$$\frac{1}{(1+z)(2+z)} = \frac{A}{1+z} + \frac{B}{2+z} \Rightarrow 1 = A(2+z) + B(1+z)$$

Putting $1+z=0 \Rightarrow z=-1$ and $z+2=0 \Rightarrow z=-2$

$1 = A(2-1) \Rightarrow \boxed{A=1}$ $\qquad 1 = 0A + B(1-2) \Rightarrow \boxed{B=-1}$

$$\int_0^1 \left(\frac{1}{1+z} - \frac{1}{2+z}\right) dz = \int_0^1 \frac{1}{z+1}\,dz - \int_0^1 \frac{dz}{2+z}$$

$$= \ln(z+1)\Big|_0^1 - \ln(2+z)\Big|_0^1 \Rightarrow \ln\left(\frac{z+1}{z+2}\right)\Big|_0^1$$

$$= \ln\left(\frac{1+1}{1+2}\right) - \ln\left(\frac{0+1}{0+2}\right) \Rightarrow \ln\left(\frac{2}{3}\right) - \ln\left(\frac{1}{2}\right) \Rightarrow \ln\left(\frac{2/3}{1/2}\right) = \ln\left(\frac{4}{3}\right)$$ Ans.

To calculate area under the Curve :-

(1) To calculate the area under the curve along x-axis find x-intercepts putting $y=0$ and area is $\int_a^b y\,dx$

(2) To calculate the area under the curve along y-axis find y-intercepts putting $x=0$ and area is $\int_a^b x\,dy$.

Exercise 3.7

Q.1 $y = x^2+1$ $\qquad x=1$ to $x=2$

Area $= \int_1^2 y\,dx$

Area $= \int_1^2 (x^2+1)\,dx$

Area $= \left[\frac{x^3}{3} + x\right]_1^2$

Area $= \left(\frac{2^3}{3} + 2\right) - \left(\frac{1^3}{3} + 1\right)$

Area $= \frac{8}{3} + 2 - \frac{1}{3} - 1 = \frac{8+6-1-3}{3}$

Area $= \frac{10}{3}$ sq. units

So Area along x-axis is $\frac{10}{3}$ sq. units

Q.2 $y = 5-x^2$ $\qquad x=-1$ to $x=2$

Area $= \int_{-1}^2 y\,dx$

Area $= \int_{-1}^2 (5-x^2)\,dx$

Area $= \left(5x - \frac{x^3}{3}\right)_{-1}^2$

Area $= \left(5(2) - \frac{2^3}{3}\right) - \left(5(-1) - \frac{(-1)^3}{3}\right)$

Area $= \left(10 - \frac{8}{3}\right) - \left(-5 + \frac{1}{3}\right) = 10 - \frac{8}{3} + 5 - \frac{1}{3}$

Area $= \frac{30-8+15-1}{3} = \frac{36}{3} = 12$ sq. units

Area along x-axis is 12 sq. units.

Q.3 $y = 3\sqrt{x}$ $\qquad x=1$ to $x=4$

Area $= \int_1^4 y\,dx$

Area $= \int_1^4 3(x)^{1/2}\,dx$

Area $= \frac{2}{3}3x^{3/2}\Big|_1^4 = \left[2x^{3/2}\right]_1^4$

Area $= 2\left[4^{3/2} - 1^{3/2}\right] = 2(8-1) = 14$

Area along x-axis is 14 sq. units.

Q.4 $y = \cos x \qquad x = \frac{-\pi}{2}$ to $x = \pi/2$

$$\text{Area} = \int_{-\pi/2}^{\pi/2} \cos x \, dx$$

$$\text{Area} = \left[\sin x\right]_{-\pi/2}^{\pi/2}$$

$$\text{Area} = \sin\frac{\pi}{2} - \sin(-\pi/2)$$

$$\text{Area} = [1-(-1)] = 1+1 = 2$$

Req. Area $= 2$ sq. units.

Q.5 $y = 4x - x^2$

Put $y=0$ to get x-intercepts

$$4x - x^2 = 0 \Rightarrow x(4-x) = 0$$

$x=0$ to $x=4$

$$\text{Area} = \int_0^4 y\,dx = \int_0^4 (4x - x^2)\,dx$$

$$\text{Area} = \left(\frac{4x^2}{2} - \frac{x^3}{3}\right)_0^4$$

$$\text{Area} = \left(2(4)^2 - \frac{(4)^3}{3} - 2(0)^2 + \frac{(0)^3}{3}\right)$$

$$\text{Area} = 32 - \frac{64}{3} - 0 + 0 = \frac{96-64}{3}$$

Req. Area $= \frac{32}{3}$ sq. units.

