Exercise 2.1

Q.1: Differentiate w.r.t x ab-initio.

i. x^3 ii. $\frac{1}{x^2}$ iii. $\frac{1}{\sqrt{x}}$

iv. $x^{\frac{2}{3}}$ v. $x^{\frac{1}{4}}$ vi. $x^{-\frac{4}{5}}$

Q.2: Find the derivative from first principle.

i. $x^3 - 3x + 4$ ii. $\sqrt{x+9}$ iii $(x+4)^{\frac{2}{3}}$

Answers 2.1

Q.1: i. $3x^2$ ii. $\frac{-2}{x^3}$ iii. $-\frac{1}{2x^{3/2}}$

iv. $\frac{2}{3x^{1/3}}$ v. $\frac{1}{4x^{3/4}}$ vi. $-\frac{4}{5}x^{-9/5}$

Q.2 i. $3x^2 - 3$ ii. $\frac{1}{2\sqrt{x+9}}$ iii $\frac{2}{3}(x+4)^{-\frac{1}{3}}$

2.4 Fundamental Rules for Differentiation

Rule 1:

Derivative of x^n (power rule)

Let $y = x^n$

$$y + \delta y = (x + \delta x)^n$$

$$\delta y = (x + \delta x)^n - x^n \quad = x^n \left(1 - \frac{\delta x}{x}\right)^n - x^n$$

$$= x^n\left[\left(1 + \frac{\delta x}{x}\right)^n - 1\right]$$

$$= x^n\left[1 + n\frac{\delta x}{x} + \frac{n(n-1)}{2!}\left(\frac{\delta x}{x}\right)^2 + \frac{n(n-1)(n-2)}{3!}\left(\frac{\delta x}{x}\right)^3 + \ldots\ldots - 1\right]$$

$$\delta y = x^n\left[n\frac{\delta x}{x} + \frac{n(n-1)}{2!}\left(\frac{\delta x}{x}\right)^2 + \frac{n(n-1)(n-2)}{3!}\left(\frac{\delta x}{x}\right)^3 + \dots\dots\dots\right]$$

$$\frac{\delta y}{\delta x} = \frac{x^n\,\delta x}{\delta x}\left[n\frac{1}{x} + \frac{n(n-1)\delta x}{2!\;\;x^2} + \dots\dots\right]$$

$$\lim_{x\to 0}\frac{\delta y}{\delta x} = x^n\lim_{x\to 0}\left[n\frac{1}{x} + \frac{n(n-1)\delta x}{2!\;\;x^2} + \dots\dots\right]$$

$$\frac{dy}{dx} = x^n\left[n\frac{1}{x} + 0 + 0\dots\dots\right]$$

$$\frac{dy}{dx} = nx^{n-1}$$

$$\therefore\quad \frac{dy}{dx} = (x^n) = n\,x^{n-1}$$

Note : if n is positive , we use Binomial theorem.

Examples 2:

i. $y = x^7$ ii. $y = x^{3/4}$ Find $\frac{dy}{dx}$

Solution:

i. $y = x^7$

$$\frac{dy}{dx} = 7\,x^{7-1} \quad = 7x^6$$

ii. $y = x^{3/4}$

$$\frac{dy}{dx} = \frac{3}{4}x^{3/4 - 1} \quad = \frac{3}{4}x^{-1/4} \quad = \frac{3}{4\,x^{1/4}}$$

Rule 2:

The differential coefficient of any constant is zero.

Let $y = c$

$$y + \delta y = c$$

$$y + \delta y - y = c - c$$

$$\delta y = 0$$

$$\lim_{x\to 0}\frac{\delta y}{\delta x} = 0 \implies \frac{dy}{dx} = 0$$

Rule 3:

Product of constant and function.

