viii) $\tan\left[\sin^{-1}\left(\frac{-1}{2}\right)\right]$

Let $y = \sin^{-1}\left(\frac{-1}{2}\right)$, $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin y = \frac{-1}{2}$

$y = -\frac{\pi}{6}$

Now

$\tan\left[\sin^{-1}\left(\frac{-1}{2}\right)\right] = \tan\left(-\frac{\pi}{6}\right)$

$\tan\left[\sin^{-1}\left(\frac{-1}{2}\right)\right] = -\tan\frac{\pi}{6}$

$\tan\left[\sin^{-1}\left(\frac{-1}{2}\right)\right] = -\frac{1}{\sqrt{3}}$

(ix) $\sin\left[\tan^{-1}(-1)\right]$

Let $y = \tan^{-1}(-1)$, $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan y = -1$

$y = -\frac{\pi}{4}$

Now

$\sin\left[\tan^{-1}(-1)\right] = \sin\left(-\frac{\pi}{4}\right)$

$\sin\left[\tan^{-1}(-1)\right] = -\sin\frac{\pi}{4}$

$\sin\left[\tan^{-1}(-1)\right] = -\frac{1}{\sqrt{2}}$

Prove the Formulae:-

(i) $\sin^{-1}A + \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$

(ii) $\sin^{-1}A - \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} - B\sqrt{1-A^2}\right)$

(iii) $\cos^{-1}A + \cos^{-1}B = \cos^{-1}\left(AB - \sqrt{(1-A^2)(1-B^2)}\right)$

(iv) $\cos^{-1}A - \cos^{-1}B = \cos^{-1}\left(AB + \sqrt{(1-A^2)(1-B^2)}\right)$

Proof:- Let $\alpha = \sin^{-1}A$ & $\beta = \sin^{-1}B$

$\sin\alpha = A$ $\quad$ $\sin\beta = B$

$\cos\alpha = \sqrt{1-\sin^2\alpha}$, $\cos\beta = \sqrt{1-\sin^2\beta}$

$= \sqrt{1-A^2}$, $= \sqrt{1-B^2}$

Now

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha+\beta) = A\sqrt{1-B^2} + B\sqrt{1-A^2}$

$\alpha+\beta = \sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$

Putting α, β, we have

$\boxed{\sin^{-1}A + \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)}$

Now

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = A\sqrt{1-B^2} - B\sqrt{1-A^2}$

$\alpha-\beta = \sin^{-1}\left(A\sqrt{1-B^2} - B\sqrt{1-A^2}\right)$

Putting α, β, we have

$\boxed{\sin^{-1}A - \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} - B\sqrt{1-A^2}\right)}$

Proof:- Let $\alpha = \cos^{-1}A$ & $\beta = \cos^{-1}B$

$\cos\alpha = A$ $\quad$ $\cos\beta = B$

$\sin\alpha = \sqrt{1-\cos^2\alpha}$, $\sin\beta = \sqrt{1-\cos^2\beta}$

$\sin\alpha = \sqrt{1-A^2}$, $\sin\beta = \sqrt{1-B^2}$

Now

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha+\beta) = AB - \sqrt{1-A^2}\cdot\sqrt{1-B^2}$

$\alpha+\beta = \cos^{-1}\left(AB - \sqrt{(1-A^2)(1-B^2)}\right)$

Putting α, β, we have

$\boxed{\cos^{-1}A + \cos^{-1}B = \cos^{-1}\left(AB - \sqrt{(1-A^2)(1-B^2)}\right)}$

Now

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = AB + \sqrt{1-A^2}\cdot\sqrt{1-B^2}$

$\alpha-\beta = \cos^{-1}\left(AB + \sqrt{(1-A^2)(1-B^2)}\right)$

Putting α, β, we have

$\boxed{\cos^{-1}A - \cos^{-1}B = \cos^{-1}\left(AB + \sqrt{(1-A^2)(1-B^2)}\right)}$

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(v) $\tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)$

