(iii) $\sin\theta - \cos\theta$

let $r\cos\phi = 1 \quad r\sin\phi = -1$

$r^2\cos^2\phi + r^2\sin^2\phi = 1+1$

$r^2(\cos^2\phi + \sin^2\phi) = 2$

$r^2 = 2 \Rightarrow r = \sqrt{2}$

$\frac{r\sin\phi}{r\cos\phi} = \frac{-1}{1}$

$\tan\phi = -1 \Rightarrow \phi = \tan^{-1}(-1)$

$\phi = 135^\circ$

Thus $\sin\theta - \cos\theta = r\sin\theta\cos\phi + r\cos\theta\sin\phi$

$= r(\sin\theta\cos\phi + \cos\theta\sin\phi)$

$= r\sin(\theta+\phi)$

where $r = \sqrt{2}$ and $\phi = 135^\circ$

(iv) $5\sin\theta - 4\cos\theta$

let $r\cos\phi = 5 \quad r\sin\phi = -4$

$r^2\cos^2\phi + r^2\sin^2\phi = 25+16$

$r^2(\cos^2\phi + \sin^2\phi) = 41$

$r^2 = 41 \Rightarrow r = \sqrt{41}$

Now $\frac{r\sin\phi}{r\cos\phi} = \frac{-4}{5}$

$\tan\phi = -4/5 \Rightarrow \phi = \tan^{-1}(-4/5)$

Thus $5\sin\theta - 4\cos\theta = r\sin\theta\cos\phi + r\cos\theta\sin\phi$

$= r(\sin\theta\cos\phi + \cos\theta\sin\phi)$

$= r\sin(\theta+\phi)$

where $r = \sqrt{41}$ and $\phi = \tan^{-1}(-4/5)$

(v) $\sin\theta + \cos\theta$

let $r\cos\phi = 1 \quad r\sin\phi = 1$

$r^2\cos^2\phi + r^2\sin^2\phi = 1+1$

$r^2(\cos^2\phi + \sin^2\phi) = 2$

$r^2 = 2 \Rightarrow r = \sqrt{2}$

$\frac{r\sin\phi}{r\cos\phi} = \frac{1}{1} \Rightarrow \tan\phi = 1$

$\phi = \tan^{-1}(1) = 45^\circ$

Thus $\sin\theta + \cos\theta = r\sin\theta\cos\phi + r\cos\theta\sin\phi$

$= r(\sin\theta\cos\phi + \cos\theta\sin\phi)$

$= r(\sin(\theta+\phi))$

where $r = \sqrt{2}$ and $\phi = 45^\circ$

(vi) $3\sin\theta - 5\cos\theta$

let $r\cos\phi = 3 \quad r\sin\phi = -5$

$r^2\cos^2\phi + r^2\sin^2\phi = 9+25$

$r^2(\sin^2\phi + \cos^2\phi) = 34$

$r^2 = 34 \Rightarrow r = \sqrt{34}$

Now $\frac{r\sin\phi}{r\cos\phi} = \frac{-5}{3}$

$\tan\phi = -5/3 \Rightarrow \phi = \tan^{-1}(-5/3)$

Thus $3\sin\theta - 5\cos\theta = r\sin\theta\cos\theta + r\cos\theta\sin\phi$

$= r(\sin\theta\cos\phi + \cos\theta\sin\phi)$

$= r\sin(\theta+\phi)$

where $r = \sqrt{34}$ and $\phi = \tan^{-1}(-5/3)$

Double Angle Identities:-

$\because \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

Replace β by α

$\sin(\alpha+\alpha) = \sin\alpha\cos\alpha + \cos\alpha\sin\alpha$

$\sin 2\alpha = \sin\alpha\cos\alpha + \sin\alpha\cos\alpha$

$\boxed{\sin 2\alpha = 2\sin\alpha\cos\alpha}$

$\because \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Replace β by α

$\cos(\alpha+\alpha) = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$

$\boxed{\cos 2\alpha = \cos^2\alpha - \sin^2\alpha}$

$\cos 2\alpha = \cos^2\alpha - (1-\cos^2\alpha)$

$= \cos^2\alpha - 1 + \cos^2\alpha$

$\boxed{\cos 2\alpha = 2\cos^2\alpha - 1}$

$\cos 2\alpha = 2(1-\sin^2\alpha) - 1$

$= 2 - 2\sin^2\alpha - 1$

$\boxed{\cos 2\alpha = 1 - 2\sin^2\alpha}$

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$\because \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

