

## Exercise: 3.8

Q.1 Check that  $y$  is solution of d.Eq: (iii)

(i)  $x \frac{dy}{dx} = 1+y$      $y = cx-1$

If  $y = cx-1$  is the solution of differential Eq so must satisfy it.

$$x \frac{d}{dx}(cx-1) = 1+(cx-1)$$

$$x(c \cdot 1 - 0) = 1+cx-1$$

$$xc = cx$$

$$\therefore LHS = RHS$$

So  $y = cx-1$  is the solution of d.Eq.

(ii)  $x^2(2y+1) \frac{dy}{dx} - 1 = 0$  — (1)

$$y^2 + y = e^{-1/x} \text{ — (2)}$$

Diff. (2) w.r.t  $x$  we have

$$2y \frac{dy}{dx} + \frac{dy}{dx} = 0 + \frac{1}{x^2}$$

$$(2y+1) \frac{dy}{dx} = \frac{1}{x^2}$$

$$x^2(2y+1) \frac{dy}{dx} = 1$$

Putting in (1), we have

$$1 - 1 = 0$$

$$0 = 0$$

$$LHS = RHS$$

So (2) is the solution of (1)

Putting in (1)  $\frac{1+y^2}{e^{-x}} = \frac{1+y^2}{e^{-x}}$

$$LHS = RHS \text{ so}$$

(2) represents the solution of (1)

$$y \frac{dy}{dx} - e^{2x} = 1 \text{ — (1)}$$

$$y^2 = e^{2x} + 2x + C \text{ — (2)}$$

$$2y \frac{dy}{dx} = 2e^{2x} + 2$$

$$y \frac{dy}{dx} = e^{2x} + 1$$

Putting in (1), we have

$$e^{2x} + 1 - e^{2x} = 1$$

$$1 = 1$$

$$LHS = RHS$$

So (2) represents the solution of (1)

(iv)  $\frac{1}{x} \frac{dy}{dx} - 2y = 0$  — (1)

$$y = ce^{2x^2} \text{ — (2)}$$

Diff. (2) w.r.t.  $x$

$$\frac{dy}{dx} = ce^{2x^2} \cdot 2x$$

$$\frac{dy}{dx} = 2xy$$

Putting in (1), we have

$$\frac{1}{x}(2xy) - 2y = 0$$

$$2y - 2y = 0 \Rightarrow 0 = 0$$

$$LHS = RHS$$

So (2) represents the solution of (1)

(v)  $\frac{dy}{dx} = \frac{y^2+1}{e^{-x}}$  — (1)     $y = \tan(e^x+c)$  — (2)

Diff. (2) w.r.t.  $x$   $\frac{dy}{dx} = \sec^2(e^x+c) e^x$

$$\frac{dy}{dx} = \frac{1 + \tan^2(e^x+c)}{e^{-x}} \quad \therefore y = \tan(e^x+c)$$

