxed{C(x,y) = C\left(11/3, 8/3\right)}$ Ans.

Thus trisection Points of A B are $(4/3, 10/3)$, $(11/3, 8/3)$.



Q.14 Find the point three-fifth of the way along segment formed by $A(-5,8)$, $B(5,3)$

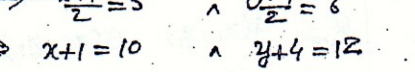
Soln:- Let $m:n = 3:2$ and $C(x,y)$ be the required point so

$C(x,y) = C\left(\frac{3(5)+2(-5)}{3+2}, \frac{3(3)+2(8)}{3+2}\right)$

$C(x,y) = C\left(\frac{15-10}{5}, \frac{9+16}{5}\right)$

$\boxed{C(x,y) = C\left(\frac{5}{5}, \frac{25}{5}\right) = C(1, 5)}$ Ans.

Q.15 (i) Let $C(x,y)$ be the required point which is twice far from A.



Now $|AB| : |BC| = 1:1$

Thus $B(5,6) = B\left[\frac{x+1}{2}, \frac{y+4}{2}\right]$

$\Rightarrow \frac{x+1}{2} = 5 \quad \wedge \quad \frac{y+4}{2} = 6$

$\Rightarrow x+1 = 10 \quad \wedge \quad y+4 = 12$

$\Rightarrow x = 9 \quad \wedge \quad y = 8$

Thus $C(x,y) = C(9,8)$ Ans.

(ii) Let $D(x,y)$ be the required point twice far from A in opposite direction to B.

Now $|AD| : |AB| = 2:1$

Thus $A(1,4) = A\left(\frac{2(5)+1(x)}{2+1}, \frac{2(6)+1(y)}{2+1}\right)$

$A(1,4) = A\left(\frac{x+10}{3}, \frac{y+12}{3}\right)$

$\Rightarrow \frac{x+10}{3} = 1 \quad \wedge \quad \frac{y+12}{3} = 4$

$\Rightarrow x+10 = 3 \quad \wedge \quad y+12 = 12$

$\Rightarrow x = -7 \quad \wedge \quad y = 0$

Thus $D(x,y) = D(-7,0)$ Ans.

Q.16 Let $P(x,y)$ be the point which is equidistant from A, B, C then

$|AP| = |BP| = |CP|$

Now $|AP| = \sqrt{(x-5)^2 + (y-3)^2}$

$|BP| = \sqrt{(x+2)^2 + (y-2)^2}$

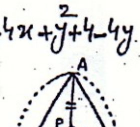
$|CP| = \sqrt{(x-4)^2 + (y-2)^2}$

Firstly let $|AP|^2 = |BP|^2$

$(x-5)^2 + (y-3)^2 = (x+2)^2 + (y-2)^2$

$\cancel{x^2} + 25 - 10x + \cancel{y^2} + 9 - 6x = \cancel{x^2} + 4 + 4x + \cancel{y^2} + 4 - 4y$

$0 = 14x + 2y - 26$



$\Rightarrow 7x + y - 13 = 0$ —— ①

Now suppose $|BP|^2 = |CP|^2$

$(x+2)^2 + (y-2)^2 = (x-4)^2 + (y-2)^2$

$\Rightarrow (x+2)^2 = (x-4)^2$

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$$x^2+4x+4 = x^2-8x+16$$

$$12x = 16-4 = 12$$

$$\Rightarrow \boxed{x=1}$$

Putting in ①, we have

$$7(1)+y = 13 \Rightarrow \boxed{y=13-7=6}$$

Thus $\boxed{P(x,y) = P(1,6)}$ Ans.

Now Circum Radius $= |AP| = |BP| = |CP|$

$$R = \sqrt{(1-5)^2+(6-3)^2} \quad (\text{for } |AP|)$$

$$R = |AP| = \sqrt{16+9} = \sqrt{25}$$

$\boxed{\text{Circum Radius } (R) = 5}$ Ans.

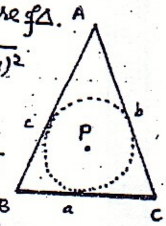
* Bisectors of a triangle are Concurrent. The point of Concurrency of bisectors is also in-centre which is $\left(\dfrac{ax_1+bx_2+cx_3}{a+b+c}, \dfrac{ay_1+by_2+cy_3}{a+b+c}\right)$ where a, b, c are the sides of triangle and $A(x_1,y_1), B(x_2,y_2)$, $C(x_3,y_3)$ are vertices of Δ

* The point where medians of triangle are Concurrent is called Centriod which is

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Q.17 let $A(4,-2)$, $B(-2,4)$, $C(5,5)$

also let P be the in-Centre of Δ.



$$|AB| = c = \sqrt{(4+2)^2+(-2-4)^2}$$

$$c = \sqrt{36+36} = 6\sqrt{2}$$

$$|BC| = a = \sqrt{(5+2)^2+(5-4)^2}$$

$$a = \sqrt{49+1} = 5\sqrt{2}$$

$$|AC| = b = \sqrt{(5-4)^2+(5+2)^2}$$

$$b = \sqrt{1+49} = 5\sqrt{2}$$

Now let $x = \dfrac{ax_1+bx_2+cx_3}{a+b+c}$

$$x = \frac{(5\sqrt{2})(4)+(5\sqrt{2})(-2)+(6\sqrt{2})(5)}{5\sqrt{2}+5\sqrt{2}+6\sqrt{2}}$$

$$x = \frac{40\sqrt{2}}{16\sqrt{2}} = \frac{5}{2}$$

Now let $y = \dfrac{ay_1+by_2+cy_3}{a+b+c}$

$$y = \frac{(5\sqrt{2})(4)+(5\sqrt{2})(-2)+(6\sqrt{2})(5)}{5\sqrt{2}+5\sqrt{2}+6\sqrt{2}}$$

$$y = \frac{40\sqrt{2}}{16\sqrt{2}} = 5/2$$

Thus in-centre $P(x,y)$ is

$$P(x,y) = P(5/2, 5/2).$$

Q: $A(x_1,y_1)$, $B(x_2,y_2)$

Let C divides $\overline{AB}$ in the ratio $1:3$

$$C(a,b) = C\left[\frac{1(x_2)+3(x_1)}{1+3}, \frac{1(y_2)+3(y_1)}{1+3}\right]$$

$$C(a,b) = C\left(\frac{3x_1+x_2}{4}, \frac{3y_1+y_2}{4}\right)$$ Ans.