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Chapter: 14 (Ist Year)

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Solution of Trigonometric Equations

Trigonometric Equations:-

"The equations which contains the trigonometric functions are called Trigonometric Equations."

e.g. $\sin x + \cos x = 2$ $2\sec x + \tan x = 4/3$

Roots of Trigonometric Equations:-

The values of variable in Circular measure (Radian) which satisfy the Trigonometric Equation are called its roots.

* Trigonometric Equations have infinite roots due to periodicity.

* The process of finding roots of equation is called its Solution.

Extraneous Roots:-

The roots which obtained from Equation but Could not satisfy Equation are called Extraneous roots.

Criteria for Solution of Trigometric Equation:-

(i) Factorize the Equation Completely.

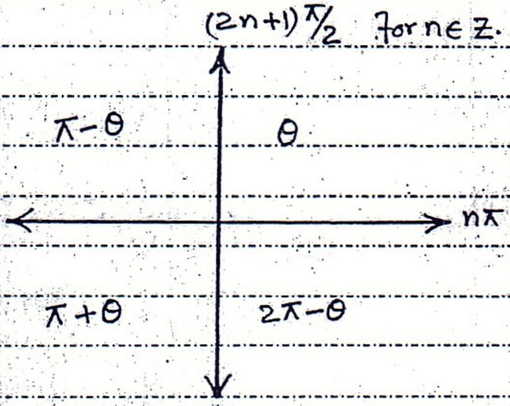
(ii) Find value of variable in the period of given function.

(iii) Check it for Sign according to Quadrent and find roots as: θ, $\pi-\theta$, $\pi+\theta$, $2\pi-\theta$ in Ist, 2nd, 3rd, 4th Quadrent.

(iv) Generalize Solution by adding periodic rotations $2n\pi$, $n\pi$
  $2n\pi$ for $\sin x$, $\cos x$, $\sec x$ and $\csc x$ where $n \in Z$.
  $n\pi$ for $\tan x$ and $\cot x$ where $n \in Z$ (Integer).

(v) Write the Solution as union of all roots set.

$f(\theta)$ \ θ	$0°$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	-1	0
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	-1	0	1
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0	$-\infty$	0
$\cot\theta$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0	$-\infty$	0	∞
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞	-1	$-\infty$	1
$\csc\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1	$-\infty$	-1	∞



Angles in each Quadrent.

Exercise: 14

Q.1 Find the solutions of the following Equations which lie in $[0, 2\pi]$

(i) $\sin x = -\frac{\sqrt{3}}{2}$

$\because \sin x$ is $-$ve in III and IV Quad.

and $\frac{\sqrt{3}}{2}$ is the value of $\frac{\pi}{3}$ of Sine

So

$x = \pi + \frac{\pi}{3}$ and $x = 2\pi - \pi/3$

$\Rightarrow x = \frac{4\pi}{3}$ and $x = \frac{5\pi}{3}$

(ii) $\operatorname{Cosec}\theta = 2$

$\Rightarrow \sin\theta = 1/2$ which is in I & II

and $\frac{1}{2}$ is the value of $\frac{\pi}{6}$ of Sine.

So

$\theta = \pi/6$ and $\theta = \pi - \pi/6$

$\Rightarrow \theta = \pi/6$ and $\theta = \frac{5\pi}{6}$

(iii) $\sec x = -2$

$\Rightarrow \cos x = \frac{-1}{2}$

$\because \cos x$ is $-$ve and in II and III

and $\frac{1}{2}$ is the value of $\frac{\pi}{3}$ of Cosine

So

$x = \pi - \frac{\pi}{3}$ and $x = \pi + \pi/3$

$\Rightarrow x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$

(iv) $\cot\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \sqrt{3}$

$\because \tan\theta$ is $+$ve in I and III and

$\sqrt{3}$ is the value of $\pi/3$ of Tangent.

So

$\theta = \pi/3$ and $\theta = \pi + \pi/3$

$\Rightarrow \theta = \pi/3$ and $\theta = \frac{4\pi}{3}$

Q.2. Solve the following Trigonometric Equations:

(i) $\tan^2\theta = 1/3$

$\Rightarrow \tan\theta = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow$

$\tan\theta = \frac{1}{\sqrt{3}}$ and $\tan\theta = \frac{-1}{\sqrt{3}}$

θ lies in I and III, θ lies in II, IV

and $\frac{1}{\sqrt{3}}$ is the value of $\pi/6$ of tangent.