Q.2  $\frac{dy}{dx} = -y$

Separating the variables

$$\frac{dy}{y} = -dx$$

Integrating both sides

$$\int \frac{dy}{y} = -\int dx$$

$$\ln y = -x + c$$

$$\ln y + x = c \quad \text{G. Solution.}$$

Q.3  $y dx + x dy = 0$

$$y dx = -x dy$$

$$-\frac{dy}{y} = +\frac{dx}{x} \Rightarrow \frac{dy}{y} = -\frac{dx}{x}$$

Integrating both sides

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\ln y = -\ln x + \ln c$$

$$\ln y + \ln x = \ln c$$

$$\ln(xy) = \ln c$$

$$xy = c \quad \text{G. Solution.}$$

Q.4  $\frac{dy}{dx} = \frac{1-x}{y}$

$$y dy = (1-x) dx$$

Integrating both sides

$$\int y dy = \int (1-x) dx$$

$$\frac{y^2}{2} = x - \frac{x^2}{2} + c$$

$$\frac{y^2}{2} + \frac{x^2}{2} = x + c$$

$$x^2 + y^2 = 2(x+c) \quad \text{G. Solution.}$$

Q.5  $\frac{dy}{dx} = \frac{y}{x^2}$

$$\frac{dy}{y} = \frac{dx}{x^2}$$

Integrating both sides

$$\int \frac{dy}{y} = \int x^{-2} dx$$

$$\ln y = \frac{x^{-1}}{-1} + c$$

$$\ln y = -\frac{1}{x} + c$$

$$\ln y + \frac{1}{x} = c \quad \text{G. Solution.}$$

Q.6  $\sin y \operatorname{cosec} x \frac{dy}{dx} = 1$

$$\sin y dy = \frac{dx}{\operatorname{cosec} x}$$

$$\sin y dy = \sin x dx$$

Integrating both sides

$$\int \sin y dy = \int \sin x dx$$

$$-\cos y = -\cos x + c$$

$$\cos x - \cos y = c \quad \text{G. Solution.}$$

Q.7  $x dy + y(x-1) dx = 0$

$$x dy = -y(x-1) dx$$

$$\frac{dy}{y} = \left(\frac{1-x}{x}\right) dx$$

Integrating both sides

$$\int \frac{dy}{y} = \int \frac{1}{x} dx - \int dx$$

$$\ln y = \ln x - x + c$$

$$\ln y - (\ln x - x) = c$$

$$\ln\left(\frac{y}{x}\right) + x = c \quad \text{G. Solution.}$$

Q.8  $\frac{x+1}{y^2+1} = \frac{x}{y} \frac{dy}{dx}$

$\frac{x+1}{x} dx = \frac{y^2+1}{y} dy$

Integrating both sides

$\int (\frac{x}{x} + \frac{1}{x}) dx = \int (\frac{y^2}{y} + \frac{1}{y}) dy$

$\int (x + \frac{1}{x}) dx = \int (y + \frac{1}{y}) dy$

$\frac{y^2}{2} + \ln y = \frac{x^2}{2} + \ln x + c$

$\frac{y^2}{2} - \frac{x^2}{2} + \ln y - \ln x = c$

$\frac{y^2 - x^2}{2} + \ln(\frac{y}{x}) = c$  Gr. Solution

Q.9  $\frac{1}{x} \frac{dy}{dx} = \frac{1}{2}(1+y^2)$

$\frac{dy}{1+y^2} = \frac{1}{2} x dx$

Integrating both sides

$\int \frac{dy}{1+y^2} = \frac{1}{2} \int x dx$

$\tan^{-1} y = \frac{1}{2} \frac{x^2}{2} + c$

$\tan^{-1} y - \frac{x^2}{4} = c$  Gr. Solution

Q.10  $2xy \frac{dy}{dx} = x^2 - 1$

$2y dy = \frac{x^2 - 1}{x^2} dx$

Integrating both sides

$\int 2y dy = \int (\frac{x^2}{x^2} - \frac{1}{x^2}) dx$

$\frac{2y^2}{2} = \int dx - \int x^{-2} dx$

$y^2 = x - \frac{x^{-1}}{-1} + c$

$y^2 = x + \frac{1}{x} + c$  Gr. Solution

$\int \frac{\sec^2 y}{\tan y} dy = - \int \frac{\sec^2 x}{\tan x} dx$

$\ln|\tan y| = -\ln|\tan x| + \ln c$

$\ln(\tan y) + \ln(\tan x) = \ln c \Rightarrow \ln(\tan y \tan x) = \ln c \Rightarrow \tan x \tan y = c$  Gr. Solution

Q.11  $\frac{dy}{dx} + \frac{\partial xy}{\partial y+1} = x$

$\frac{dy}{dx} = x - \frac{\partial xy}{\partial y+1}$

$\frac{dy}{dx} = \frac{\partial yx + x - \partial xy}{\partial y+1} = \frac{x}{\partial y+1}$