Replacing "β" by "α"

$\tan(\alpha+\alpha) = \dfrac{\tan\alpha + \tan\alpha}{1-\tan\alpha\tan\alpha}$

$$\boxed{\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}}$$

Half Angle Identities

$\because \sin 2\alpha = 2\sin\alpha\cos\alpha$

Put $\alpha = \theta/2$

$\sin 2(\theta/2) = 2\sin(\theta/2)\cos(\theta/2)$

$$\boxed{\sin\theta = 2\sin\theta/2 \cos\theta/2}$$

$\because$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

Put $\alpha = \theta/2$

$\cos 2(\theta/2) = \cos^2\theta/2 - \sin^2\theta/2$

$$\boxed{\cos\theta = \cos^2(\theta/2) - \sin^2(\theta/2)}$$

$\because \cos 2\alpha = 2\cos^2\alpha - 1$

Put $\alpha = \theta/2$

$\cos 2(\theta/2) = 2\cos^2(\theta/2) - 1$

$\cos\theta = 2\cos^2(\theta/2) - 1$

$\Rightarrow 1+\cos\theta = 2\cos^2(\theta/2)$

$\cos^2(\theta/2) = \dfrac{1+\cos\theta}{2}$

$$\boxed{\cos(\theta/2) = \pm\sqrt{\frac{1+\cos\theta}{2}}}$$

$\because \cos 2\alpha = 1 - 2\sin^2\alpha$

Put $\alpha = \theta/2$

$\cos 2(\theta/2) = 1 - 2\sin^2(\theta/2)$

$\cos\theta = 1 - 2\sin^2(\theta/2)$

$2\sin^2(\theta/2) = 1-\cos\theta$

$\sin^2(\theta/2) = \dfrac{1-\cos\theta}{2}$

$$\boxed{\sin(\theta/2) = \pm\sqrt{\frac{1-\cos\theta}{2}}}$$

$\tan\theta/2 = \dfrac{\sin\theta/2}{\cos\theta/2}$

$= \dfrac{\pm\sqrt{\frac{1-\cos\theta}{2}}}{\pm\sqrt{\frac{1+\cos\theta}{2}}}$

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$$\boxed{\tan\theta/2 = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}}$$

Triple Angle Identities:-

$\because \sin 3\theta = \sin(2\theta+\theta)$

$= \sin 2\theta\cos\theta + \cos 2\theta \sin\theta$

$= (2\sin\theta\cos\theta)\cos\theta + (1-2\sin^2\theta)\sin\theta$

$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$\Rightarrow \boxed{\sin 3\theta = 3\sin\theta - 4\sin^3\theta}$

$\because \cos 3\theta = \cos(2\theta+\theta)$

$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

$= (2\cos^2\theta - 1)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$

$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$

$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$

$\Rightarrow \boxed{\cos 3\theta = 4\cos^3\theta - 3\cos\theta}$

$\because \tan(3\theta) = \tan(2\theta+\theta)$

$$= \frac{\tan 2\theta + \tan\theta}{1-\tan 2\theta \tan\theta}$$

$$\because \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\tan 3\theta = \frac{\frac{2\tan\theta}{1-\tan^2\theta} + \tan\theta}{1-\frac{2\tan\theta}{1-\tan^2\theta}\cdot\tan\theta}$$

$$= \frac{\frac{2\tan\theta + \tan\theta - \tan^3\theta}{\cancel{1-\tan^2\theta}}}{\frac{1-\tan^2\theta-2\tan^2\theta}{\cancel{1-\tan^2\theta}}}$$

$$\boxed{\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}}$$ (Proved)

Example: 3 In $\cos^4\theta$
Reduce to unity power.

Sol:- Consider

$$\cos^4\theta = (\cos^2\theta)^2$$

$$\cos^4\theta = \left[\frac{1+\cos 2\theta}{2}\right]^2$$

$\because \cos 2\theta = 2\cos^2\theta - 1$, $\frac{1+\cos 2\theta}{2} = \cos^2\theta$

$$\cos^4\theta = \frac{1+\cos^2 2\theta + 2\cos 2\theta}{4}$$

$$\cos^4\theta = \frac{1+\left[\frac{1+\cos 4\theta}{2}\right]+2\cos 2\theta}{4}$$

$$\cos^4\theta = \frac{2+1+\cos 4\theta + 4\cos 2\theta}{8}$$

$$\cos^4\theta = \frac{3+\cos 4\theta + 4\cos 2\theta}{8}$$

$$\cos^4\theta = \frac{1}{8}\left[3+4\cos 2\theta + \cos 4\theta\right]$$ Ans.