So

$\theta = \pi/6$, $\theta = \pi + \pi/6$, $\theta = \pi - \pi/6$, $\theta = 2\pi - \pi/6$

$\theta = \pi/6$, $\theta = \frac{7\pi}{6}$, $\theta = \frac{5\pi}{6}$, $\theta = \frac{11\pi}{6}$

Thus Solutions are:

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

(ii) $\operatorname{Cosec}^2\theta = 4/3$

$\Rightarrow \sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow$

$\sin\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = -\frac{\sqrt{3}}{2}$

θ lies in I and II, θ lies in III and IV.

and $\frac{\sqrt{3}}{2}$ is the value of $\frac{\pi}{3}$ of Sine.

So

$\theta = \pi/3$, $\theta = \pi - \pi/3$, $\theta = \pi + \pi/3$, $\theta = 2\pi - \pi/3$

$\theta = \pi/3$, $\theta = \frac{2\pi}{3}$, $\theta = \frac{4\pi}{3}$, $\theta = \frac{5\pi}{3}$

Thus Solutions are:

$\pi/3, \frac{2\pi}{3}, 4\pi/3, 5\pi/3$

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Chapter (14) (Ist Year) ③

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(iii) $\sec^2\theta = 4/3$

$\Rightarrow \cos^2\theta = 3/4 \Rightarrow \cos\theta = \pm\sqrt{3}/2$

$\cos\theta = \frac{\sqrt{3}}{2}$ and $\cos\theta = -\frac{\sqrt{3}}{2}$

θ lies in I and IV ∧ θ lies in II and III

and $\frac{\sqrt{3}}{2}$ is the value $\pi/6$ of Cosine.

So

$\theta = \frac{\pi}{6}, \theta = 2\pi - \pi/6, \theta = \pi - \pi/6 \quad \theta = \pi + \pi/6$

So

$\theta = \pi/6, \theta = \frac{11\pi}{6}, \theta = \frac{5\pi}{6}, \theta = \frac{7\pi}{6}$

Thus solutions are:

$\pi/6, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(iv) $\cot^2\theta = \frac{1}{3}$

$\Rightarrow \tan^2\theta = 3 \Rightarrow \tan\theta = \pm\sqrt{3}$

$\tan\theta = \sqrt{3}$ and $\tan\theta = -\sqrt{3}$

θ lies in I and III ∧ θ lies in II and IV

So $\sqrt{3}$ is the value of $\pi/3$ of tangent.

So $\theta = \frac{\pi}{3}, \theta = \pi + \pi/3, \theta = \pi - \pi/3, \theta = 2\pi - \pi/3$

$\theta = \pi/3, \theta = \frac{4\pi}{3}, \theta = 2\pi/3, \theta = 5\pi/3$

Thus Solutions are:

$\pi/3, 2\pi/3, 4\pi/3, 5\pi/3$.

Find the Values of θ for Followings:

Q.3 $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$

$(\sqrt{3}\tan\theta)^2 + 2(\sqrt{3}\tan\theta)(1) + (1)^2 = 0$

$\Rightarrow (\sqrt{3}\tan\theta + 1)^2 = 0$

$\Rightarrow \sqrt{3}\tan\theta + 1 = 0$

$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$

Sol. is in 2nd and 4th Quadrent

and $\frac{1}{\sqrt{3}}$ is the value of $\frac{\pi}{6}$ of tangent.

So $\theta = \pi - \pi/6$ ∧ $\theta = 2\pi - \pi/6$

$\theta = 5\pi/6 \quad \theta = 11\pi/6$

Thus Solutions are:

$\frac{5\pi}{6}, 11\pi/6$

Q.4 $\tan^2\theta - \sec\theta - 1 = 0$

$(\sec^2\theta - 1) - (\sec\theta + 1) = 0$

$(\sec\theta - 1)(\sec\theta + 1) - (\sec\theta + 1) = 0$

$(\sec\theta + 1)[\sec\theta - 1 - 1] = 0$

$(\sec\theta + 1)(\sec\theta - 2) = 0$

$\Rightarrow \sec\theta + 1 = 0$ ∧ $\sec\theta - 2 = 0$

$\sec\theta = -1$ ∧ $\sec\theta = 2$

$\cos\theta = -1$ ∧ $\cos\theta = 1/2$

Consider $\cos\theta = -1$

$\Rightarrow \theta = \pi$

Now $\cos\theta = 1/2$

∵ $\cos\theta$ is +ve in I and IV and

$\frac{1}{2}$ is the value of $\pi/3$ of Cosine.

So $\theta = \pi/3$ and $\theta = 2\pi - \pi/3$

$= \frac{5\pi}{3}$

Thus Solutions are:

$\pi, \pi/3, 5\pi/3$.