Q.6 $y = x^2 + 2x - 3$

Let $y=0$ to get x-intercepts.

$$x^2 + 2x - 3 = 0 \Rightarrow x^2 + 3x - x - 3 = 0$$

$$(x+3)(x-1) = 0 \Rightarrow x = 1, -3$$

$$\text{Area} = \int_{-3}^1 y\,dx$$

$$\text{Area} = \int_{-3}^1 (x^2 + 2x - 3)\,dx$$

$$\text{Area} = \left(\frac{x^3}{3} + x^2 - 3x\right)_{-3}^1$$

$$\text{Area} = \left(\frac{1}{3} + 1 - 3\right) - \left(\frac{(-3)^3}{3} + (-3)^2 - 3(-3)\right)$$

$$\text{Area} = \left(\frac{1}{3} - 2\right) - (-\cancel{9} + \cancel{9} + 9)$$

$$\text{Area} = \frac{1}{3} - 2 - 9 = \frac{1-6-27}{3}$$

$$\text{Area} = \frac{-32}{3}$$

Req. Area $= \left|\frac{-32}{3}\right| = \frac{32}{3}$ sq. units

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Q.7 $y = x^3 + 1 \qquad x = 2$

Putting $y=0$ to x intercept

$$x^3 + 1 = 0 \Rightarrow (x+1)(x^2 - x + 1) = 0$$

$x+1=0 \qquad x^2 - x + 1$ gives

$x=-1 \qquad$ imaginary values

$$\text{Area} = \int_{-1}^2 y\,dx$$

$$\text{Area} = \int_{-1}^2 (x^3+1)\,dx$$

$$\text{Area} = \left(\frac{x^4}{4} + x\right)_{-1}^2$$

$$\text{Area} = \left[\frac{(2)^4}{4} + 2\right] - \left[\frac{(-1)^4}{4} - 1\right]$$

$$\text{Area} = \frac{16}{4} + 2 - \frac{1}{4} + 1$$

$$\text{Area} = \frac{16+8-1+4}{4} = \frac{27}{4} \text{ sq. unit}$$

Req. Area $= \frac{27}{4}$ sq. units.

Q.8 $y = x^3 - 2x + 4 \qquad x = 1$

Putting $y=0$ to get x-intercept

$$x^3 - 2x + 4 = 0$$

	1	0	−2	4
−2		−2	4	−4
	1	−2	2	0 ← Remainder

$$(x+2)(x^2 - 2x + 2) = 0$$

$x+2=0 \quad \wedge \quad x^2 - 2x + 2 = 0$

$x=-2 \quad \wedge \quad$ gives imaginary value

$$\text{Area} = \int_{-2}^{1} y\,dx = \int_{-2}^{1} (x^3-2x+4)\,dx$$

$$\text{Area} = \left(\frac{x^4}{4} - x^2 + 4x\right]_{-2}^{1}$$

$$\text{Area} = \left(\frac{1}{4} - 1 + 4\right) - \left(\frac{16}{4} - 4 - 8\right)$$

$$\text{Area} = \frac{1}{4} + 3 - \frac{\cancel{16}^{4}}{\cancel{4}} + 12 = \frac{1}{4} + 11$$

$$\text{Area} = \frac{1+44}{4} = \left|-\frac{45}{4}\right| = \frac{45}{4}$$

$$\therefore \text{Area} = \frac{45}{4} \text{ Sq. Units.}$$

Q.9 $y = x^3 - 4x$

Putting $y=0$ to get x-intercepts

$$x^3 - 4x = 0 \Rightarrow x(x^2-4) = 0$$

$$x = 0 \quad x = \pm 2 \Rightarrow x = -2 \text{ to } x = 2$$

$$\text{Area} = \int_{-2}^{2} y\,dx = \int_{-2}^{0} y\,dx + \int_{0}^{2} y\,dx$$

$$\text{Area} = \int_{-2}^{0} (x^3-4x)\,dx + \int_{0}^{2} (x^3-4x)\,dx$$

$$\text{Area} = \left(\frac{x^4}{4} - 2x^2\right)\Big|_{-2}^{0} + \left(\frac{x^4}{4} - 2x^2\right)\Big|_{0}^{2}$$

$$\text{Area} = \left[(0-0) - \left(\frac{\cancel{16}^{4}}{\cancel{4}} - 8\right)\right] + \left[\left(\frac{16}{4} - 8\right) - (0-0)\right]$$

$$\text{Area} = 0 + 4 + |-4| - 0 = 8 \text{ Sq. Units}$$

Req. Area $= 8$ Sq. Units.

