16. If $y=\sqrt{x}-\frac{1}{\sqrt{x}}$ then prove that

$$2x\frac{dy}{dx}+y=2\sqrt{x}$$

Proof: $y=\sqrt{x}-\frac{1}{\sqrt{x}}$

Diff. w.r.t. x

$$\frac{dy}{dx}=\frac{d}{dx}[\sqrt{x}]-\frac{d}{dx}[x^{-1/2}]$$

$$\frac{dy}{dx}=\frac{1}{2\sqrt{x}}+\frac{1}{2}x^{-3/2}=\frac{1}{2\sqrt{x}}+\frac{1}{2x^{3/2}}$$

$$\frac{dy}{dx}=\frac{1}{2\sqrt{x}}\left[1+\frac{1}{x}\right]$$

Multiplying both sides by 2x

$$2x\frac{dy}{dx}=\frac{\cancel{2x}}{\cancel{2}\sqrt{x}}\left[\frac{x+1}{\cancel{x}}\right]$$

$$2x\frac{dy}{dx}=\frac{x}{\sqrt{x}}+\frac{1}{\sqrt{x}}=\sqrt{x}+\frac{1}{\sqrt{x}}$$

$$2x\frac{dy}{dx}=\sqrt{x}+\frac{1}{\sqrt{x}}$$

Adding y in both sides

$$2x\frac{dy}{dx}+y=\sqrt{x}+\frac{1}{\sqrt{x}}+y$$

$\because y=\sqrt{x}-\frac{1}{\sqrt{x}}$

$$2x\frac{dy}{dx}+y=\sqrt{x}+\cancel{\frac{1}{\sqrt{x}}}+\left(\sqrt{x}-\cancel{\frac{1}{\sqrt{x}}}\right)$$

$$2x\frac{dy}{dx}+y=2\sqrt{x}\quad\text{(Proved)}.$$

Q.17 If $y=x^4+2x^2+2$, prove that

$$\frac{dy}{dx}=4x\sqrt{y-1}$$

$\because y=x^4+2x^2+2$

Diff w.r.t. x

$$4x^3+4x+0=4x(x^2+1)$$

$$x\sqrt{(x^2+1)^2}\qquad(\because a=\sqrt{a^2})$$

$$x^4+2x^2+1$$

$$(2x^2+1)+1-1$$

$$\frac{dy}{dx}=4x\sqrt{x^4+2x^2+2-1}$$

$$\frac{dy}{dx}=4x\sqrt{y-1}\quad\text{(Proved)}$$

Implicit function:-

The function in which x, y are not expressed in terms of dependent and independent variable are called implicit functions.

Such as $x^2+xy=2xy^3$

Implicit Differentiation:-

"The process of finding dy/dx (derivative) from implicit functions is called implicit differentiation."

Exercise 2.4

Q.1 Find dy/dx in the functions:

(i) $y=\sqrt{\frac{1-x}{1+x}}$

Diff. w.r.t. x

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1-x}{1+x}\right)^{1/2}$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2}\cdot\frac{d}{dx}\left(\frac{1-x}{1+x}\right)$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{1+x}{1-x}\right)^{1/2}\cdot\frac{(1+x)\frac{d}{dx}(1-x)-(1-x)\frac{d}{dx}(1+x)}{(1+x)^2}$$

$$\frac{dy}{dx}=\frac{1}{2}\frac{(1+x)^{1/2}}{(1-x)^{1/2}}\cdot\frac{(1+x)(0-1)-(0+1)(1-x)}{(1+x)^2}$$

$$\frac{dy}{dx}=\frac{1}{2}\frac{(x+1)^{1/2}}{\sqrt{1-x}}\cdot\frac{-1-\cancel{x}-1+\cancel{x}}{(1+x)^2}$$

$$\frac{dy}{dx}=\frac{-\cancel{2}}{\cancel{2}\sqrt{1-x}\,(1+x)^{2-1/2}}$$

$$\frac{dy}{dx}=\frac{-1}{\sqrt{1-x}\,(1+x)^{3/2}}\quad\text{Ans.}$$

(ii) $y=\sqrt{x+\sqrt{x}}$

Diff. w.r.t. x

$$\frac{dy}{dx}=\frac{d}{dx}\left(x+\sqrt{x}\right)^{1/2}$$

$$\frac{dy}{dx}=\frac{1}{2}(x+\sqrt{x})^{-1/2}\ \frac{d}{dx}(x+\sqrt{x})$$

$$\frac{dy}{dx}=\frac{1}{2(x+\sqrt{x})^{1/2}}\cdot\left(1+\frac{1}{2\sqrt{x}}\right)$$

$$\frac{dy}{dx}=\frac{1}{2\sqrt{x+\sqrt{x}}}\cdot\left(\frac{2\sqrt{x}+1}{2\sqrt{x}}\right)$$

$$\frac{dy}{dx}=\frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}\quad \text{Ans.}$$