Q.5 $2\sin\theta + \cos^2\theta - 1 = 0$

$2\sin\theta + (1 - \sin^2\theta) - 1 = 0$

$2\sin\theta - \sin^2\theta = 0$

$\Rightarrow \sin\theta(2 - \sin\theta) = 0$

$\sin\theta = 0$ or $2 - \sin\theta = 0$

Consider $\sin\theta = 0$

$\Rightarrow \theta$ lies on x-axis so

$\theta = 0, \pi$

Now $2 - \sin\theta = 0 \Rightarrow \sin\theta = 2 \notin [-1, 1]$

So neglecting it

Thus Solutions are:

0 and π.

Q.6 $2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0$ or $2\sin\theta - 1 = 0$

Consider $\sin\theta = 0$

$\Rightarrow \theta$ lies on x-axis

So $\theta = 0$ and π

Now $2\sin\theta - 1 = 0$

$\Rightarrow \sin\theta = 1/2$

$\because \sin\theta$ is +ve in I and II.

and $1/2$ is the value of $\pi/6$ of sine.

so $\theta = \pi/6$ and $\theta = \pi - \pi/6$

$\theta = \pi/6$, $\theta = 5\pi/6$

Thus Solutions are:

$0, \pi, \pi/6, 5\pi/6$.

Q.7 $3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

Dividing by $\cos^2\theta$

$3 - 2\sqrt{3}\tan\theta - 3\tan^2\theta = 0$

Multiplying by -1, we have

$3\tan^2\theta + 2\sqrt{3}\tan\theta - 3 = 0$

which is quadratic in $\tan\theta$.

$$\tan\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\tan\theta = \frac{-2\sqrt{3} \pm \sqrt{12 + 36}}{2(3)}$$

$$\tan\theta = \frac{-2\sqrt{3} \pm \sqrt{48}}{6}$$

$$\tan\theta = \frac{-2\sqrt{3} \pm 4\sqrt{3}}{2}$$

$$\tan\theta = \frac{-2\sqrt{3} + 4\sqrt{3}}{6}, \frac{-2\sqrt{3} - 4\sqrt{3}}{6}$$

$$\tan\theta = \frac{2\sqrt{3}}{6}, \frac{-6\sqrt{3}}{6}$$

$$\tan\theta = \frac{1}{\sqrt{3}} \text{ or } \tan\theta = -\sqrt{3}$$

Consider $\tan\theta = 1/\sqrt{3}$

$\because \tan\theta$ is +ve in I and III and $1/\sqrt{3}$ is the value of $\pi/6$ of tangent

So $\theta = \pi/6$ and $\theta = \pi + \pi/6$

$\Rightarrow \theta = \pi/6$, $\theta = 7\pi/6$

Now $\tan\theta = -\sqrt{3}$

$\because \tan\theta$ is −ve in II and IV and $\sqrt{3}$ is the value of $\pi/3$ of tangent.

So $\theta = \pi - \pi/3$, $\theta = 2\pi - \pi/3$

$\Rightarrow \theta = 2\pi/3$, $\theta = 5\pi/3$

Thus Solutions are:

$\pi/6, 7\pi/6, 2\pi/3, 5\pi/3$.

Q.8 $4\sin^2\theta - 8\cos\theta + 1 = 0$

$4(1 - \cos^2\theta) - 8\cos\theta + 1 = 0$

$4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$

$\Rightarrow -4\cos^2\theta - 8\cos\theta + 5 = 0$

Multiplying by -1, we have

$4\cos^2\theta + 8\cos\theta - 5 = 0$

$\Rightarrow 4\cos^2\theta + 10\cos\theta - 2\cos\theta - 5 = 0$

$\Rightarrow 2\cos\theta(2\cos\theta + 5) - 1(2\cos\theta + 5) = 0$

$\Rightarrow (2\cos\theta + 5)(2\cos\theta - 1) = 0$

$2\cos\theta - 1 = 0$ or $2\cos\theta + 5 = 0$

$\cos\theta = 1/2$ or $\cos\theta = -5/2 \notin [-1, 1]$

Now So neglecting.

$\cos\theta$ is +ve in I and IV and $1/2$ is the value of $\pi/3$ for Cosine

So $\theta = \pi/3$, $\theta = 2\pi - \pi/3$

$\theta = \pi/3$, $\theta = 5\pi/3$

Thus Solutions are:

$\pi/3$ and $5\pi/3$.