(vi) $\tan^{-1}A-\tan^{-1}B=\tan^{-1}\left(\frac{A-B}{1-AB}\right)$

Proof:- Let $\alpha=\tan^{-1}A$, $\beta=\tan^{-1}B$

$\tan\alpha=A$, $\tan\beta=B$

Now

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha+\beta)=\frac{A+B}{1-AB}$$

$$\alpha+\beta=\tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

Putting α,β, we have

$$\boxed{\tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)}$$

Now

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{A-B}{1+AB}$$

$$\alpha-\beta=\tan^{-1}\left(\frac{A-B}{1+AB}\right)$$

Putting α,β, we have

$$\boxed{\tan^{-1}A-\tan^{-1}B=\tan^{-1}\left(\frac{A-B}{1+AB}\right)}$$

Put $A=B$ in (v)

$$\tan^{-1}A+\tan^{-1}A=\tan^{-1}\left(\frac{A+A}{1-A\cdot A}\right)$$

$$\boxed{2\tan^{-1}A=\tan^{-1}\left(\frac{2A}{1-A^2}\right)}$$

EXERCISE : 13.2

Show that:

Q.1 $\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{7}{25}\right)=\cos^{-1}\left(\frac{253}{325}\right)$

Let $\alpha=\sin^{-1}(5/13)$, $\beta=\sin^{-1}\left(\frac{7}{25}\right)$

$\sin\alpha=5/13$ $\quad\sin\beta=7/25$

$\cos\alpha=\sqrt{1-\sin^2\alpha}$, $\cos\beta=\sqrt{1-\sin^2\beta}$

$=\sqrt{1-\frac{25}{169}}$ $\quad=\sqrt{1-\frac{49}{625}}$

$=\sqrt{\frac{169-25}{169}}$ $\quad=\sqrt{\frac{625-49}{625}}$

$\cos\alpha=\sqrt{\frac{144}{169}}=\frac{12}{13}$, $\cos\beta=\sqrt{\frac{576}{625}}=\frac{24}{25}$

Consider

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\cos(\alpha+\beta)=\frac{12}{13}\times\frac{24}{25}-\frac{5}{13}\times\frac{7}{25}$$

$$=\frac{288}{325}-\frac{35}{325}=\frac{288-35}{325}$$

$$\alpha+\beta=\cos^{-1}\left(\frac{253}{325}\right)$$

Putting α and β

$$\sin^{-1}(5/13)+\sin^{-1}(7/25)=\cos^{-1}\left(\frac{253}{325}\right)$$

(Proved)

Q.2 $\tan^{-1}(1/4)+\tan^{-1}(1/5)=\tan^{-1}\left(\frac{9}{19}\right)$

LHS $=\tan^{-1}(1/4)+\tan^{-1}(1/5)$

$=\tan^{-1}\left(\frac{1/4+1/5}{1-1/4\cdot1/5}\right)$ $\quad\because \tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)$

$=\tan^{-1}\left(\frac{5+4/20}{20-1/20}\right)$

$=\tan^{-1}\left(\frac{9}{19}\right)=$ RHS

So $\tan^{-1}(1/4)+\tan^{-1}(1/5)=\tan^{-1}\left(\frac{9}{19}\right)$ (Proved)

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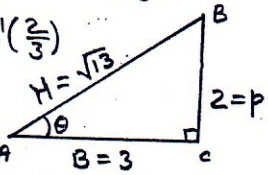
Q.3 $2\tan^{-1}\left(\frac{2}{3}\right) = \sin^{-1}\left(\frac{12}{13}\right)$

Let $\theta = \tan^{-1}\left(\frac{2}{3}\right)$

$\tan\theta = \frac{2}{3}$



$\because P = 2,\ B = 3,\ H = ?$

$H^2 = P^2 + B^2 = 4 + 9 = 13$

$H = \sqrt{13}$

$\sin\theta = \frac{2}{\sqrt{13}},\quad \cos\theta = \frac{3}{\sqrt{13}}$

Now

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = 2 \times \frac{2}{\sqrt{13}} \times \frac{3}{\sqrt{13}} = \frac{12}{13}$

$2\theta = \sin^{-1}\left(\frac{12}{13}\right)$

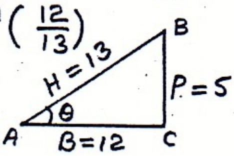
Putting θ, we have

$2\tan^{-1}\left(\frac{2}{3}\right) = \sin^{-1}\left(\frac{12}{13}\right)$. *(Proved)*

Q.4 $\tan^{-1}\left(\frac{120}{119}\right) = 2\cos^{-1}\left(\frac{12}{13}\right)$

