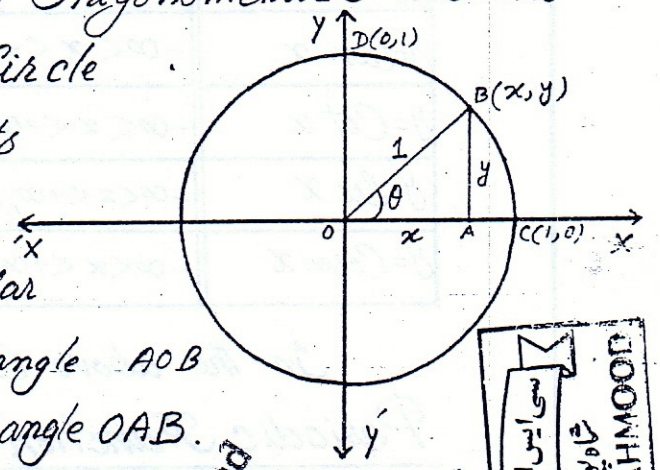
Trigonometric Functions and their Graphs

Trigonometric Functions:-

"The functions relating to the right angle triangle are called Trigonometric Functions."

These functions are $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\operatorname{cosec}\theta$, $\sec\theta$. The $\sin\theta$ and $\cos\theta$ are called basic or fundamental trigonometric functions while remaining four are called Derived Trigonometric Functions.

Consider a unit Circle having a Point $B(x,y)$ on its Circumference. Joint $B(x,y)$ by Origion and Draw perpendicular from B to $\overrightarrow{OX}$ at A. The angle AOB is θ. From the right triangle OAB.



$$\sin\theta = \frac{y}{1} \Rightarrow y = \sin\theta$$

$$\cos\theta = \frac{x}{1} \Rightarrow x = \cos\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \tan\theta = \frac{y}{x}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \Rightarrow \cot\theta = \frac{x}{y}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \Rightarrow \operatorname{cosec}\theta = \frac{1}{y}$$

$$\sec\theta = \frac{1}{\cos\theta} \Rightarrow \sec\theta = \frac{1}{x}$$

The above relations show that $\sin\theta$ and $\cos\theta$ has domain all the real numbers $\mathbb{R}$ but others depend upon their denominator if Denominator approaches to

Zero, then the function has not definate value (Range).

Domain and Range of Trigonometric Functions:-

The Following table represents the domain and range of Trigonometric Functions:

Functions	Domain	Range
$y=\sin x$	$-\infty < x < +\infty$	$-1 \leq y \leq 1$
$y=\cos x$	$-\infty < x < +\infty$	$-1 \leq y \leq 1$
$y=\tan x$	$-\infty < x < +\infty \wedge x \neq (2n+1)\frac{\pi}{2}$	$-\infty < y < \infty$
$y=\cot x$	$-\infty < x < +\infty \wedge x \neq n\pi$	$-\infty < y < \infty$
$y=\sec x$	$-\infty < x < +\infty \wedge x \neq (2n+1)\frac{\pi}{2}$	$y \geq 1 \vee y \leq -1$
$y=\operatorname{Cosec} x$	$-\infty < x < +\infty \wedge x \neq n\pi$	$y \geq 1 \vee y \leq -1$

In the above table n is an integer ie $n \in Z$

Periodic Function:-

"The Function which repeats its values for some unique domain is called Periodic Function." i.e. $f(x+p) = f(x)$ (regular) then "p" is period of $f(x)$.

For example every trigonometric Function repeats its value either increases or decreases 2π

So every trigonometric Function is periodic Function having Period 2π

Tangent and Cotangent ($\tan\theta$ and $\cot\theta$) are also periodic Functions of Period π.