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Find the Solution Sets of the Followings:

Q.9/ $\sqrt{3}\tan x - \sec x - 1 = 0$

$\sqrt{3}\tan x = \sec x + 1$

Squaring both Sides:

$3\tan^2 x = \sec^2 x + 1 + 2\sec x$

$3(\sec^2 x - 1) - \sec^2 x - 2\sec x - 1 = 0$

$3\sec^2 x - 3 - \sec^2 x - 2\sec x - 1 = 0$

$2\sec^2 x - 2\sec x - 4 = 0$

$\sec^2 x - \sec x - 2 = 0 \quad \because 2 \neq 0$

$\sec^2 x - 2\sec x + \sec x - 2 = 0$

$\sec x(\sec x - 2) + 1(\sec x - 2) = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x - 2 = 0$ or $\sec x + 1 = 0$

$\sec x = 2$ or $\sec x = -1$

$\cos x = \frac{1}{2}$ or $\cos x = -1$

Consider $\cos x = -1$

$\Rightarrow x = \pi$

General value of x is $\{\pi + 2n\pi\}$

Now $\cos x = \frac{1}{2}$

$\because \cos x$ is +ve in I and IV and $\frac{1}{2}$ is the value of $\pi/3$ of Cosine

So $x = \pi/3$ or $x = 2\pi - \pi/3$

$\Rightarrow x = \pi/3$ or $x = 5\pi/3$

But $x = \frac{5\pi}{3}$ is Extraneous root

so General value of x is $\{\pi/3 + 2n\pi\}$

Thus Solution Set is:

$\{\pi + 2n\pi\} \cup \{\frac{\pi}{3} + 2n\pi\}$.

Q.10/ $\cos 2x = \sin 3x$

$1 - 2\sin^2 x = 3\sin x - 4\sin^3 x$

$4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$

$\sin x = 1$ is its one root.

Using Synthetic Division

1	4	−2	−3	1
		4	2	−1
	4	2	−1	0

$\Rightarrow \sin x = 1$ and $4\sin^2 x + 2\sin x - 1 = 0$

Consider $\sin x = 1$

$\Rightarrow x = \pi/2$

So general value of x is $\{\frac{\pi}{2} + 2n\pi\}$.

Now $4\sin^2 x + 2\sin x - 1 = 0$

$$\sin x = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{2(4)}$$

$$\sin x = \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-2 \pm 2\sqrt{5}}{8}$$

$$\sin x = \frac{-1 \pm \sqrt{5}}{4} = \frac{-1 \pm 2.24}{4}$$

$\sin x = \frac{-1+2.24}{4}$ or $\sin x = \frac{-1-2.24}{4}$

$\sin x = 0.31 \qquad \sin x = -0.81$

let $\sin x = 0.31$

$\because \sin x$ is +ve in I and II and 0.31 is the value of $\pi/10$ of sine

$x = \pi/10$ and $x = \pi - \pi/10 = \frac{9\pi}{10}$

So $x = \{\frac{\pi}{10} + 2n\pi\}$ or $x = \{\frac{9\pi}{10} + 2n\pi\}$.

Now $\sin x = -0.81$

$\because \sin x$ is −ve in III and IV and -0.81 is the value $\frac{3\pi}{10}$ of sine

So $x = \pi + \frac{3\pi}{10}$ or $x = 2\pi - \frac{3\pi}{10}$

$x = \frac{13\pi}{10}$ or $x = \frac{17\pi}{10}$

$\Rightarrow x = \{\frac{13\pi}{10} + 2n\pi\}$ or $x = \{\frac{17\pi}{10} + 2n\pi\}$

Sol. Set is $\{\frac{\pi}{2} + 2n\pi\} \cup \{\frac{\pi}{10} + 2n\pi\} \cup \{\frac{9\pi}{10} + 2n\pi\} \cup \{\frac{13\pi}{10} + 2n\pi\} \cup \{\frac{17\pi}{10} + 2n\pi\}$.

Q.11 $\sec 3\theta = \sec\theta$

$1/\cos 3\theta = 1/\cos\theta$

$\cos 3\theta = \cos\theta$

$4\cos^3\theta - 3\cos\theta = \cos\theta$

$4\cos^3\theta - 4\cos\theta = 0$

$4\cos\theta\{\cos^2\theta - 1\} = 0$

$\cos\theta = 0$ or $\cos^2\theta - 1 = 0$

Consider $\cos\theta = 0$

$\Rightarrow \theta = \{(2n+1)\pi/2\}$ for $n \in Z$

Now $\cos^2\theta = 1$

$\Rightarrow \cos\theta = \pm 1$

$\Rightarrow \theta = \{n\pi\}$ for $n \in Z$

Thus solution set is:

$\{n\pi\} \cup \{(2n+1)\pi/2\}$

$= \{n\pi\} \cup \{n\pi + \pi/2\}$

Q.12 $\tan 2\theta + \cot\theta = 0$

$\Rightarrow \dfrac{\sin 2\theta}{\cos 2\theta} + \dfrac{\cos\theta}{\sin\theta} = 0$

$\Rightarrow \dfrac{\sin 2\theta \sin\theta + \cos 2\theta\cos\theta}{\cos 2\theta \sin\theta} = 0$

$\Rightarrow \cos 2\theta\cos\theta + \sin 2\theta \sin\theta = 0$

$\Rightarrow \cos(2\theta - \theta) = 0$

$\because \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = \{(2n+1)\pi/2\}$ for $n \in Z$

Thus Solution Set is:

$\{(2n+1)\pi/2\} = \{n\pi + \pi/2\}$

Q.13 $\sin 2x + \sin x = 0$

$2\sin x\cos x + \sin x = 0$

$\sin x[2\cos x + 1] = 0$.