Let $y = cv$

Where c is a constant and v is a function of x.

$$y + \delta y = c(v + \delta v)$$

$$\delta y = c(v + \delta v) - cv$$

$$\delta y = cv + c\delta v) - cv$$

$$\delta y = c\,\delta\, v$$

$$\frac{\delta y}{\delta x} = c\,\frac{\delta v}{\delta x}$$

$$\lim_{x\to 0}\frac{\delta y}{\delta x} = \lim_{x\to 0}\left(c.\frac{\delta v}{\delta\, x}\right)$$

$$\frac{dy}{dx} = c\,\frac{dv}{dx}$$

Example 3:

If $y = 3x^2$, find $\frac{dy}{dx}$

Solution:

$$y = 3x^2$$

$$\frac{dy}{dx} = \frac{d}{dx}(3x^2) = 3.\frac{d}{dx}(x^2) = 3.2x. = 6x$$

Rule 4 : Differential coefficient of a sum.

The differential coefficient of the sum of a set of functions of x is the sum of the differential coefficients of the several functions.

Let $y = u + v + w + \ldots\ldots\ldots\ldots$

Which are all finite in number and function of x, therefore, y is also function of x.

Let the increments of u, v, w..........and y be respectively.

$\delta u, \delta v, \delta w, \ldots\ldots\ldots\ldots\ldots\ldots, \delta y$

$$\therefore \quad y + \delta y = (u + \delta u) + (v + \delta v) + (w + \delta w)\ldots\ldots\ldots\ldots$$

$$\delta y = \delta u + \delta v + \delta w + \ldots\ldots\ldots\ldots\ldots$$

$$\frac{\delta y}{\delta x} = \frac{\delta u}{\delta x} + \frac{\delta v}{\delta x} + \frac{\delta w}{\delta x} + \ldots\ldots\ldots\ldots\ldots$$

$$\therefore \quad \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{\delta u}{\delta x} + \lim_{\delta x \to 0} \frac{\delta v}{\delta x} + \lim_{\delta x \to 0} \frac{\delta w}{\delta x}$$

Hence $$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} + \ldots\ldots\ldots\ldots$$

Example 4:

If $y = x^3 + x^2 + 2x + 3$ find $\frac{dy}{dx}$

Solution:

$$y = x^3 + x^2 + 2x + 3$$

$$\frac{dy}{dn} = \frac{d}{dn}(x^3 + x^2 + 2x + 3)$$

$$= \frac{dy}{dn}(x^3) + \frac{d}{dn}(x^2) + 2\frac{d}{dn}(x) + \frac{d}{dn}(3)$$

$$= 3x^2 + 2x + 2$$

Rule 5 (Chain rule):

Differentiation of a function of function or composite functions:

When y is a function of u and u is further a function of x. then y is also function of x and called function of function or composite function. The derivative of y w.r.t. x is product of derivative of y w.r.t. u and the derivative of u w.r.t. x. (This is sometimes called the chain rule of differentiation)

Since $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du}\ \frac{du}{dx}$

Proof:

Let δx, δu and δy be the increments of x, u and y then

$$\frac{\delta y}{\delta x} = \frac{\delta y}{\delta u}\ \frac{\delta u}{\delta x}$$

In the limiting case as $\delta x \to 0$ also $\delta u \to 0$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta u \to 0} \frac{\delta y}{\delta u} \cdot \lim_{\delta x \to 0} \frac{\delta u}{\delta x}$$

$$\frac{dy}{dx} = \frac{dy}{du}\ \frac{du}{dx}$$

Example 5:

If $y = (3x^2 + 2x + 9)^7$ Find $\frac{dy}{dx}$

Solution:

Let $u = 3x^2 + 2x + 9$

Then $y = u^7$

$$\frac{dy}{du} = 7u^6$$

also $u = 3x^2 + 2x + 9$

$$\frac{du}{dx} = 6x + 2$$

Using Chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 7u^6 (6x + 2)$$

$$= 7(3x^2 + 2x + 9)^6 (6x + 2)$$

Rule 6:

Generalization of Power rule

Let $y = [f(x)]^n$

Put $u = f(x)$ then $y = u^n$

And $\frac{du}{dx} = f'(x)$ $\frac{dy}{du} = nu^{n-1}$

By chain rule $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

$$= nu^{n-1} f'(x)$$

$$\frac{dy}{dx} = n[f(x)]^{n-1} f'(x)$$

Example 6:

Find the derivative of $(ax + b)^n$

Solution:

Let $y = (ax + b)^n$

$$y + \delta y = [a(x + \delta x) + b]^n$$

$$= [a(x + \delta x) + b]^n = [\,(ax + b) + a\,\delta x]^n$$

$$= (ax + b)^n\left(1 + \frac{a\delta x}{ax + b}\right)^n$$

$$y + \delta y - y = (ax + b)^n\left(1 + \frac{a\delta x}{ax + b}\right)^n - (ax + b)^n$$

$$\delta y = (ax + b)^n\left[\left(1 + \frac{a\delta x}{ax + b}\right)^n - 1\right]$$

$$= (ax + b)^n\left[1 + n\frac{a\delta x}{ax + b} + \frac{n(n-1)}{2!}\left(\frac{a\delta x}{ax + b}\right)^2 + \ldots\ldots -1\right]$$

$$\delta y = (ax + b)^n\left[n\frac{a\delta x}{ax + b} + \frac{n(n-1)}{2!}\left(\frac{a\delta x}{ax + b}\right)^2 + \ldots\ldots\right]$$

Divide both sides by δx and taking limit.

$$\lim_{\delta x \to 0}\frac{\delta y}{\delta x} = \lim_{\delta x \to 0}(ax + b)^n\left[n\left(\frac{a}{ax + b}\right) + \frac{n(n-1)}{2!}\frac{a^2\delta x}{(ax + b)^2} + \ldots\ldots\right]$$

$$\frac{dy}{dx} = (ax + b)^n\left[\frac{na}{ax + b} + 0 + \ldots\ldots\right]$$

$$\frac{dy}{dx} = n(ax + b)^{n-1}\,a$$

Note : if n is positive , we use binomial theorem.

OR

$y = (ax + b)^n$

Apply the generalization of power rule.

$$\frac{dy}{dx} = (ax + b)^{n-1} \cdot \frac{d}{dx}(ax + b)$$

$$\frac{dy}{dx} = (ax + b)^{n-1}(a + 0)$$

$$= a(ax + b)^{n-1}$$

Example 7:

$$y = \sqrt{x^2 + 1} \quad \text{find} \quad \frac{dy}{dx}$$

Solution:

$$\therefore \qquad y = (x^2 + 1)^{1/2}$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + 1)^{1/2}$$

$$= \frac{1}{2}(x^2 + 1)^{-1/2}\frac{d}{dx}(x^2 + 1)$$

$$= \frac{1}{2}(x^2 + 1)^{1/2}\, 2x$$

$$= \frac{x}{\sqrt{x^2 + 1}}$$

Rule 7:

The Differential Coefficient of the Product of Two Functions:

The product of two derivable functions is itself derivable and its derivative equal to (first function. derivative of second function) + (Second function. derivative of first function)

Let $y = uv$

Where u, v are two derivable function of x.

Let δu, δv, δy be the increments in u, v and y respectively corresponding to the increment δx in x. We have

$$y + \delta y = (u + \delta u)(v + \delta v)$$

$$= u\,v + u\delta v + v\delta u + \delta u\ \delta v$$

$$y + \delta y - y = uv + u\delta v + u\delta u + \delta u.\ \delta v - uv$$

$$\delta y = u.\delta v + v.\delta u + \delta u.\ \delta v$$

$$\frac{\delta y}{\delta x} = u.\frac{\delta v}{\delta x} + v.\frac{\delta u}{\delta x} - \delta u.\frac{\delta v}{\delta x}$$

When $\delta x \rightarrow 0$. Then also $\delta u \rightarrow 0$

$$\lim_{\delta\to 0} \frac{\delta y}{\delta x} = \lim_{\delta x\to 0} \left[u \frac{\delta v}{\delta x} + v.\frac{\delta u}{\delta x} + \delta u.\frac{\delta v}{\delta x} \right]$$

$$\frac{dy}{dx} = u. \lim_{\delta x\to 0} \frac{\delta v}{\delta x} + v \lim_{x\to 0} \frac{\delta u}{\delta x} + \lim_{\delta x\to 0} \delta u.\frac{\delta v}{\delta x}$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v.\frac{du}{dx}$$

$$\therefore \quad \frac{dy}{dx}(uv) = u.\frac{dv}{dx} + v.\frac{du}{dx}$$

Note: Extension rule of product: The result may be extended for the product of more than two functions.