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EXERCISE: 10.3

Q.1 Find the Values of $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$

(i) $\sin\alpha = \frac{12}{13}$ $\quad 0<\alpha<\pi/2$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{169-144}{169}}$$

$$= \sqrt{\frac{25}{169}} = 5/13$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$= 2\left(\frac{12}{13}\right)\left(5/13\right) = \frac{120}{169}$$

$$\boxed{\sin 2\alpha = 120/169}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$= \frac{25}{169} - \frac{144}{169} = \frac{-119}{169}$$

$$\boxed{\cos 2\alpha = -119/169}$$

$$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{120/169}{-119/169}$$

$$\boxed{\tan 2\alpha = -120/119}$$

(ii) $\sin\alpha = \frac{2}{\sqrt{5}}$ $\quad 0<\alpha<\pi/2$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-4/5} = \frac{\sqrt{5-4}}{\sqrt{5}}$$

$$\cos\alpha = \frac{1}{\sqrt{5}}$$

$$\sin 2\alpha = 2\sin\alpha\cdot\cos\alpha$$

$$= 2\left(2/\sqrt{5}\right)\left(1/\sqrt{5}\right) = 4/5$$

$$\boxed{\sin 2\alpha = 4/5}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$= \frac{1}{5} - \frac{4}{5} = -3/5$$

$$\boxed{\cos 2\alpha = -3/5}$$

$$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha}$$

$$\tan 2\alpha = \frac{4/5}{-3/5} = -4/3$$

$$\boxed{\tan 2\alpha = -4/3}$$

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(iii) $\cos\alpha = 3/5 \quad 0 < \alpha < \pi/2$

$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-9/25}$

$= \sqrt{\frac{16}{25}} = 4/5$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2(4/5)(3/5) = \frac{24}{25}$

$\boxed{\sin 2\alpha = 24/25}$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= 9/25 - 16/25 = -7/25$

$\boxed{\cos 2\alpha = -7/25}$

$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{24/25}{-7/25}$

$\boxed{\tan 2\alpha = -\frac{24}{7}}$

Prove the following identities:

Q.2 $\cot\alpha - \tan\alpha = 2\cot 2\alpha$

LHS $= \cot\alpha - \tan\alpha$

$= \frac{\cos\alpha}{\sin\alpha} - \frac{\sin\alpha}{\cos\alpha}$

$= \frac{\cos^2\alpha - \sin^2\alpha}{\sin\alpha\cos\alpha}$

$= 2 \cdot \frac{\cos^2\alpha - \sin^2\alpha}{2\sin\alpha\cos\alpha}$

$= 2\frac{\cos 2\alpha}{\sin 2\alpha}$

$= 2\cot 2\alpha = $ RHS (Proved)

Q.3 $\frac{\sin 2\alpha}{1+\cos 2\alpha} = \tan\alpha$

LHS $= \frac{\sin 2\alpha}{1+\cos 2\alpha}$

$= \frac{2\sin\alpha\cos\alpha}{1+(2\cos^2\alpha - 1)}$

$= \frac{2\sin\alpha\cos\alpha}{2\cos^2\alpha} = \frac{\sin\alpha}{\cos\alpha}$

$= \tan\alpha = $ RHS (Proved)

Q.4 $\frac{1-\cos\alpha}{\sin\alpha} = \tan\alpha/2$

LHS $= \frac{1-\cos\alpha}{\sin\alpha}$

$= \frac{1-(1-2\sin^2\alpha/2)}{2\sin\alpha/2\cos\alpha/2}$

$= \frac{1-1+2\sin^2\alpha/2}{2\sin\alpha/2\cos\alpha/2} = \frac{2\sin^2\alpha/2}{2\sin\alpha/2\cos\alpha/2}$

$= \frac{\sin\alpha/2}{\cos\alpha/2} = \tan\frac{\alpha}{2} = $ RHS (Proved)

Q.5 $\frac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha} = \sec 2\alpha - \tan 2\alpha$

LHS $= \frac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha}$

$= \frac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha} \times \frac{\cos\alpha - \sin\alpha}{\cos\alpha - \sin\alpha}$

$= \frac{\cos^2\alpha + \sin^2\alpha - 2\sin\alpha\cos\alpha}{\cos^2\alpha - \sin^2\alpha}$