$\Rightarrow \sin x = 0$ or $2\cos x + 1 = 0$

Consider $\sin x = 0$

$\Rightarrow x = \{n\pi\}$ for $n \in Z$.

Now $2\cos x + 1 = 0$

$\Rightarrow \cos x = -1/2$

$\because \cos x$ is $-$ve in II and III and $1/2$ is the value of $\pi/3$ of Cosine so

$x = \pi - \pi/3$ or $x = \pi + \pi/3$

$= 2\pi/3$ $\quad x = 4\pi/3$

$\Rightarrow$

$x = \{\frac{2\pi}{3} + 2n\pi\}$ $\quad x = \{\frac{4\pi}{3} + 2n\pi\}$

Thus Solution Set is:

$\{n\pi\} \cup \{\frac{2\pi}{3} + 2n\pi\} \cup \{\frac{4\pi}{3} + 2n\pi\}$.

Q.14 $\sin 4x - \sin 2x = \cos 3x$

$2\cos\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = \cos 3x$

$\because \sin P - \sin Q = 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$.

$2\cos 3x\sin x - \cos 3x = 0$

$\Rightarrow \cos 3x[2\sin x - 1] = 0$

$\cos 3x = 0$ or $2\sin x - 1 = 0$

Consider $2\sin x - 1 = 0$

$\sin x = 1/2$

$\because \sin x$ is +ve in I and II and $1/2$ is the value of $\pi/6$ of sine so

$x = \pi/6$ and $x = \pi - \pi/6 = \frac{5\pi}{6}$

$\Rightarrow x = \{\pi/6 + 2n\pi\}$ $\quad x = \{\frac{5\pi}{6} + 2n\pi\}$

Now $\cos 3x = 0$

$\Rightarrow 3x = \pi/2$

$\Rightarrow 3x = \frac{\pi}{2} + 2n\pi$

$x = \left\{\frac{\pi}{6} + \frac{2n\pi}{3}\right\}$ for $n \in Z$

Thus Solutionset is:

$\{\frac{\pi}{6} + \frac{2n\pi}{3}\} \cup \{\pi/6 + 2n\pi\} \cup \{\frac{5\pi}{6} + 2n\pi\}$.

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Q.15 $\sin x + \cos 3x = \cos 5x$

$\sin x = \cos 5x - \cos 3x$

$\sin x = -2\sin\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)$

$\sin x = -2\sin 4x \sin x$

$\sin x + 2\sin 4x \sin x = 0$

$\sin x\,[1+2\sin 4x] = 0$

$\Rightarrow \sin x = 0$ or $1+2\sin 4x = 0$

Consider $\sin x = 0$

$\Rightarrow x = \{n\pi\}$ for $n \in Z$

Now $1+2\sin 4x = 0$

$\Rightarrow \sin 4x = -1/2$

Now $\sin x$ is -ve in III and IV and $1/2$ is the value of $\pi/6$ of sine

so $4x = \pi + \pi/6$ $\quad$ $4x = 2\pi - \pi/6$

$4x = \frac{7\pi}{6}$ $\quad$ $4x = \frac{11\pi}{6}$

$4x = \frac{7\pi}{6} + 2n\pi$ $\quad$ $4x = \frac{11\pi}{6} + 2n\pi$

$x = \left\{\frac{7\pi}{24} + \frac{n\pi}{2}\right\}$ $\quad$ $x = \left\{\frac{11\pi}{24} + \frac{n\pi}{2}\right\}$

The Solution set is:

$\{n\pi\} \cup \left\{\frac{7\pi}{24} + \frac{n\pi}{2}\right\} \cup \left\{\frac{11\pi}{24} + \frac{n\pi}{2}\right\}$

Q.16 $\sin 3x + \sin 2x + \sin x = 0$

$(\sin 3x + \sin x) + \sin 2x = 0$

$2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) + \sin 2x = 0$

$2\sin 2x \cos x + \sin 2x = 0$

$\sin 2x(2\cos x + 1) = 0$

$\Rightarrow \sin 2x = 0$ or $2\cos x + 1 = 0$

Now $2\cos x + 1 = 0 \Rightarrow \cos x = -1/2$

$\because \cos x$ is -ve in II and III and $\frac{1}{2}$ is the value of $\pi/3$ of Cosine so

$x = \pi - \pi/3$, $x = \pi + \pi/3$

$x = \frac{2\pi}{3}$ $\quad$ $x = \frac{4\pi}{3}$

$x = \left\{\frac{2\pi}{3} + 2n\pi\right\}$ $\quad$ $x = \left\{\frac{4\pi}{3} + 2n\pi\right\}$

Now $\sin 2x = 0$

$\Rightarrow 2x = n\pi$ for $n \in Z$

$x = \left\{\frac{n\pi}{2}\right\}$

Thus Solution set is:

$\left\{\frac{n\pi}{2}\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\}$.