Let $y = uvw$

Then $\frac{dy}{dx} = uv\frac{dw}{dx} + uw.\frac{dv}{dx} + vw.\frac{du}{dx}$

Example 8:

Find the derivatives of $y = (x^2 + 2)(x^3 + 3)$

Solution:

Here $u = x^2 + 2$ and $v = x^3 + 3$. Then $y = uv$

$\frac{du}{dx} = 2x$ $\frac{dv}{dx} = 3x^2$

$$\frac{dy}{dx} = u.\frac{dv}{dx} + v.\frac{du}{dx}$$

$$= (x^2 + 2)(3x^2) + (x^3 + 3)(2x)$$

$$= 3x^4 + 6x^2 + 2x^4 + 6x$$

$$= 5x^4 + 6x^2 + 6x$$

Rule 9:

The differential Coefficient of a Quotient of Two Function:

Let $y = \frac{u}{v}$

Where u, v are two derivable function of x also v is not zero for all values under consideration.

Then $y + \delta y = \frac{u + \delta u}{v + \delta v}$

$$y + \delta y - y = \frac{u + \delta u}{v + \delta v} - \frac{u}{v}$$

$$\Rightarrow \qquad \delta y = \frac{v.\delta u - u.\,\delta u}{v(v + \delta v)}$$

$$\frac{\delta y}{\delta x} = \left[\frac{v\,\frac{\delta u}{\delta x} - u.\frac{\delta v}{\delta x}}{v(v - \delta v)}\right]$$

When $\delta x \rightarrow 0$ also $\delta v \rightarrow 0$:

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \left[\frac{v\,\frac{\delta u}{\delta x} - u.\,\frac{\delta v}{\delta x}}{v^2}\right]$$

$$\frac{d}{dx} = \frac{v\,\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$$

$$\therefore \qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$$

Example 9:

$\Rightarrow$ If $y = \dfrac{x}{1 + x}$ find $\dfrac{dy}{dx}$

Solution:

$$\frac{dy}{dx} = \frac{(1 + x)\,\frac{d}{dx}(x) - x.\,\frac{d}{dx}(1 + x)}{(1 + x)^2}$$

$$= \frac{(1 + x)\,1 - x.1}{(1 + x)^2}$$

$$= \frac{1 + x - x}{(1 + x)^2}$$

$$= \frac{1}{(1 + x)^2}$$

Illustrative Examples:

1. Find the derivatives of the following:

i. $x^3 + 4x + 9$ ii. $x^{3/2} + x^{1/2} + 1$

Solution:

i. Let $y = x^3 + 4x + 9$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 + 4x + 9)$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) + 4\frac{d}{dx}(x) + \frac{d}{dx}(9)$$

$$\frac{dy}{dx} = 3x^3 + 4 + 0$$

$$\frac{dy}{dx} = 3x^2 + 4$$

ii. Let $y = x^{3/2} + x^{½} + 1$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{3/2} + x^{½} + 1\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{3/2}\right) + \frac{d}{dx}\left(x^{½}\right) + \frac{d}{dx}(1)$$

$$= \frac{3}{2}x^{3/2-1} + \frac{1}{2}x^{½-1} + 0$$

$$= \frac{3}{2}x^{½} + \frac{1}{2x^{½}}$$

2. i. Find $\frac{dy}{dx}$ at $x = 2$ When $y = \sqrt{u^2 - 2}$ and $u = 4\sqrt{x}$

ii. if $y = \sqrt{ax^2 + 2bx + c}$ Find $\frac{dy}{dx}$

Solution:

i. $y = \sqrt{u^2 - 2}$, $u = 4\sqrt{x}$

$$\frac{dy}{du}=\frac{1}{2}\,(u^2-2)^{-1/2}(2u)\quad,\quad \frac{du}{dx}=4\,\frac{1}{2}\,x^{-1/2}$$

$$\frac{dy}{du}=\frac{u}{\sqrt{u^2-2}}\quad,\quad \frac{du}{dx}=\frac{2}{\sqrt{x}}$$