Let $\theta = \cos^{-1}\left(\frac{12}{13}\right)$

$\cos\theta = \frac{12}{13}$



$P = ?\quad B = 12,\ H = 13$

$H^2 = P^2 + B^2 \Rightarrow P^2 = H^2 - B^2$

$P^2 = 169 - 144 = 25 \Rightarrow P = 5$

$\tan\theta = \frac{5}{12}$

Now

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

$\tan 2\theta = \frac{2(5/12)}{1 - \frac{25}{144}} = \frac{5/6}{119/144}$

$2\theta = \tan^{-1}\left(\frac{5}{6} \times \frac{144}{119}\right) = \tan^{-1}\left(\frac{120}{119}\right)$

$\Rightarrow 2\cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{120}{119}\right)$ *(Proved)*

Q.5 $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + \cot^{-1}(3) = \pi/4$

Let $\alpha = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ $\quad \beta = \cot^{-1}(3)$

$\sin\alpha = 1/\sqrt{5}\quad , \quad \cot\beta = 3$

$\cos\alpha = \sqrt{1 - \sin^2\alpha}\quad , \quad \tan\beta = \frac{1}{3}$

$= \sqrt{1 - 1/5}$

$\cos\alpha = \sqrt{4/5} = \frac{2}{\sqrt{5}}$

So $\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{1/\sqrt{5}}{2/\sqrt{5}} = \frac{1}{2}$

Using

$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\cdot\frac{1}{3}} = \frac{\frac{3+2}{6}}{\frac{6-1}{6}}$

$\alpha + \beta = \tan^{-1}\left(\frac{5}{5}\right) = \tan^{-1}(1) = \pi/4$

Putting α, β

$\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + \cot^{-1}(3) = \frac{\pi}{4}$ *(Proved)*

Q.6 $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\left(\frac{77}{85}\right)$

$\text{LHS} = \sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right)$

$= \sin^{-1}\left(\frac{3}{5}\sqrt{1 - \frac{64}{289}} + \frac{8}{17}\sqrt{1 - \frac{9}{25}}\right)$

$\because \sin^{-1}A + \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$

$= \sin^{-1}\left(\frac{3}{5}\sqrt{\frac{289-64}{289}} + \frac{8}{17}\sqrt{\frac{25-9}{25}}\right)$

$= \sin^{-1}\left(\frac{3}{5}\sqrt{\frac{225}{289}} + \frac{8}{17}\sqrt{\frac{16}{25}}\right)$

$= \sin^{-1}\left(\frac{3}{5}\cdot\frac{15}{17} + \frac{8}{17}\cdot\frac{4}{5}\right)$

$= \sin^{-1}\left(\frac{45}{85} + \frac{32}{85}\right) = \sin^{-1}\left(\frac{77}{85}\right) = \text{RHS}.$

So

$\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\left(\frac{77}{85}\right)$ *(Proved)*

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Q.7 $\sin^{-1}\left(\frac{77}{85}\right) - \sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1}\left(\frac{15}{17}\right)$

Let $\alpha = \sin^{-1}\left(\frac{77}{85}\right)$, $\beta = \sin^{-1}\left(\frac{3}{5}\right)$

$\sin\alpha = \frac{77}{85}$ $\sin\beta = \frac{3}{5}$

$\cos\alpha = \sqrt{1-\sin^2\alpha}$, $\cos\beta = \sqrt{1-\sin^2\beta}$

$= \sqrt{1-\frac{5929}{7225}}$ $= \sqrt{1-\frac{9}{25}}$

$= \sqrt{\frac{1296}{7225}}$ $= \sqrt{\frac{16}{25}}$

$\cos\alpha = \frac{36}{85}$, $\cos\beta = \frac{4}{5}$

Using

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \frac{36}{85}\times\frac{4}{5} + \frac{77}{85}\times\frac{3}{5}$

$\cos(\alpha-\beta) = \frac{144+231}{425} = \frac{375}{425} = \frac{15}{17}$

$\alpha-\beta = \cos^{-1}\left(\frac{15}{17}\right)$

Putting α, β, we have:

$\sin^{-1}\left(\frac{77}{85}\right) - \sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1}\left(\frac{15}{17}\right)$

(Proved)

Q.8 $\cos^{-1}\left(\frac{63}{65}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) = \sin^{-1}\left(\frac{3}{5}\right)$.