Q.17 $\sin 7x - \sin x = \sin 3x$

$2\cos\left(\frac{7x+x}{2}\right)\sin\left(\frac{7x-x}{2}\right) = \sin 3x$

$2\cos 4x \sin 3x - \sin 3x = 0$

$\sin 3x\,[2\cos 4x - 1] = 0$

$\Rightarrow \sin 3x = 0$ or $2\cos 4x - 1 = 0$

Consider $\sin 3x = 0$

$\Rightarrow 3x = n\pi$ for $n \in Z$

$\Rightarrow x = \left\{\frac{n\pi}{3}\right\}$

Now $2\cos 4x - 1 = 0 \Rightarrow \cos 4x = 1/2$

$\because \cos x$ is +ve in I and IV and $1/2$ is the value of $\pi/3$ of Cosine so

$4x = \pi/3$ $\quad$ $4x = 2\pi - \pi/3 = \frac{5\pi}{3}$

$4x = \frac{\pi}{3} + 2n\pi$, $4x = \frac{5\pi}{3} + 2n\pi$

$x = \left\{\frac{\pi}{12} + \frac{n\pi}{2}\right\}$, $x = \left\{\frac{5\pi}{12} + \frac{n\pi}{2}\right\}$

Thus solution set is:

$\left\{\frac{n\pi}{3}\right\} \cup \left\{\frac{\pi}{12} + \frac{n\pi}{2}\right\} \cup \left\{\frac{5\pi}{12} + \frac{n\pi}{2}\right\}$

Q.18 $\sin x + \sin 3x + \sin 5x = 0$

$\Rightarrow (\sin 5x + \sin x) + \sin 3x = 0$

$\Rightarrow 2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right) + \sin 3x = 0$

$\Rightarrow 2\sin 3x \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x[2\cos 2x+1]=0$

$\Rightarrow \sin 3x=0$ or $2\cos 2x+1=0$

Consider $\sin 3x=0$

$\Rightarrow 3x=n\pi$ For $n\in Z$

$x=\left\{\frac{n\pi}{3}\right\}$

Now $2\cos 2x+1=0$

$\Rightarrow \cos 2x=-1/2$

$\because \cos x$ is $-$ve in II and III and

$1/2$ is the value of $\pi/3$ of Cosine so

$2x=\pi-\pi/3$, $2x=\pi+\pi/3$

$2x=\frac{2\pi}{3}+2n\pi$, $2x=\frac{4\pi}{3}+2n\pi$

$x=\left\{\frac{\pi}{3}+n\pi\right\}$, $x=\left\{\frac{2\pi}{3}+n\pi\right\}$

Thus Solution Set is:

$\left\{\frac{n\pi}{3}\right\}\cup\left\{\frac{\pi}{3}+n\pi\right\}\cup\left\{\frac{2\pi}{3}+n\pi\right\}$.

Q.19/ $\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta=0$

$(\sin 7\theta+\sin\theta)+(\sin 5\theta+\sin 3\theta)=0$

$2\sin\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)+2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)=0$

$2\sin 4\theta\cos 3\theta+2\sin 4\theta\cos\theta=0$

$2\sin 4\theta[\cos 3\theta+\cos\theta]=0$

$2\sin 4\theta\left[2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right]=0$

$4\sin 4\theta\cos 2\theta\cdot\cos\theta=0$

$\Rightarrow \sin 4\theta=0$ $\cos 2\theta=0$ $\cos\theta=0$

If $\sin 4\theta=0$

$\Rightarrow 4\theta=n\pi$ For $n\in Z$

$\theta=\left\{\frac{n\pi}{4}\right\}$

For $\cos 2\theta=0$

$\Rightarrow 2\theta=(2n+1)\frac{\pi}{2}$

$\theta=\left\{(2n+1)\frac{\pi}{4}\right\}$

$=\left\{\frac{n\pi}{2}+\frac{\pi}{4}\right\}$

For $\cos\theta=0$

$\Rightarrow \theta=(2n+1)\pi/2$

$\theta=\{n\pi+\pi/2\}$

Thus Solution Set is:

$\left\{\frac{n\pi}{4}\right\}\cup\left\{\frac{n\pi}{2}+\frac{\pi}{4}\right\}\cup\{n\pi+\pi/2\}$