$(2y+1) dy = x dx$

Integrating both sides

$\int (2y+1) dy = \int x dx$

$\frac{2y^2}{2} + y = \frac{x^2}{2} + c$

$y^2 + y - \frac{x^2}{2} = c$  Gr. Solution

Q.12  $(x^2 - y^2) \frac{dy}{dx} + y^2 + xy^2 = 0$

$x^2(1-y) \frac{dy}{dx} = -y^2 - xy^2$

$x^2(1-y) \frac{dy}{dx} = -y^2(1+x)$

$\frac{(1-y) dy}{y^2} = \frac{x-1}{x^2} dx$

Integrating both sides

$\int (\frac{1}{y^2} - \frac{y}{y^2}) dy = \int (\frac{x}{x^2} - \frac{1}{x^2}) dx$

$\int y^{-2} dy - \int \frac{1}{y} dy = \int \frac{1}{x} dx - \int x^{-2} dx$

$\frac{y^{-1}}{-1} - \ln y = \ln x - \frac{x^{-1}}{-1} + c$

$-\frac{1}{y} - \ln y = \ln x + \frac{1}{x} + c$

$\ln x + \ln y + \frac{1}{x} + \frac{1}{y} + c = 0$

$\ln(xy) + \frac{1}{x} + \frac{1}{y} + c = 0$  Gr. Solution

Q.13  $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

$-\sec^2 x \tan y dx = \sec^2 y \tan x dy$

$\frac{\sec^2 y}{\tan y} dy = - \frac{\sec^2 x dx}{\tan x}$

Integrating both sides

$\ln|\tan y| = -\ln|\tan x| + \ln c$

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$$Q-14 \quad (y - x \frac{dy}{dx}) = 2(y^2 + \frac{dy}{dx})$$

$$y - x \frac{dy}{dx} = 2y^2 + 2 \frac{dy}{dx}$$

$$y - 2y^2 = 2 \frac{dy}{dx} + x \frac{dy}{dx}$$

$$(2+x) \frac{dy}{dx} = (y-2y^2)$$

$$\frac{dy}{y(1-2y)} = \frac{dx}{2+x}$$

Integrating both sides.

$$\frac{1}{y(1-2y)} = \frac{A}{y} + \frac{B}{1-2y}$$

$$1 = A(1-2y) + B(y)$$

Putting  $y=0$  and  $1-2y=0 \Rightarrow y=1/2$

$$\boxed{1 = A}$$

$$1 = B \frac{1}{2} \Rightarrow \boxed{B=2}$$

$$\int (\frac{1}{y} + \frac{2}{1-2y}) dy = \int \frac{dx}{x+2}$$

$$\int \frac{1}{y} dy - \int \frac{-2}{1-2y} dy = \int \frac{dx}{x+2}$$

$$\ln y - (\ln(1-2y)) = \ln(x+2) + \ln c$$

$$\ln y - (\ln(1-2y)) - \ln(x+2) = \ln c$$

$$\ln \left( \frac{y}{(x+2)(1-2y)} \right) = \ln c$$

$$\frac{y}{(x+2)(1-2y)} = c$$

$$y = (x+2)(1-2y)c \quad \text{G-Solution}$$

$$Q-15 \quad 1 + \cos x \tan y \frac{dy}{dx} = 0$$

$$\cos x \tan y \frac{dy}{dx} = -1$$

$$\tan y dy = \frac{-dx}{\cos x}$$

$$\tan y dy = -\sec x dx$$

Integrating both sides.