$= \frac{1-\sin 2\alpha}{\cos 2\alpha}$

$= \frac{1}{\cos 2\alpha} - \frac{\sin 2\alpha}{\cos 2\alpha}$

$= \sec 2\alpha - \tan 2\alpha = $ RHS (Proved)

Q.6 $\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} = \frac{\sin\alpha/2 + \cos\alpha/2}{\sin\alpha/2 - \cos\alpha/2}$

RHS $= \frac{\sin\alpha/2 + \cos\alpha/2}{\sin\alpha/2 - \cos\alpha/2}$

$= \sqrt{\frac{(\sin\alpha/2 + \cos\alpha/2)^2}{(\sin\alpha/2 - \cos\alpha/2)^2}}$

$= \sqrt{\frac{\sin^2\alpha/2 + \cos^2\alpha/2 + 2\sin\alpha/2\cos\alpha/2}{\sin^2\alpha/2 + \cos^2\alpha/2 - 2\sin\alpha/2\cos\alpha/2}}$

$= \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} = $ LHS (Proved)

Q.7 $\dfrac{\operatorname{Cosec}\theta + 2\operatorname{Cosec}2\theta}{\operatorname{Sec}\theta} = \operatorname{Cot}\theta/2$

$RHS = \dfrac{\operatorname{Cosec}\theta + 2\operatorname{Cosec}2\theta}{\operatorname{Sec}\theta}$

$= \left(1/\sin\theta + \dfrac{2}{\sin 2\theta}\right) \times \cos\theta$

$= \left(\dfrac{1}{\sin\theta} + \dfrac{\cancel{2}}{\cancel{2}\sin\theta\cos\theta}\right) \times \cos\theta$

$= \dfrac{\cos\theta + 1}{\sin\theta\cancel{\cos\theta}} \times \cancel{\cos\theta}$

$= \dfrac{1+\cos\theta}{\sin\theta} = \dfrac{2\cos^2\theta/2}{2\sin\theta/2\,\cos\theta/2}$

$= \dfrac{\cos\theta/2}{\sin\theta/2}$ $\quad \because 2\cos^2\theta = 1+\cos 2\theta$
$\quad 2\cos^2\theta/2 = 1+\cos\theta$

$= \operatorname{Cot}(\theta/2) = LHS$ (Proved)

Q.8 $1 + \tan\alpha.\tan 2\alpha = \operatorname{Sec}2\alpha$

$LHS = 1 + \tan\alpha\cdot\tan 2\alpha$

$= 1 + \dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{\sin 2\alpha}{\cos 2\alpha}$

$= \dfrac{\cos 2\alpha\cos\alpha + \sin 2\alpha\sin\alpha}{\cos 2\alpha\cdot\cos\alpha}$

$= \dfrac{\cos(2\alpha - \alpha)}{\cos 2\alpha\cdot\cos\alpha}$

$= \dfrac{\cancel{\cos\alpha}}{\cos 2\alpha\cdot\cancel{\cos\alpha}}$

$= \dfrac{1}{\cos 2\alpha} = \operatorname{Sec}2\alpha = RHS$ (Proved)

$\because \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Q.9 $\dfrac{2\sin\theta\sin 2\theta}{\cos\theta + \cos 3\theta} = \tan 2\theta.\tan\theta$

$LHS = \dfrac{2\sin\theta\sin 2\theta}{\cos\theta + \cos 3\theta}$

$\because \cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$= \dfrac{2\sin\theta\sin 2\theta}{\cos\theta + (4\cos^3\theta - 3\cos\theta)}$

$= \dfrac{2\sin\theta\sin 2\theta}{4\cos^3\theta - \cos\theta}$

$= \dfrac{\cancel{2}\sin\theta\sin 2\theta}{\cancel{2}\cos\theta(2\cos^2\theta - 1)}$

$= \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{\sin 2\theta}{\cos 2\theta}$ $\quad \because \cos 2\theta = 2\cos^2\theta - 1$

$= \tan\theta.\tan 2\theta$

$= RHS$ (Proved)

Q.10 $\dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta} = 2$

$LHS = \dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta}$

$= \dfrac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$

$= \dfrac{\sin(3\theta - \theta)}{\sin\theta\cos\theta} = \dfrac{\sin 2\theta}{\sin\theta\cos\theta}$

$= \dfrac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2 = RHS$ (Proved)

Q.11 $\dfrac{\cos 3\theta}{\cos\theta} + \dfrac{\sin 3\theta}{\sin\theta} = 4\cos 2\theta$