Q.20 $\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta=0$

$(\cos 7\theta+\cos\theta)+(\cos 5\theta+\cos 3\theta)=0$

$2\cos\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)+2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)=0$

$2\cos 4\theta\cos 3\theta+2\cos 4\theta\cos\theta=0$

$2\cos 4\theta[\cos 3\theta+\cos\theta]=0$

$2\cos 4\theta\left[2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right]=0$

$2\cos 4\theta[2\cos 2\theta\cos\theta]=0$

$4\cos 4\theta\cos 2\theta\cos\theta=0$

$\Rightarrow \cos 4\theta=0$, $\cos 2\theta=0$, $\cos\theta=0$

Consider $\cos 4\theta=0$

$\Rightarrow 4\theta=(2n+1)\pi/2$

$\theta=\{(2n+1)\pi/8\}=\left\{\frac{n\pi}{4}+\frac{\pi}{8}\right\}$

For $\cos 2\theta=0$

$2\theta=(2n+1)\pi/2$

$\theta=\{(2n+1)\pi/4\}=\left\{\frac{n\pi}{2}+\pi/4\right\}$

For $\cos\theta=0$

$\theta=\{(2n+1)\pi/2\}$

$\theta=\{n\pi+\pi/2\}$

Thus Solution Set is:

$\left\{\frac{n\pi}{4}+\frac{\pi}{8}\right\}\cup\left\{\frac{n\pi}{2}+\pi/4\right\}\cup\{n\pi+\pi/2\}$

SOLVED EXAMPLES.

Example ① Solve $\sin x=1/2$

Sol:- Consider $\sin x=1/2$

$\because \sin x$ is +ve in I and II and

$1/2$ is the value of $\pi/6$ of Sine

so $x=\pi/6$ and $x=\pi-\pi/6=5\pi/6$

$x=\{\pi/6+2n\pi\}$ $x=\left\{\frac{5\pi}{6}+2n\pi\right\}$

Thus Solution Set is:

$\left\{\frac{\pi}{6}+2n\pi\right\}\cup\left\{\frac{5\pi}{6}+2n\pi\right\}$.

Example 2

Solve: $1+\cos x=0$

Sol:- $1+\cos x=0$

$\Rightarrow \cos x=-1$

$\Rightarrow x=\pi$

Solution Set is:

$\{\pi+2n\pi\}$

Imp

Example 3

Solve $4\cos^2 x-3=0$

Sol:- $4\cos^2 x-3=0$

$4\cos^2 x=3$

$\cos^2 x=3/4$

$\cos x=\pm\frac{\sqrt{3}}{2}$

$\Rightarrow$

$\cos x=\frac{\sqrt{3}}{2}$ and $\cos x=-\frac{\sqrt{3}}{2}$

Consider

$\cos x=\frac{\sqrt{3}}{2}$

$\because \cos x$ is +ve in I and IV and

$\sqrt{3}/2$ is the value of $\pi/6$ of Cosine

So

$x=\pi/6$ and $x=2\pi-\pi/6=\frac{11\pi}{6}$

$x=\{\frac{\pi}{6}+2n\pi\}$, $x=\{\frac{11\pi}{6}+2n\pi\}$

Now $\cos x=-\sqrt{3}/2$

$\because \cos x$ is -ve in II and III and

$\sqrt{3}/2$ is the value of $\pi/6$ for Cosine so

$x=\pi-\pi/6=5\pi/6$ $\quad x=\pi+\pi/6=7\pi/6$

$x=\{\frac{5\pi}{6}+2n\pi\}$, $x=\{\frac{7\pi}{6}+2n\pi\}$

Thus Solution Set is:

$\{\frac{\pi}{6}+2n\pi\}\cup\{\frac{5\pi}{6}+2n\pi\}\cup\{\frac{7\pi}{6}+2n\pi\}$

$\cup\{\frac{11\pi}{6}+2n\pi\}$

V.Imp

Example 4

Solve: $\sin x+\cos x=0$

Sol:- $\because \sin x+\cos x=0$

$\Rightarrow \sin x=-\cos x$

$\Rightarrow \frac{\sin x}{\cos x}=-1$

$\Rightarrow \tan x=-1$

$\because \tan x$ is -ve in II during period

and 1 is the value of $\pi/4$ for tangent

So

$x=\pi-\pi/4=\frac{3\pi}{4}$

Thus Solution Set is:

$\{\frac{3\pi}{4}+n\pi\}$

V.Imp

Example 5

Find the Solution Set: $\sin x\cos x=\frac{\sqrt{3}}{4}$

Sol:- $\sin x\cos x=\sqrt{3}/4$

$2\sin x\cos x=2\sqrt{3}/4$

$\sin 2x=\sqrt{3}/2$

$\because \sin x$ is +ve in I and II and $\sqrt{3}/2$ is the value of $\pi/3$ for Sine so