Q.10 $y = x(x-1)(x+1)$

Putting $y=0$ to get x-intercepts

$$x(x-1)(x+1) = 0$$

$$x = 0 \quad x-1 = 0 \quad x+1 = 0$$

$$x = 0, 1, -1$$

$$\text{Area} = \int_{-1}^{1} y\,dx$$

$$\text{Area} = \int_{-1}^{0} y\,dx + \int_{0}^{1} y\,dx$$

$$\text{Area} = \int_{-1}^{0} (x^3-x)\,dx + \int_{0}^{1} (x^3-x)\,dx$$

$$\text{Area} = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)\Big|_{-1}^{0} + \left(\frac{x^4}{4} - \frac{x^2}{2}\right)\Big|_{0}^{1}$$

$$\text{Area} = \left[(0-0) - \left(\frac{1}{4} - \frac{1}{2}\right)\right] + \left[\left(\frac{1}{4} - \frac{1}{2}\right) - (0-0)\right]$$

$$\text{Area} = \left|-\left(\frac{1-2}{4}\right)\right| + \left|\left(\frac{1-2}{4}\right)\right|$$

$$\text{Area} = \frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4}$$

$$\therefore \text{Area} = \frac{1}{2} \text{ Sq Units.}$$

Q.11 $y^2 = 3 - x \qquad x = -1 \text{ to } x = 2$

$$\text{Area} = \int_{-1}^{2} y\,dx = \int_{-1}^{2} \sqrt{3-x}\,dx$$

$$\text{Area} = -1\int_{-1}^{2} (3-x)^{1/2}(-1)\,dx$$

$$\text{Area} = \frac{-2}{3}(3-x)^{3/2}\Big]_{-1}^{2}$$

$$\text{Area} = \frac{-2}{3}\left[(3-2)^{3/2} - (3+1)^{3/2}\right]$$

$$\text{Area} = \frac{-2}{3}\left((1)^{3/2} - (4)^{3/2}\right)$$

$$\text{Area} = \frac{-2}{3}(1-8) = \frac{-2}{3}(-7)$$

$$\text{Area} = \frac{14}{3} \text{ Sq. Units.}$$

Q.12 $y = \cos\frac{x}{2} \qquad x = -\pi \text{ to } x = \pi$

$$\text{Area} = \int_{-\pi}^{\pi} y\,dx = \int_{-\pi}^{\pi} \cos\frac{x}{2}\,dx$$

$$\text{Area} = 2\left[\sin\frac{x}{2}\right]_{-\pi}^{\pi}$$

$$\text{Area} = 2\left(\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right)$$

$$\text{Area} = 2\left[1 - (-1)\right] = 2\left[1+1\right] = 2(2)$$

$$\text{Area} = 4 \text{ Sq. Units}$$

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Q.13 $y = \sin 2x \qquad x=0$ to $x=\frac{\pi}{3}$

$$Area = \int_0^{\pi/3} y\,dx = \int_0^{\pi/3} \sin 2x\, dx$$

$$Area = \left[\frac{-\cos 2x}{2}\right]_0^{\pi/3}$$

$$Area = \frac{-1}{2}\left[\cos\frac{2\pi}{3} - \cos 0\right]$$

$$Area = \frac{-1}{2}\left(\frac{-1}{2} - 1\right) = \frac{-1}{2}\left(\frac{-3}{2}\right) = \frac{3}{4}$$

$$Area = \frac{3}{4} \text{ sq. units.}$$

Q.14 $y = \sqrt{2ax - x^2}$

let $y=0$ to get x-intercepts

$$\sqrt{2ax - x^2} = 0 \Rightarrow 2ax - x^2 = 0$$

$$x(2a - x) = 0 \Rightarrow x=0 \wedge 2a - x = 0$$

$$x = 0 \text{ to } x = 2a$$

$$Area = \int_0^{2a} \sqrt{2ax - x^2}\, dx$$

$$Area = \int_0^{2a} \sqrt{a^2 - a^2 + 2ax - x^2}\, dx$$

$$Area = \int_0^{2a} \sqrt{a^2 - (x-a)^2}\, dx$$

$$Area = \left[\frac{(x-a)\sqrt{a^2-(x-a)^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right)\right]_0^{2a}$$

$$Area = \left[\frac{(2a-a)\sqrt{a^2-(2a-a)^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{2a-a}{a}\right)\right]$$
$$- \left[\frac{-a\sqrt{a^2-a^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{-a}{a}\right)\right]$$

$$Area = \frac{a\sqrt{0}}{2} + \frac{a^2}{2}\left[\frac{\pi}{2}\right] - 0 + \frac{a^2}{2}\frac{\pi}{2}$$

$$Area = \frac{\pi a^2}{4} + \frac{\pi a^2}{4} = \frac{\pi a^2 + \pi a^2}{4}$$

$$Area = \frac{2\pi a^2}{4} = \frac{1}{2}\pi a^2$$

$$Area = \frac{1}{2}\pi a^2 \text{ sq. units.}$$

Differential Equation:-

"The equation containing atleast one derivative is called differential Equation."

e.g $\frac{dy}{dx} + y = 5$

Order of Differential Equation:

"The order of highest derivative in the equation is called the order of differential Equation."

e.g $\frac{dy}{dx} + 5 = y$ 1st order

$\frac{d^2y}{dx^2} + y = x$ 2nd order

Solution of differential Equation:-

The values of variable involved in the equation for which differential Equation is true is called solution of differential Equation.

General Solution:-

"The solution containing arbitrary (not fixed) constant is called general solution."

"Particular Solution."

The solution containing fixed constants is called particular