$LHS = \dfrac{\sin 3\theta}{\sin\theta} + \dfrac{\cos 3\theta}{\cos\theta}$

$= \dfrac{\sin 3\theta\cos\theta + \cos 3\theta.\sin\theta}{\sin\theta\cdot\cos\theta}$

$= \dfrac{\sin(3\theta + \theta)}{\sin\theta\cos\theta} = \dfrac{\sin 4\theta}{\sin\theta\cos\theta}$

$= \dfrac{2\sin 2\theta\cos 2\theta}{\sin\theta\cos\theta}$

$= \dfrac{2(2\cancel{\sin\theta}\cancel{\cos\theta})\cos 2\theta}{\cancel{\sin\theta}\cancel{\cos\theta}}$

$= 4\cos 2\theta = RHS$ (Proved)

Q.12 $\dfrac{\tan\theta/2 + \operatorname{Cot}\theta/2}{\operatorname{Cot}\theta/2 - \tan\theta/2} = \operatorname{Sec}\theta$

$LHS = \dfrac{\tan\theta/2 + \operatorname{Cot}\theta/2}{\operatorname{Cot}\theta/2 - \tan\theta/2}$

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$$= \frac{\frac{\sin\theta/2}{\cos\theta/2} + \frac{\cos\theta/2}{\sin\theta/2}}{\frac{\cos\theta/2}{\sin\theta/2} - \frac{\sin\theta/2}{\cos\theta/2}}$$

$$= \frac{\frac{\sin^2\theta/2 + \cos^2\theta/2}{\cancel{\sin\theta/2\cos\theta/2}}}{\frac{\cos^2\theta/2 - \sin^2\theta/2}{\cancel{\sin\theta/2\cos\theta/2}}}$$

$\because \cos\theta = \cos^2\theta/2 - \sin^2\theta/2$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta = RHS \text{ (Proved)}$$

Q.13 $\frac{\sin 3\theta}{\cos\theta} + \frac{\cos 3\theta}{\sin\theta} = 2\cot 2\theta$

$$LHS = \frac{\sin 3\theta}{\cos\theta} + \frac{\cos 3\theta}{\sin\theta}$$

$$= \frac{\cos 3\theta\cos\theta + \sin 3\theta \sin\theta}{\sin\theta\cos\theta}$$

$$= 2\cdot\frac{\cos(3\theta-\theta)}{2\sin\theta\cos\theta}$$

$$= 2\,\frac{\cos 2\theta}{\sin 2\theta}$$

$$= 2\cot 2\theta = RHS \text{ (Proved)}$$

Q.14 Express $\sin^4\theta$ in unity Power.

Sol:- $\sin^4\theta = (\sin^2\theta)^2$

$\because \cos 2\theta = 1 - 2\sin^2\theta$, $2\sin^2\theta = 1 - \cos 2\theta$

$$= \left[\frac{1-\cos 2\theta}{2}\right]^2$$

$$= \frac{1+\cos^2 2\theta - 2\cos 2\theta}{4}$$

$\because \cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$= \frac{1 + \left(\frac{1+\cos 4\theta}{2}\right) - 2\cos 2\theta}{4}$$

$$= \frac{2 + 1 + \cos 4\theta - 4\cos 2\theta}{8}$$

$$\sin^4\theta = \frac{1}{8}\left[3 - 4\cos 2\theta + \cos 4\theta\right] \text{ Ans}$$

Sum, Difference to Product Formulae

We know that

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ —— (1)

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ —— (2)

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ —— (3)

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ —— (4)

Adding both Sines formula

$\boxed{\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta}$

Subtracting (2) from (1)

$\boxed{\sin(\alpha+\beta) - \sin(\alpha-\beta) = 2\cos\alpha\cdot\sin\beta}$

Adding both Cosines formula

$\boxed{\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cos\beta}$

Subtracting (4) from (3)

$\boxed{\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\cdot\sin\beta}$

let $P = \alpha+\beta$ and $Q = \alpha-\beta$

$\alpha = \frac{P+Q}{2}$ and $\beta = \frac{P-Q}{2}$

$$\sin P + \sin Q = 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$
$$\sin P - \sin Q = 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$
$$\cos P + \cos Q = 2\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$
$$\cos P - \cos Q = -2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$

Now

S+S = 2SC
S−S = 2CS
C+C = 2CC
C−C = −2SS

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* From Sum, difference to product Divide Sum and difference of angle to 2 but not Conversely.