Using chain rule

$$\frac{dy}{dx}=\frac{dy}{du}\,\frac{du}{dx}$$

$$=\frac{u}{\sqrt{u^2-2}}\,\frac{2}{\sqrt{x}}$$

$$=\frac{4\sqrt{x}}{\sqrt{16x-2}}\,\frac{2}{\sqrt{x}}$$

$$=\frac{8}{\sqrt{16x-2}}$$

at $x=2$

$$\therefore\quad\left(\frac{dy}{dx}\right)=\frac{8}{\sqrt{32-2}}=\frac{8}{\sqrt{30}}$$

ii. Let $u=ax^2+2bx+c$

$$y=\sqrt{u}\quad,\quad u=ax^2+2bx+c$$

$$\Rightarrow\quad \frac{dy}{du}=\frac{1}{2}(u)^{-1/2}\quad,\quad \frac{du}{dx}=2ax+2b$$

$$=\frac{1}{2\sqrt{u}}$$

Using chain rule

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$$

$$=\frac{1}{2\sqrt{u}}\times(2ax+2b)$$

$$=\frac{ax+b}{\sqrt{ax^2+2bx+c}}$$

Summary

1. (i) $\frac{d}{dx}(x^n) = nx^{n-1}$, for all constant values of n (ii) $\frac{d}{dx}(x) = 1$.

2. $\frac{d}{dx}(c) = 0$, where c is any constant.

3. $\frac{d}{dx}(u+v-w+...) = \frac{du}{dx} + \frac{dv}{dx} - \frac{dw}{dx} + ...$

4. (i) $\frac{d}{dx}(cu) = c\frac{du}{dx}$ (ii) $\frac{d}{dx}(\frac{u}{c}) = \frac{1}{c}\frac{du}{dx}$

5. $\frac{d}{dx}(ax+b)^n = na\,(ax+b)^{n-1}$, for all constant values of n.

6. $\frac{d}{dx}(v)^n = nv^{n-1}\frac{dv}{dx}$, where v is a function of x.

7. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

8. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

9. $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$, y being a function of u, u being function of x.

10. $\frac{dy}{dx} = \frac{1}{dx/dy}$, y being a function of x.

In above results u, v, w, y are functions of x whereas a, b, c and n are constants.

Exercise 2.2

Q.1 Differentiate the following w.r.t.x.

i. $7x^7$ ii. $-5 + 3x - \frac{3}{2}x^2 - 7x^3$ iii. $2x^3 + 4x^2 - 5x + 8$

iv. $3x^2 + x^{-1/2}$ v $\frac{6}{x} + \frac{4}{x^2} - \frac{3}{x^3}$ vi. $\frac{1}{5}x^{5/2} + \frac{1}{3}x^{3/2}$

vii $x^2 - x^{-2}$ viii $\sqrt{x} + \frac{1}{\sqrt{x}}$ ix $\sqrt{x}\,(x^3 - 1)$

Q.2 Differentiate w.r.t.x

i. $\sqrt{x^2+1}$ ii. $(x^2+3x+9)^{3/2}$ iii. $(ax^p+bx^q)^{p/q}$

iv. $\sqrt[3]{x^2+9x+8}$ v. $\frac{1}{\sqrt{x+b}}$ vi. $\frac{1}{\sqrt{a^2-x^2}}$

vii. $(x+x^{-1})^2$ viii. $(1+x+x^2)^3$ ix $(1-x^2)^{1/2}$

x $(2x^2+4x-5)^6$

Q.3 Find the derivative w.r.t.x

i. $(4x^3+7)(3x^2+2)$ ii. $(ax^2+b)(cx^2+d)$ iii. $x\sqrt{x+1}$

iv. $(x-1)^3(x+2)^4$ v. $(x+1)^2(x^2+1)^{-3}$ vi. $(a+x)\sqrt{a-x}$

vii $(3-x^2)(x^3-x+1)$ viii $(x-1)(x^2+x+1)$

ix $(x+3)(2x+3)(x^2+1)$

Q.4. Differentiate w.r.t.x.