(iii) $y=x\sqrt{\frac{a+x}{a-x}}$

already solved in Last Exercise

(iv) $y=(3x^2-2x+7)^6$

Diff. w.r.t. x

$$\frac{dy}{dx}=\frac{d}{dx}(3x^2-2x+7)^6.$$

$$\frac{dy}{dx}=6(3x^2-2x+7)^5\ \frac{d}{dx}(3x^2-2x+7)$$

$$\frac{dy}{dx}=6(3x^2-2x+7)^5.(6x-2)$$

$$dy/dx=6(6x-2)(3x^2-2x+7)^5\quad \text{Ans.}$$

(v) $y=\sqrt{\frac{a^2+x^2}{a^2-x^2}}$

Diff. w.r.t. x

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{a^2+x^2}{a^2-x^2}\right)^{1/2}$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{a^2+x^2}{a^2-x^2}\right)^{-1/2}\ \frac{d}{dx}\left(\frac{a^2+x^2}{a^2-x^2}\right)$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{a^2-x^2}{a^2+x^2}\right)^{1/2}\cdot\frac{(a^2-x^2)(0+2x)-(a^2+x^2)(0-2x)}{(a^2-x^2)^2}$$

$$\frac{dy}{dx}=\frac{1}{2}\frac{(a^2-x^2)^{1/2}}{(a^2+x^2)^{1/2}}\cdot\frac{2a^2x-\cancel{2x^3}+2a^2x+\cancel{2x^3}}{(a^2-x^2)^2}$$

$$\frac{dy}{dx}=\frac{2\cancel{4}a^2x}{\cancel{2}\sqrt{a^2+x^2}\cdot(a^2-x^2)^{2-1/2}}$$

$$\frac{dy}{dx}=\frac{2a^2x}{\sqrt{a^2+x^2}\ (a^2-x^2)^{3/2}}\quad \text{Ans.}$$

Q.2 Find $\frac{dy}{dx}$ if

(i) $3x+4y+7=0$

Diff. w.r.t. x

$$3\frac{dx}{dx}+4\frac{dy}{dx}+\frac{d(7)}{dx}=0$$

$$3.1+4\frac{dy}{dx}+0=0$$

$$4\frac{dy}{dx}=-3\Rightarrow\frac{dy}{dx}=-3/4\quad \text{Ans.}$$

(ii) $xy+y^2=2$

Diff. w.r.t. x

$$\frac{d}{dx}(xy)+\frac{d}{dx}(y^2)=\frac{d(2)}{dx}$$

$$x\frac{dy}{dx}+y\frac{dx}{dx}+2y\frac{dy}{dx}=0$$

$$x\frac{dy}{dx}+y.1+2y\frac{dy}{dx}=0$$

$$(x+2y)\frac{dy}{dx}=-y$$

$$\frac{dy}{dx}=\frac{-y}{x+2y}\quad \text{Ans.}$$

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(iii) $x^2-4xy-5y=0$

Diff w.r.t. x

$$\frac{d}{dx}(x^2)-4\frac{d}{dx}(xy)-5\frac{dy}{dx}=0$$

$$2x.\frac{dx}{dx}-4\left(x\frac{dy}{dx}+y\frac{dx}{dx}\right)-5\frac{dy}{dx}=0$$

$$2x-4x\frac{dy}{dx}-4y-5\frac{dy}{dx}=0$$

$$2x-4y=(4x+5)\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{2(x-2y)}{(4x+5)}\quad \text{Ans.}$$