Istly $2\tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{2(1/5)}{1-1/25}\right)$

$\because 2\tan^{-1}A = \tan^{-1}\left(\frac{2A}{1-A^2}\right)$

so $2\tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{2/5}{24/25}\right)$

$2\tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{5}{12}\right)$

Now let

$\alpha = \cos^{-1}\left(\frac{63}{65}\right)$ $\beta = \tan^{-1}\left(\frac{5}{12}\right)$

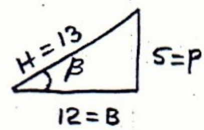
$\cos\alpha = \frac{63}{65}$ $\tan\beta = \frac{5}{12}$

$\sin\alpha = \sqrt{1-\cos^2\alpha}$

$= \sqrt{1-\frac{3969}{4225}}$

$= \sqrt{\frac{256}{4225}}$

$\sin\alpha = \frac{16}{65}$



$\sin\beta = \frac{5}{13}$

$\cos\beta = \frac{12}{13}$

Using $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha+\beta) = \frac{16}{65}\times\frac{12}{13} + \frac{63}{65}\times\frac{5}{13}$

$\sin(\alpha+\beta) = \frac{192+315}{845} = \frac{507}{845} = \frac{3}{5}$

Dividing N and D by 169

$\alpha+\beta = \sin^{-1}\left(\frac{3}{5}\right)$

Putting α, β, we have

$\cos^{-1}\left(\frac{63}{65}\right) + \tan^{-1}\left(\frac{5}{12}\right) = \sin^{-1}\left(\frac{3}{5}\right)$

(Proved)

Q.9 $\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{8}{19}\right) = \frac{\pi}{4}$

$LHS = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{8}{19}\right)$

$= \tan^{-1}\left[\frac{3/4 + 3/5}{1-\frac{3}{4}\cdot\frac{3}{5}}\right] - \tan^{-1}\left(\frac{8}{19}\right)$

$= \tan^{-1}\left(\frac{\frac{15+12}{20}}{\frac{20-9}{20}}\right) - \tan^{-1}\left(\frac{8}{19}\right)$

$= \tan^{-1}\left(\frac{27}{11}\right) - \tan^{-1}\left(\frac{8}{19}\right)$

$= \tan^{-1}\left[\frac{\frac{27}{11}-\frac{8}{19}}{1+\frac{27}{11}\cdot\frac{8}{19}}\right] = \tan^{-1}\left[\frac{\frac{513-88}{209}}{\frac{209+216}{209}}\right]$

$= \tan^{-1}\left[\frac{425}{425}\right] = \tan^{-1}(1) = \frac{\pi}{4} = RHS$

so

$\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{8}{19}\right) = \frac{\pi}{4}$

(Proved)

Q.10 $\sin^{-1}(4/5)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$

$$L.H.S = \sin^{-1}(4/5)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)$$
$$= \sin^{-1}\left[4/5\sqrt{1-\frac{25}{169}}+\frac{5}{13}\sqrt{1-\frac{16}{25}}\right]+\sin^{-1}\left(\frac{16}{65}\right)$$
$$= \sin^{-1}\left(\frac{4}{5}\sqrt{\frac{144}{169}}+\frac{5}{13}\sqrt{9/25}\right)+\sin^{-1}\left(\frac{16}{65}\right)$$
$$= \sin^{-1}\left(\frac{4}{5}\times\frac{12}{13}+\frac{5}{13}\times\frac{3}{5}\right)+\sin^{-1}\left(\frac{16}{65}\right)$$
$$= \sin^{-1}\left(\frac{48+15}{65}\right)+\sin^{-1}\left(\frac{16}{65}\right)$$
$$= \sin^{-1}\left(\frac{63}{65}\right)+\sin^{-1}\left(\frac{16}{65}\right)$$
$$= \sin^{-1}\left[\frac{63}{65}\sqrt{1-\frac{256}{4225}}+\frac{16}{65}\sqrt{1-\frac{3969}{4225}}\right]$$
$$= \sin^{-1}\left[\frac{63}{65}\sqrt{\frac{3969}{4225}}+\frac{16}{65}\sqrt{\frac{256}{4225}}\right]$$
$$= \sin^{-1}\left[\frac{63}{65}\cdot\frac{63}{65}+\frac{16}{65}\cdot\frac{16}{65}\right]$$
$$= \sin^{-1}\left[\frac{3969+256}{4225}\right]$$
$$= \sin^{-1}\left(\frac{4225}{4225}\right)=\sin^{-1}(1)$$
$$= \frac{\pi}{2} = RHS.$$