i. $\frac{x}{x^2+1}$ ii. $\frac{x+1}{x^2+2x+2}$ iii. $\frac{x^2}{1+x^2}$

iv. $\sqrt{\frac{1+x}{1-x}}$ v. $\sqrt{\frac{a+x}{a-x}}$ vi. $x\sqrt{\frac{a+x}{a-x}}$

vii. $\frac{\sqrt{x}}{\sqrt{x+1}}$ viii. $\frac{x}{(a^2+x^2)^{3/2}}$ ix. $\left(\frac{x+1}{x-1}\right)^2$

x. $\frac{(x^2+1)^3}{(x^2+2)^2}$

Q.5 Find dy/dx at the given point if:

(i) $y=x^{2/3}$ at $x=8$ (ii) $y=8x-3x^2$ at $x=2$

(iii) $y=x^4-x^2+2$ at $x=-1$ (iv) $y=x+2x^{-1}$ at $x=2$

Answers 2.2

Q.1

(i) $49x^6$ (ii) $3-3x-21x^2$ (iii) $6x^2+8x-5$

(iv) $6x-\frac{1}{2}x^{-3/2}$ (v) $-\frac{6}{x^2}-\frac{8}{x^3}+\frac{9}{x^4}$ (vi) $\frac{1}{2}x^{3/2}+\frac{1}{2}x^{1/2}$

vii $2x+2/x^3$ (viii) $\frac{x-1}{2x^{3/2}}$ (ix) $\frac{7}{2}x^{5/2}-\frac{1}{2\sqrt{x}}$

Q.2

(i) $\frac{x}{\sqrt{x^2+1}}$ (ii) $\frac{3}{2}(x^2+3x+9)^{\frac{1}{2}}(3x+9)$

(iii) $\frac{p}{q}(ax^p+by^q)^{p/q-1}(ap.x^{p-1}+bqx^{q-1})$

(iv) $\frac{1}{3}(x^2+9x+8)^{-2/3}(2x+9)$

(v) $\frac{-1}{2(x+b)^{3/2}}$ (vi) $\frac{x}{(a^2-x^2)^{3/2}}$

(vii) $\frac{2(x^4-1)}{x^3}$ (viii) $3(1+2x)(1+x+x^2)^2$

(ix) $\frac{-x}{\sqrt{1-x^2}}$ (x) $24(x+1)(2x^2+4-5)^5$

Q.3

(i) $6x(10x^3+4x+7)$ (ii) $2x(2acx^2+bc+ad)$ (iii) $\frac{3x+2}{2\sqrt{x+1}}$

(iv) $(x-1)^2(x+2)^3(7x+2)$ (v) $\frac{-2(2x^3+5x^2+2x-1)}{(x^2+1)^4}$

(vi) $\frac{a-3x}{2\sqrt{a-x}}$ (vii) $-5x^4+12x^2-2x-3$ (viii) $3x^2$

(ix) $8x^3+27x^2+22x+9$

Q.4

(i) $\frac{1-x^2}{(1+x^2)^2}$ (ii) $\frac{-x(x+2)}{(x^2+2x+2)^2}$ (iii) $\frac{2x}{(1+x^2)^2}$

(iv) $\frac{1}{\sqrt{1+x}\,(1-x)^{3/2}}$ (v) $\frac{a}{(a-x)\sqrt{a^2-x^2}}$

(vi) $\frac{-x^2+ax+a^2}{(a+x)^{1/2}(a-x)^{3/2}}$ (vii) $\frac{1}{2\sqrt{x}(x+1)^{3/2}}$ (viii) $\frac{a^2-2x^2}{(a^2+x^2)^{5/2}}$

(ix) $\frac{-4(x+1)}{(x-1)^3}$ (x) $\frac{2x(x^2+1)^3(x^2+4)}{(x^2+2)^3}$

Q.5: (i) 1/3 (ii) - 4 (iii) - 2 (iv) 1/2