(vi) $4x^2+2hxy+by^2+2gx+2fy+c=0$

Diff. w.r.t. 'x', we have

$$4\frac{d(x^2)}{dx}+2h\frac{d}{dx}(xy)+b\frac{d}{dx}(y^2)+2g\frac{d}{dx}(x)+2f\frac{d}{dx}(y)+\frac{d}{dx}(c)=0$$

$$8x+2h\left(x\frac{dy}{dx}+y\frac{dx}{dx}\right)+b\,2y\frac{dy}{dx}+2g\cdot\frac{dx}{dx}+2f\frac{dy}{dx}+0=0$$

$$8x+2hx\frac{dy}{dx}+2hy+2yb\frac{dy}{dx}+2g+2f\frac{dy}{dx}=0$$

$$(8x+2hy+2g)+(2hx+2yb+2f)\frac{dy}{dx}=0$$

$$2(hx+yb+f)\frac{dy}{dx}=-2(4x+hy+g)$$

$$\frac{dy}{dx}=\frac{-\cancel{2}(4x+hy+g)}{\cancel{2}(hx+yb+f)}$$

$$\frac{dy}{dx}=\frac{-(4x+hy+g)}{(hx+yb+f)}\quad \text{Ans.}$$

(v) $x\sqrt{1+y}+y\sqrt{1+x}=0$

Diff. w.r.t. 'x', we have

$$\frac{d}{dx}\left[x\sqrt{1+y}\right]+\frac{d}{dx}\left[y\sqrt{1+x}\right]=0$$

$$x\frac{d}{dx}(\sqrt{1+y})+\sqrt{1+y}\frac{dx}{dx}+y\frac{d}{dx}\sqrt{1+x}+\sqrt{1+x}\frac{dy}{dx}=0$$

$$x\cdot\frac{1}{2\sqrt{1+y}}\frac{dy}{dx}+\sqrt{1+y}+y\cdot\frac{1}{2\sqrt{1+x}}+\sqrt{1+x}\frac{dy}{dx}=0$$

$$\left(\sqrt{1+y}+\frac{y}{2\sqrt{1+x}}\right)+\left(\frac{x}{2\sqrt{1+y}}+\sqrt{1+x}\right)\frac{dy}{dx}=0$$

$$\left(\frac{x}{2\sqrt{1+y}}+\sqrt{1+x}\right)\frac{dy}{dx}=-\left(\sqrt{1+y}+\frac{y}{2\sqrt{1+x}}\right)$$

$$\frac{dy}{dx}=\frac{-\left(\sqrt{1+y}+\frac{y}{2\sqrt{1+x}}\right)}{\left(\sqrt{1+x}+\frac{x}{2\sqrt{x+1}}\right)}\quad \text{Ans.}$$

(vi) $y(x^2-1)=x\sqrt{x^2+4}$

Diff. w.r.t. 'x', we have

$$\frac{d}{dx}\left[y(x^2-1)\right]=\frac{d}{dx}\left[x\sqrt{x^2+4}\right]$$

$$y\frac{d}{dx}(x^2-1)+(x^2-1)\frac{dy}{dx}=x\frac{d}{dx}\sqrt{x^2+4}+\sqrt{x^2+4}\frac{dx}{dx}$$

$$y(2x-0)+(x^2-1)\frac{dy}{dx}=\frac{x\cdot\cancel{2}x}{\cancel{2}\sqrt{x^2+4}}+\sqrt{x^2+4}\cdot 1$$

$$2xy+(x^2-1)\frac{dy}{dx}=\frac{x^2+(\sqrt{x^2+4})^2}{\sqrt{x^2+4}}$$

$$(x^2-1)\frac{dy}{dx}=\frac{x^2+x^2+4}{\sqrt{x^2+4}}-2xy$$

$$(x^2-1)\frac{dy}{dx}=\frac{2x^2+4}{\sqrt{x^2+4}}-2xy$$

$$\frac{dy}{dx}=\frac{2x^2+4-2xy\sqrt{x^2+4}}{\sqrt{x^2+4}\cdot(x^2-1)}\quad \text{Ans.}$$

