TAHIR MAHMOOD

(ii) $\frac{1}{\sqrt{x+a}} = (x+a)^{-1/2}$

Let $y = (x+a)^{-1/2}$ —— (1)

$y+\delta y = (x+\delta x+a)^{-1/2}$ —— (2)

Subtracting Eq (1) from Eq (2)

$$\delta y = \left[(x+a)+\delta x\right]^{-1/2} - (x+a)^{-1/2}$$

$$\delta y = (x+a)^{-1/2}\left[1+\frac{\delta x}{(x+a)}\right]^{-1/2} - (x+a)^{-1/2}$$

Using binomial theorem for $\left[1+\frac{\delta x}{x+a}\right]^{-1/2}$

$$\delta y = (x+a)^{-1/2}\left[1-\frac{\delta x}{2(x+a)} - \frac{\frac{1}{2}(-\frac{1}{2}-1)}{2!}\frac{\delta x^2}{(x+a)^2}+\cdots\right]-(x+a)^{-1/2}$$

$$\delta y = \cancel{(x+a)^{-1/2}} - \frac{\delta x}{2(x+a)^{3/2}} + \frac{3\,\delta x^2}{8(x+a)^{5/2}}+\cdots - \cancel{(x+a)^{-1/2}}$$

$$\delta y = \delta x\left[\frac{-1}{2(x+a)^{3/2}} + \frac{3\,\delta x}{8(x+a)^{5/2}}+\cdots\right]$$

Dividing by δx where $\lim\limits_{\delta x\to 0}$

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x} = \lim_{\delta x\to 0}\frac{\cancel{\delta x}\left(\frac{-1}{2(x+a)^{3/2}}+\frac{3\,\delta x}{8(x+a)^{5/2}}+\cdots\right)}{\cancel{\delta x}}$$

$$\frac{dy}{dx} = \lim_{\delta x\to 0}\left(\frac{-1}{2(x+a)^{3/2}} + \frac{3\,\delta x}{8(x+a)^{5/2}}+\cdots\right)$$

By Applying Limit

$$\frac{dy}{dx} = \frac{-1}{2(x+a)^{3/2}} + 0 + \cdots$$

$$\boxed{\frac{dy}{dx} = \frac{-1}{2(x+a)^{3/2}}}$$

Exercise 2.2

(ii) $(2x+3)^5$

Let $y = (2x+3)^5$ —— (1)

$y+\delta y = \left[2(x+\delta x)+3\right]^5$ —— (2)

Subtracting Eq (1) from Eq (2)

$$\delta y = \left[(2x+3)+2\delta x\right]^5 - (2x+3)^5$$

$$\delta y = (2x+3)^5\left[1+\frac{2\delta x}{(2x+3)}\right]^5 - (2x+3)^5$$

Using binomial theorem

$$\delta y = (2x+3)^5\left[1+\frac{10\,\delta x}{(2x+3)} + \frac{5\cdot 4}{2!}\frac{4\,\delta x^2}{(2x+3)^2}+\cdots\right]-(2x+3)^5$$

$$\delta y = \cancel{(2x+3)^5} + 10\,\delta x(2x+3)^4 + 40\,\delta x^2(2x+3)^3+\cdots-\cancel{(2x+3)^5}$$

$$\delta y = \delta x\left[10(2x+3)^4 + 40\,\delta x(2x+3)^3+\cdots\right]$$

Dividing by δx where $\lim\limits_{\delta x\to 0}$

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x} = \lim_{\delta x\to 0}\frac{\cancel{\delta x}\left(10(2x+3)^4+40\,\delta x(2x+3)^3+\cdots\right)}{\cancel{\delta x}}$$

$$\frac{dy}{dx} = \lim_{\delta x\to 0}\left(10(2x+3)^4 + 40\,\delta x(2x+3)^3+\cdots\right)$$

By Applying Limit

$$\frac{dy}{dx} = 10(2x+3)^4 + 40(0)(2x+3)^3+\cdots$$

$$\boxed{\frac{dy}{dx} = 10(2x+3)^4}$$

(v) $\frac{1}{(az-b)^7} = (az-b)^{-7}$

Let $y = (az-b)^{-7}$ —— (1)

$y+\delta y = \left[a(z+\delta z)-b\right]^{-7}$ —— (2)

Subtracting Eq (1) from Eq (2)

$$\delta y = \left[(az-b)+\delta z\right]^{-7} - (az-b)^{-7}$$

$$\delta y = (az-b)^{-7}\left[1+\frac{\delta z}{az-b}\right]^{-7} - (az-b)^{-7}$$

Using binomial theorem

$$\delta y = (az-b)^{-7}\left[1-\frac{7\,\delta z}{(az-b)} + \frac{7\cdot 8}{2!}\frac{\delta z^2}{(az-b)^2}+\cdots\right]-(az-b)^{-7}$$

$$\delta y = \cancel{(az-b)^{-7}} - 7\,\delta z(az-b)^{-8} + 28\,\delta z^2(az-b)^{-9}+\cdots - \cancel{(az-b)^{-7}}$$

$$\delta y = \delta z\left[-7(az-b)^{-8} + 28\,\delta z(az-b)^{-9}+\cdots\right]$$

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779