$$\int \tan y dy = -\int \sec x dx$$

$$\ln |\sec y| = -(\ln |\sec x + \tan x|) + \ln c$$

$$\ln(\sec x) + \ln(\sec x + \tan x) = \ln c$$

$$\ln(\sec x (\sec x + \tan x)) = \ln c$$

$$\sec x (\sec x + \tan x) = c \quad \text{G-Solution}$$

$$Q-16 \quad y - x \frac{dy}{dx} = 3(1 + x \frac{dy}{dx})$$

$$y - x \frac{dy}{dx} = 3 + 3x \frac{dy}{dx}$$

$$y - 3 = (x + 3x) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y-3}{4x}$$

$$\frac{dy}{y-3} = \frac{dx}{4x}$$

Integrating both sides

$$\int \frac{dy}{y-3} = \frac{1}{4} \int \frac{dx}{x}$$

$$\ln(y-3) = \frac{1}{4} \ln x + \ln c$$

$$\ln(y-3) - \ln x^{1/4} = \ln c$$

$$\ln((y-3)x^{1/4}) = \ln c$$

$$x^{1/4}(y-3) = c \quad \text{G-Solution}$$

$$Q-17 \quad \sec x + \tan y \frac{dy}{dx} = 0$$

$$\tan y \frac{dy}{dx} = -\sec x$$

$$\tan y dy = -\sec x dx$$

Integrating both sides

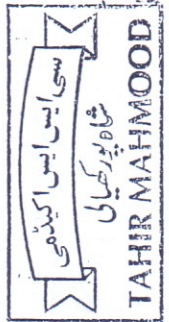
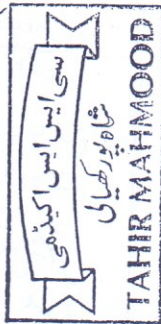
$$\ln |\sec y| = -(\ln |\sec x + \tan x|) + \ln c$$

$$\ln(\sec y) + \ln(\sec x + \tan x) = \ln c$$

$$\ln(\sec y (\sec x + \tan x)) = \ln c$$

$$\sec y (\sec x + \tan x) = c$$

G-Solution



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$$Q.18 \quad (e^x + e^{-x}) \frac{dy}{dx} = e^x - e^{-x}$$

$$\frac{dy}{dx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$dy = \left( \frac{e^x - e^{-x}}{e^x + e^{-x}} \right) dx$$

Integrating both sides

$$\int dy = \int \left( \frac{e^x - e^{-x}}{e^x + e^{-x}} \right) dx$$

$$y = \ln(e^x + e^{-x}) + c \quad \text{G. Solution.}$$

$$Q.19 \quad \frac{dy}{dx} - x = xy^2$$

$$\frac{dy}{dx} = xy^2 + x$$

$$\frac{dy}{dx} = x(y^2 + 1)$$

$$\frac{dy}{1+y^2} = x dx$$

Integrating both sides

$$\int \frac{dy}{1+y^2} = \int x dx$$

$$\tan^{-1} y = \frac{x^2}{2} + c \quad \text{G. Solution.}$$

For particular solution  $y=1, x=0$

$$\tan^{-1}(1) = 0 + c \Rightarrow c = \frac{\pi}{4}$$

$$\text{Thus } \tan^{-1} y = \frac{x^2}{2} + \frac{\pi}{4} \quad \text{P. Solution.}$$

$$Q.20 \quad \frac{dx}{dt} = 2x$$

$$\frac{dx}{x} = 2 dt$$

Integrating both sides

$$\int \frac{dx}{x} = \int 2 dt$$

$$\ln x = 2t + c \quad \text{G. Solution}$$

For particular solution

$$x=4 \quad \text{and } t=0$$

$$\ln 4 = 0 + c \Rightarrow c = \ln 4$$

$$\ln x = 2t + \ln 4$$

$$\ln x - \ln 4 = 2t$$

$$\ln \left( \frac{x}{4} \right) = 2t$$

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Particular Solution.