Q.3 Find $\frac{dy}{dx}=?$ if

(i) $x=\theta+\frac{1}{\theta}$ and $y=\theta+1$

$x=\theta+\theta^{-1}$ $\qquad$ $y=\theta+1$

Diff. w.r.t θ

$$\frac{dx}{d\theta}=\frac{d\theta}{d\theta}+\frac{d\theta^{-1}}{d\theta}$$

$$\frac{dx}{d\theta}=1-1\cdot\theta^{-2}$$

$$\frac{dx}{d\theta}=1-\frac{1}{\theta^2}$$

$$\frac{dx}{d\theta}=\frac{\theta^2-1}{\theta^2}$$

$$\frac{dy}{d\theta}=\frac{d\theta}{d\theta}+\frac{d1}{d\theta}$$

$$\frac{dy}{d\theta}=1+0$$

$$\frac{dy}{d\theta}=1$$

$$\frac{dy}{d\theta}=1$$

Now using chain rule

$$\frac{dy}{dx}=\frac{dy}{d\theta}\cdot\frac{d\theta}{dx}$$

$$\frac{dy}{dx}=1\cdot\frac{\theta^2}{\theta^2-1}=\frac{\theta^2}{\theta^2-1}$$

$$\boxed{\frac{dy}{dx}=\frac{\theta^2}{\theta^2-1}}\quad \text{Ans.}$$

(ii) $x = \dfrac{a(1-t^2)}{1+t^2}$ ^ $y = \dfrac{2bt}{1+t^2}$

$$x = \frac{a(1-t^2)}{1+t^2}$$

Diff. w.r.t. 't', we have

$$\frac{dx}{dt} = a\frac{d}{dt}\left[\frac{1-t^2}{1+t^2}\right]$$

$$\frac{dx}{dt} = a\cdot\frac{(1+t^2)(0-2t)-(1-t^2)(0+2t)}{(1+t^2)^2}$$

$$\frac{dx}{dt} = a\cdot\frac{-2t-\cancel{2t^3}-2t+\cancel{2t^3}}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{-4at}{(1+t^2)^2} \quad \text{—— (1)}$$

$$y = \frac{2bt}{1+t^2}$$

Diff. w.r.t. "t". we have

$$\frac{dy}{dt} = 2b\frac{d}{dt}\left[\frac{t}{1+t^2}\right]$$

$$\frac{dy}{dt} = 2b\left[\frac{(1+t^2).1-t(2t+0)}{(1+t^2)^2}\right]$$

$$\frac{dy}{dt} = 2b\cdot\frac{1+t^2-2t^2}{(1+t^2)^2}$$

$$\frac{dy}{dt} = 2b\frac{(1-t^2)}{(1+t^2)^2} \quad \text{—— (2)}$$

Now using Chain Rule

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{\cancel{2}b(1-t^2)}{\cancel{(1+t^2)^2}}\times\frac{\cancel{(1+t^2)^2}}{-\cancel{4}_2at}$$

$$\frac{dy}{dx} = \frac{-b(1-t^2)}{2at}$$

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$$\boxed{\frac{dy}{dx} = \frac{b(t^2-1)}{2at}}$$ Ans.

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(4) Prove that $y\dfrac{dy}{dx} + x = 0$ if $x = \dfrac{1-t^2}{1+t^2}$ and $y = \dfrac{2t}{1+t^2}$

Proof:

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$$x = \frac{1-t^2}{1+t^2}$$

Diff. w.r.t "t"

$$\frac{dx}{dt} = \frac{d}{dt}\left[\frac{1-t^2}{1+t^2}\right]$$

$$\frac{dx}{dt} = \frac{(1+t^2).(0-2t)-(1-t^2)(0+2t)}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{-2t-\cancel{2t^3}-2t+\cancel{2t^3}}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{-4t}{(1+t^2)^2} \quad \text{—— (1)}$$

$$y = \frac{2t}{1+t^2}$$

Diff. w.r.t. "t"

$$\frac{dy}{dt} = 2\frac{d}{dt}\left[\frac{t}{1+t^2}\right]$$

$$dy/dt = 2\cdot\frac{(1+t^2).1-t(0+2t)}{(1+t^2)^2}$$

$$= 2\left[\frac{1+t^2-2t^2}{(1+t^2)^2}\right] = 2\left[\frac{1-t^2}{(1+t^2)^2}\right]$$

$$\frac{dy}{dt} = \frac{2(1-t^2)}{(1+t^2)^2} \quad \text{—— (2)}$$

Now Using chain Rule.