$2x=\pi/3$ $\quad 2x=\pi-\pi/3=\frac{2\pi}{3}$

$2x=\pi/3+2n\pi$ $\quad 2x=\frac{2\pi}{3}+2n\pi$

$x=\{\frac{\pi}{6}+n\pi\}$, $x=\{\frac{\pi}{3}+n\pi\}$

Thus Solution Set is:

$\{\frac{\pi}{6}+n\pi\}\cup\{\frac{\pi}{3}+n\pi\}$

Example 6

Solve: $\sin 2x = \cos x$

Sol:- $\sin 2x = \cos x$

$2\sin x\cos x - \cos x = 0$

$\cos x[2\sin x - 1] = 0$

$\cos x = 0 \quad \wedge \quad 2\sin x - 1 = 0$

$\cos x = 0$

$\Rightarrow x = \{(2n+1)\pi/2\}$ for $n \in Z$

$x = \{\frac{\pi}{2} + n\pi\}$

Now

$2\sin x - 1 = 0$

$\sin x = 1/2$

$\because \sin x$ is +ve in I and II and $1/2$ is the value of $\pi/6$ for sine

So

$x = \pi/6 \qquad x = \pi - \pi/6$

$x = \pi/6 \qquad x = 5\pi/6$

$x = \{\pi/6 + 2n\pi\}, \quad x = \{\frac{5\pi}{6} + 2n\pi\}$

Thus Solution Set is:

$\{n\pi + \pi/2\} \cup \{\frac{\pi}{6} + 2n\pi\} \cup \{\frac{5\pi}{6} + 2n\pi\}$.

v.Imp

Example 7

Solve $\sin^2 x + \cos x = 1$

Sol:- $\sin^2 x + \cos x = 1$

$(1 - \cos^2 x) + \cos x - 1 = 0$

$-\cos^2 x + \cos x = 0$

$\cos x(1 - \cos x) = 0$

For $\cos x = 0$

$x = \{(2n+1)\pi/2\}$ for $n \in Z$

$x = \{n\pi + \pi/2\}$

Now $1 - \cos x = 0$

$\cos x = 1$

$\Rightarrow x = 0$

So $x = \{0 + 2n\pi\} = \{2n\pi\}$

Thus Solution Set is:

$\{2n\pi\} \cup \{n\pi + \pi/2\}$

v.v.Imp.

Example 8

Solve: $\operatorname{cosec} x = \sqrt{3} + \cot x$

Sol:- $\operatorname{cosec} x = \sqrt{3} + \cot x$

$\Rightarrow \frac{1}{\sin x} = \sqrt{3} + \frac{\cos x}{\sin x}$

$\Rightarrow 1 = \sqrt{3}\sin x + \cos x$

$\Rightarrow 1 - \cos x = \sqrt{3}\sin x$

$\Rightarrow (1 - \cos x)^2 = (\sqrt{3}\sin x)^2$

$\Rightarrow 1 + \cos^2 x - 2\cos x = 3\sin^2 x$

$\Rightarrow 1 + \cos^2 x - 2\cos x = 3(1 - \cos^2 x)$

$\Rightarrow 1 + \cos^2 x - 2\cos x = 3 - 3\cos^2 x$

$\Rightarrow 1 + \cos^2 x - 2\cos x - 3 + 3\cos^2 x = 0$

$\Rightarrow 4\cos^2 x - 2\cos x - 2 = 0$

$\Rightarrow 2\cos^2 x - \cos x - 1 = 0 \qquad \because 2 \neq 0$

$\Rightarrow 2\cos^2 x - 2\cos x + \cos x - 1 = 0$

$\Rightarrow 2\cos x(\cos x - 1) + 1(\cos x - 1) = 0$

$\Rightarrow (\cos x - 1)(2\cos x + 1) = 0$

$\Rightarrow \cos x - 1 = 0$ or $2\cos x + 1 = 0$

Consider $\cos x - 1 = 0$

$\Rightarrow \cos x = 1$

$\Rightarrow x = 0$

But at $x = 0$ $\operatorname{cosec} x$ and $\cot x$ are undefined

Now $2\cos x + 1 = 0$

$\Rightarrow \cos x = -1/2$

$\because \cos x$ is -ve in II and III and $1/2$ is the value of $\pi/3$ so

$x = \pi - \pi/3 \qquad x = \pi + \pi/3$

$x = 2\pi/3 \qquad x = 4\pi/3$

But $x = 4\pi/3$ is extraneous root

So $x = \{\frac{2\pi}{3} + 2n\pi\}$

So Solution set is:

$\{\frac{2\pi}{3} + 2n\pi\}$.