So
$\sin^{-1}(4/5)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$ (Proved)

Q.11 $\tan^{-1}\left(\frac{1}{11}\right)+\tan^{-1}\left(\frac{5}{6}\right)=\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)$

$$L.H.S = \tan^{-1}\left(\frac{1}{11}\right)+\tan^{-1}\left(\frac{5}{6}\right)$$
$$= \tan^{-1}\left[\frac{1/11+5/6}{1-1/11\cdot 5/6}\right]$$
$$= \tan^{-1}\left[\frac{6+55/66}{66-5/66}\right]$$
$$= \tan^{-1}\left(\frac{61}{61}\right)$$
$$= \tan^{-1}(1)$$
$$= \frac{\pi}{4}$$

$$RHS = \tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)$$
$$= \tan^{-1}\left[\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}\right]$$
$$= \tan^{-1}\left[\frac{\frac{3+2}{6}}{\frac{6-1}{6}}\right]$$
$$= \tan^{-1}\left[\frac{5}{5}\right]=\tan^{-1}(1)$$
$$= \frac{\pi}{4}$$

Thus LHS = RHS

So $\tan^{-1}\left(\frac{1}{11}\right)+\tan^{-1}\left(\frac{5}{6}\right)=\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)$

Q.12 $2\tan^{-1}(1/3)+\tan^{-1}\left(\frac{1}{7}\right)=\frac{\pi}{4}$

$$L.H.S = 2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)$$
$$\because 2\tan^{-1}A=\tan^{-1}\left(\frac{2A}{1-A^2}\right)$$
$$= \tan^{-1}\left[\frac{2(1/3)}{1-1/9}\right]+\tan^{-1}\left(\frac{1}{7}\right)$$
$$= \tan^{-1}\left(\frac{2/3}{8/9}\right)+\tan^{-1}(1/7)$$
$$= \tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}(1/7)$$
$$= \tan^{-1}\left[\frac{3/4+1/7}{1-3/4\cdot 1/7}\right]$$
$$= \tan^{-1}\left[\frac{21+4}{28}\Big/\frac{28-3}{28}\right]$$
$$= \tan^{-1}\left[\frac{25/28}{25/28}\right]$$
$$= \tan^{-1}(1)=\frac{\pi}{4}=RHS.$$

Thus

$$2\tan^{-1}(1/3)+\tan^{-1}(1/7)=\frac{\pi}{4}$$

"Victory belongs to that who Dares!"

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Q.13 Show that $\cos[\sin^{-1}x] = \sqrt{1-x^2}$

Let $\theta = \sin^{-1}x$

$\Rightarrow \sin\theta = x$

Now

$\cos[\sin^{-1}x] = \cos\theta$

$\cos[\sin^{-1}x] = \sqrt{1-\sin^2\theta}$

$\cos[\sin^{-1}x] = \sqrt{1-x^2}$ (Proved)

Q.14 Show that $\sin[2\cos^{-1}x] = 2x\sqrt{1-x^2}$

Let $\theta = \cos^{-1}x$

$\cos\theta = x$

Now

$\sin[2\cos^{-1}x] = \sin 2\theta$

$\sin[2\cos^{-1}x] = 2\sin\theta\cos\theta$

$\sin[2\cos^{-1}x] = 2\cos\theta\sqrt{1-\cos^2\theta}$

$\sin[2\cos^{-1}x] = 2x\sqrt{1-x^2}$ (Proved)

$\because \sin 2\theta = 2\sin\theta\cos\theta$

Q.15 Show that $\cos[2\sin^{-1}x] = 1-2x^2$

Let $\theta = \sin^{-1}x$

$\sin\theta = x$

Now

$\cos[2\sin^{-1}x] = \cos 2\theta$

$\cos[2\sin^{-1}x] = 1-2\sin^2\theta$

$\cos[2\sin^{-1}x] = 1-2x^2$

$\because \cos 2\theta = 1-2\sin^2\theta$

Q.16 Show that $\sin^{-1}(-x) = -\sin^{-1}x$.

Let $\theta = \sin^{-1}(-x)$

$\Rightarrow \sin\theta = -x$

$\Rightarrow -\sin\theta = x$

$\Rightarrow \sin(-\theta) = x$

$\Rightarrow -\theta = \sin^{-1}x$

$\Rightarrow \theta = -\sin^{-1}x$

$\Rightarrow \sin^{-1}(-x) = -\sin^{-1}x$ (Proved)

$\because \sin(-\theta) = -\sin\theta$

Q.17 Show that $\tan^{-1}(-x) = -\tan^{-1}x$.