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{\cancel{2}(1-t^2)}{\cancel{(1+t^2)^2}}\times\frac{\cancel{(1+t^2)^2}}{-\cancel{4}_2t} = \frac{(1-t^2)}{-2t}$$

$$\frac{dy}{dx} = \frac{(t^2-1)}{2t}$$

Multiplying both sides by y,

$$y\frac{dy}{dx} = y\cdot\frac{(t^2-1)}{2t} \qquad \left(\because y = \frac{2t}{1+t^2}\right)$$

$$y\frac{dy}{dx} = \frac{\cancel{2t}}{1+t^2}\cdot\frac{t^2-1}{\cancel{2t}}$$

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$$y\frac{dy}{dx} = \frac{t^2-1}{1+t^2}$$

Adding x in both sides

$$y\frac{dy}{dx}+x = \frac{t^2-1}{1+t^2}+x$$

$$y\frac{dy}{dx}+x = \frac{t^2-1}{1+t^2}+\frac{1-t^2}{1+t^2} \qquad \left(\because x = \frac{1-t^2}{1+t^2}\right)$$

$$y\frac{dy}{dx}+x = \frac{t^2-1+1-t^2}{1+t^2} = \frac{0}{1+t^2}$$

$$y\frac{dy}{dx}+x = 0 \qquad \text{(Proved)}$$

Q.5 Differentiate

(i) $x^2 - \frac{1}{x^2}$ w.r.t x^4

Let $y = x^2 - \frac{1}{x^2}$ and $u = x^4$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2) - \frac{d}{dx}(x^{-2}) \qquad \frac{du}{dx} = \frac{d}{dx}(x^4)$$

$$\frac{dy}{dx} = 2x + 2x^{-3}\cdot 1 \qquad \frac{du}{dx} = 4x^3\cdot 1$$

$$\frac{dy}{dx} = 2x + \frac{2}{x^3} \qquad \frac{du}{dx} = 4x^3$$

$$\frac{dy}{dx} = \frac{2x^4+2}{x^3}$$

Now using chain Rule

$$\frac{dy}{du} = \frac{dy}{dx}\cdot\frac{dx}{du}$$

$$\frac{dy}{du} = \frac{\cancel{2}(x^4+1)}{x^3}\cdot\frac{1}{{}^{2}\cancel{4}x^3} = \frac{x^4+1}{2x^6}$$

$$\boxed{\frac{dy}{du} = \frac{x^4+1}{2x^6}} \qquad \text{Ans.}$$

(ii) $(1+x^2)^n$ w.r.t. x^2

Let $y = (1+x^2)^n$ and $u = x^2$

$$\frac{dy}{dx} = \frac{d}{dx}(1+x^2)^n \qquad \frac{du}{dx} = \frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = n(1+x^2)^{n-1}\cdot 2x \qquad \frac{du}{dx} = 2x\cdot 1$$

$$\frac{dy}{dx} = 2nx(1+x^2)^{n-1} \qquad \frac{du}{dx} = 2x$$

Now using chain Rule.

$$\frac{dy}{du} = \frac{dy}{dx}\cdot\frac{dx}{du}$$

$$\frac{dy}{du} = \cancel{2}n\cancel{x}(1+x^2)^{n-1}\cdot\frac{1}{\cancel{2}\cancel{x}}$$

$$\boxed{\frac{dy}{du} = n(1+x^2)^{n-1}} \qquad \text{Ans.}$$

(iii) $\frac{x^2+1}{x^2-1}$ w.r.t. $\frac{x-1}{x+1}$

Let $y = \frac{x^2+1}{x^2-1}$ and $u = \frac{x-1}{x+1}$

$$\frac{dy}{dx} = \frac{(x^2-1)(2x+0)-(x^2+1)(2x-0)}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{\cancel{2x^3}-2x-\cancel{2x^3}-2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{-4x}{(x^2-1)^2} \text{ ——— (1)}$$

$$\frac{du}{dx} = \frac{(x+1)(1-0)-(x-1)(1+0)}{(x+1)^2}$$

$$\frac{du}{dx} = \frac{\cancel{x}+1-\cancel{x}+1}{(x+1)^2} = \frac{2}{(x+1)^2}$$