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Find the Periods of the followings :

Q.1 $\sin 3x$

∵ Period of $\sin x$ is 2π so

$\sin 3x = \sin(3x + 2\pi)$

$\sin 3x = \sin 3\left(x + \frac{2\pi}{3}\right)$

Hence period of $\sin 3x$ is $\frac{2\pi}{3}$

Q.2 $\cos 2x$

∵ Period of $\cos x$ is 2π so

$\cos 2x = \cos(2x + 2\pi)$

$\cos 2x = \cos 2(x + \pi)$

Hence period of $\cos 2x$ is π

Q.3 $\tan 4x$

∵ Period of $\tan x$ is π so

$\tan 4x = \tan(4x + \pi)$

$\tan 4x = \tan 4(x + \pi/4)$

Hence period of $\tan 4x$ is $\pi/4$

Q.4 $\cot x/2$

∵ Period of $\cot x$ is π so

$\cot \frac{x}{2} = \cot\left(\frac{x}{2} + \pi\right)$

$\cot \frac{x}{2} = \cot \frac{1}{2}(x + 2\pi)$

Hence period of $\cot \frac{x}{2}$ is 2π

Q.5 $\sin \frac{x}{3}$

∵ Period of $\sin x$ is 2π so

$\sin \frac{x}{3} = \sin\left(\frac{x}{3} + 2\pi\right)$

$\sin \frac{x}{3} = \sin \frac{1}{3}(x + 6\pi)$

Hence period of $\sin \frac{x}{3}$ is 6π

Q.6 $\operatorname{Cosec} x/4$

∵ Period of $\operatorname{Cosec} x$ is 2π so

$\operatorname{Cosec} x/4 = \operatorname{Cosec}\left(\frac{x}{4} + 2\pi\right)$

$\operatorname{Cosec} x/4 = \operatorname{Cosec} \frac{1}{4}(x + 8\pi)$

Hence period of $\operatorname{Cosec} x/4$ is 8π

Q.7 $\sin \frac{x}{5}$

∵ Period of $\sin x$ is 2π so

$\sin \frac{x}{5} = \sin\left(\frac{x}{5} + 2\pi\right)$

$\sin \frac{x}{5} = \sin \frac{1}{5}(x + 10\pi)$

Hence period of $\sin x/5$ is 10π

Q.8 $\cos \frac{x}{6}$

∵ Period of $\cos x$ is 2π so

$\cos \frac{x}{6} = \cos\left(\frac{x}{6} + 2\pi\right)$

$\cos \frac{x}{6} = \cos \frac{1}{6}(x + 12\pi)$

Hence period of $\cos \frac{x}{6}$ is 12π

Q.9 $\tan\frac{x}{7}$

∵ Period of $\tan x$ is π so

$\tan\frac{x}{7} = \tan\left(\frac{x}{7}+\pi\right)$

$\tan\frac{x}{7} = \tan\frac{1}{7}(x+7\pi)$ ∵

Hence period of $\tan\frac{x}{7}$ is 7π

Q.10 $\cot 8x$

∵ Period of $\cot x$ is π so

$\cot 8x = \cot(8x+\pi)$

$\cot 8x = \cot 8\left(x+\frac{\pi}{8}\right)$

Hence period of $\cot 8x$ is $\frac{\pi}{8}$

Q.11 $\sec 9x$

∵ Period of $\sec x$ is 2π so

$\sec 9x = \sec(9x+2\pi)$

$\sec 9x = \sec 9\left(x+\frac{2\pi}{9}\right)$ ∵

Hence period of $\sec 9x$ is $\frac{2\pi}{9}$

Q.12 $\operatorname{Cosec} 10x$

∵ Period of $\operatorname{Cosec} x$ is 2π so

$\operatorname{Cosec} 10x = \operatorname{Cosec}(10x+2\pi)$

$\operatorname{Cosec} 10x = \operatorname{Cosec} 10\left(x+\frac{\pi}{5}\right)$

Hence period of $\operatorname{Cosec} 10x$ is $\frac{\pi}{5}$

Q.13 $3\sin x$

∵ Period of $\sin x$ is 2π so

$3\sin x = 3\sin(x+2\pi)$

Hence period of $3\sin x$ is 2π

Q.14 $2\cos x$

∵ Period of $\cos x$ is 2π so

$2\cos x = 2\cos(x+2\pi)$

Hence period of $2\cos x$ is 2π

Q.15 $3\cos x/5$

∵ Period of $\cos x$ is 2π so

$3\cos x/5 = 3\cos\left(\frac{x}{5}+2\pi\right)$

$3\cos x/5 = 3\cos\frac{1}{5}(x+10\pi)$

Hence period of $3\cos x/5$ is 10π