Let $\theta = \tan^{-1}(-x)$

$\Rightarrow \tan\theta = -x$

$\Rightarrow -\tan\theta = x$

$\Rightarrow \tan(-\theta) = x$

$\Rightarrow -\theta = \tan^{-1}x$

$\Rightarrow \theta = -\tan^{-1}x$

$\Rightarrow \tan^{-1}(-x) = -\tan^{-1}x$ (Proved)

$\because \tan(-\theta) = -\tan\theta$

Q.18 Show that $\cos^{-1}(-x) = \pi - \cos^{-1}x$

Let $\theta = \cos^{-1}(-x)$

$\Rightarrow \cos\theta = -x$

$\Rightarrow -\cos\theta = x$

$\Rightarrow \cos(\pi-\theta) = x$

$\Rightarrow \pi-\theta = \cos^{-1}x$

$\Rightarrow \theta = \pi - \cos^{-1}x$

$\Rightarrow \cos^{-1}(-x) = \pi - \cos^{-1}x$ (Proved)

$\because \cos(\pi-\theta) = -\cos\theta$

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Q.19 Show that $\tan[\sin^{-1}x] = \dfrac{x}{\sqrt{1-x^2}}$

Let $\theta = \sin^{-1}x$

$\sin\theta = x$

Now

$\tan[\sin^{-1}x] = \tan\theta$

$\tan[\sin^{-1}x] = \dfrac{\sin\theta}{\cos\theta}$

$\tan[\sin^{-1}x] = \dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$

$\tan[\sin^{-1}x] = \dfrac{x}{\sqrt{1-x^2}}$ (Proved)

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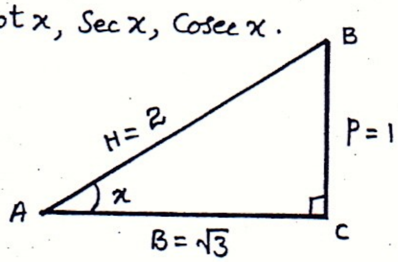
Mathematics: CH # 13

Q.20 Given that $x = \sin^{-1}\left(\frac{1}{2}\right)$. Find the values of the following functions: $\sin x$, $\cos x$, $\tan x$, $\cot x$, $\sec x$, $\operatorname{cosec} x$.



Sol:- $\because\ x = \sin^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \boxed{\sin x = \frac{1}{2}}$

$\because\ \sin x = \frac{P}{H}$

So $P = 1 \qquad H = 2 \qquad B = ?$

$\because\ H^2 = P^2 + B^2 \Rightarrow B^2 = H^2 - P^2 = (2)^2 - (1)^2 = 4 - 1 = 3$

$B = \sqrt{3}$

Now

$\sin x = \frac{P}{H} = \frac{1}{2}$ $\qquad \operatorname{cosec} x = \frac{H}{P} = 2$

$\cos x = \frac{B}{H} = \frac{\sqrt{3}}{2}$ $\qquad \sec x = \frac{H}{B} = \frac{2}{\sqrt{3}}$

$\tan x = \frac{P}{B} = \frac{1}{\sqrt{3}}$ $\qquad \cot x = \frac{B}{P} = \sqrt{3}$

SUMMARY

i) $\sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$ $\qquad$ ii) $\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

iii) $\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}x$ $\qquad$ iv) $\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$

v) $\sec^{-1}x = \frac{\pi}{2} - \operatorname{cosec}^{-1}x$ $\qquad$ vi) $\operatorname{cosec}^{-1}x = \frac{\pi}{2} - \sec^{-1}x$

vii) $\sin^{-1}A + \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$

viii) $\sin^{-1}A - \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} - B\sqrt{1-A^2}\right)$

ix) $\cos^{-1}A + \cos^{-1}B = \cos^{-1}\left(AB - \sqrt{(1-A^2)(1-B^2)}\right)$

x) $\cos^{-1}A - \cos^{-1}B = \cos^{-1}\left(AB + \sqrt{(1-A^2)(1-B^2)}\right)$

xi) $\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$ $\qquad$ xii) $\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$

xiii) $2\tan^{-1}A = \tan^{-1}\left(\frac{2A}{1-A^2}\right)$

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