Now using chain Rule

$$\frac{dy}{du} = \frac{dy}{dx}\cdot\frac{dx}{du}$$

$$\frac{dy}{du} = \frac{-{}^{2}\cancel{4}x}{(x^2-1)^2}\cdot\frac{(x+1)^2}{\cancel{2}}$$

$$\boxed{\frac{dy}{du} = \frac{-2x(x+1)^2}{(x^2-1)^2}} \qquad \text{Ans.}$$

(iv) $\dfrac{ax+b}{cx+d}$ w.r.t $\dfrac{ax^2+b}{ax^2+d}$

Let $y = \dfrac{ax+b}{cx+d}$ and $u = \dfrac{ax^2+b}{ax^2+d}$

$$\frac{dy}{dx} = \frac{(cx+d)(a.1+0)-(ax+b)(c.1+0)}{(cx+d)^2}$$

$$\frac{dy}{dx} = \frac{\cancel{acx}+da-\cancel{acx}-bc}{(cx+d)^2}$$

$$\frac{dy}{dx} = \frac{ad-bc}{(cx+d)^2} \quad \text{—— (1)}$$

$$\frac{du}{dx} = \frac{(ax^2+d)(2ax+0)-(ax^2+b)(2ax+0)}{(ax^2+d)^2}$$

$$\frac{du}{dx} = \frac{\cancel{2a^2x^3}+2adx-\cancel{2a^2x^3}-2abx}{(ax^2+b)^2}$$

$$\frac{du}{dx} = \frac{2ax(d-b)}{(ax^2+b)^2} \quad \text{—— (2)}$$

Now using Chain Rule.

$$\frac{dy}{du} = \frac{dy}{dx}\cdot\frac{dx}{du}$$

$$\frac{dy}{du} = \frac{ad-bc}{(cx+d)^2} \times \frac{(ax^2+b)^2}{2ax(d-b)}$$

$$\boxed{\frac{dy}{du} = \frac{(ad-bc)(ax^2+b)^2}{2ax(cx+d)^2(d-b)}} \quad \text{Ans.}$$

(v) $\dfrac{x^2+1}{x^2-1}$ w.r.t x^3

Let $y = \dfrac{x^2+1}{x^2-1}$ and $u = x^3$

$$\frac{dy}{dx} = \frac{(x^2-1)(2x+0)-(x^2+1)(2x-0)}{(x^2-1)^2}.$$

$$\frac{dy}{dx} = \frac{\cancel{2x^3}-2x-\cancel{2x^3}-2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

$$\frac{du}{dx} = 3x^2.1 = 3x^2$$

using chain Rule

$$\frac{dy}{du} = \frac{dy}{dx}\cdot\frac{dx}{du} = \frac{-4x}{(x^2-1)^2}\cdot\frac{1}{3x^2}$$

$$\boxed{\frac{dy}{du} = \frac{-4}{3x(x^2-1)^2}} \quad \text{Ans.}$$

Derivatives of Trigonometric functions.

(1) $\dfrac{d}{dx}[\sin x] = \cos x \cdot 1$

(2) $\dfrac{d}{dx}[\cos x] = -\sin x \cdot 1$

(3) $\dfrac{d}{dx}[\tan x] = \sec^2 x \cdot 1$

(4) $\dfrac{d}{dx}[\cot x] = -\operatorname{cosec}^2 x \cdot 1$

(5) $\dfrac{d}{dx}[\sec x] = \sec x.\tan x \cdot 1$

(6) $\dfrac{d}{dx}[\operatorname{cosec} x] = -\operatorname{cosec} x.\cot x \cdot 1$

Derivatives of Inverse Trigonometric Functions:

(1) $\dfrac{d}{dx}[\sin^{-1}x] = \dfrac{1}{\sqrt{1-x^2}} \cdot 1$

(2) $\dfrac{d}{dx}[\cos^{-1}x] = \dfrac{-1}{\sqrt{1-x^2}} \cdot 1$

(3) $\dfrac{d}{dx}[\tan^{-1}x] = \dfrac{1}{1+x^2} \cdot 1$

(4) $\dfrac{d}{dx}[\cot^{-1}x] = \dfrac{-1}{1+x^2} \cdot 1$

(5) $\dfrac{d}{dx}[\sec^{-1}x] = \dfrac{1}{x\sqrt{x^2-1}} \cdot 1$

(6) $\dfrac{d}{dx}[\operatorname{cosec}^{-1}x] = \dfrac{-1}{x\sqrt{x^2-1}} \cdot 1$

* Important to Note that all the functions beginning with "C" have -ve Sign with the derivative terms.

* In all the trigonometric functions, angle is taken as in radian measurement.