$$Q.21 \quad \frac{ds}{dt} + 2st = 0$$

$$\frac{ds}{dt} = -2st$$

$$\frac{ds}{s} = -2t dt$$

Integrating both sides

$$\int \frac{ds}{s} = -2 \int t dt$$

$$\ln(s) = -\frac{2t^2}{2} + c$$

$$\ln s = -t^2 + c$$

$$\ln s + t^2 = c \quad \text{G. Solution.}$$

For particular solution

$$s=4e \quad t=0$$

$$\ln(4e) + 0 = c \Rightarrow c = \ln 4 + \ln e$$

$$c = \ln 4 + 1$$

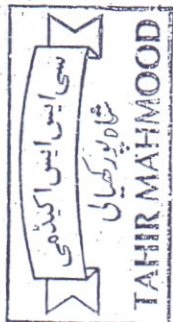
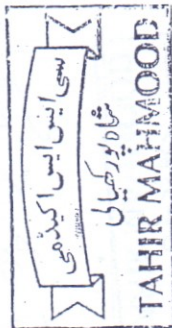
$$\ln e = 1$$

Thus particular solution is

$$\ln s + t^2 = \ln 4 + 1$$

$$\ln s - (\ln 4 + t^2) = 1$$

$$\ln \left( \frac{s}{4} \right) + t^2 = 1 \quad \text{P. Solution.}$$



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Q.22 Let N be the number of bacteria at present then

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = \mu N \quad (\mu \text{ is constant})$$

$$\frac{dN}{N} = \mu dt$$

Integrating both sides

$$\int \frac{dN}{N} = \mu \int dt$$

$$\ln|N| = \mu t + C$$

$$N = e^{\mu t + C}$$

$$N = e^{\mu t} \cdot e^C$$

Applying initial conditions.

$$t=0, N=200$$

$$200 = e^0 \cdot e^C \Rightarrow e^C = 200$$

$$N = 200 e^{\mu t}$$

When  $t=2$  hour  $N=400$

$$400 = 200 e^{r(2)}$$

$$\frac{400}{200} = e^{2r} \Rightarrow e = \frac{400}{200} = 2$$

$$2r(\ln e) = \ln(2)$$

$$r = \frac{1}{2} \ln(2)$$

$$r = \ln(2)^{1/2}$$

$N=?$  when  $t=4$

$$N = 200 e^{4(\frac{1}{2} \ln 2)}$$

$$N = 200 e^{2 \ln 2}$$

$$N = 200 e^{(\ln 2)^2} = 200 e^{\ln 4}$$

$$N = 200 \times 4 = 800 \quad \text{Ans}$$

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Q.23  $V = 2450 \text{ cm/sec}$

By the definition

$$\frac{dv}{dt} = -g$$

because body is thrown upward against gravitational field

$$dv = -g dt$$

Integrating both sides

$$\int dv = -g \int dt$$

$$v = -gt + C_1$$

Applying initial conditions.

$$v = 2450 \quad t = 0$$

$$C_1 = 2450$$

$$\therefore g = 980 \text{ cm/sec}^2$$

$$\Rightarrow v = 2450 - 980 t$$

$$\text{Also } v = \frac{dh}{dt} \quad \therefore s = h$$

$$dh = v dt$$

Integrating both sides

$$\int dh = \int (2450 - 980t) dt$$

$$h = 2450t - \frac{980t^2}{2} + C_2$$

$$\Rightarrow h = 2450t - 490t^2$$

Maximum height:-

For maximum height

$$\frac{dh}{dt} = 0 \Rightarrow 2450 - 980t = 0$$

$$t = \frac{2450}{980} = 2.5$$

$$\frac{d^2h}{dt^2} = 0 - 980 < 0$$

$h$  is maximum at  $t = 2.5$

$$h(\text{max}) = 2450(2.5) - 490(2.5)^2$$

$$h(\text{max}) = 6125 - 3062.5 = 3062.5 \text{ cm}$$

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$C_2 = 0$   
 $h = 0$  at  $t = 0$   
 $\therefore v = 2450 - 980t$

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