Theorems of Definate Integration:-

There are the following theorems:

(1) **Fundamental Theorem of integral Calculus:**

$$\int_a^b f'(x)\,dx = F(b) - F(a)$$

(2) **Order Theorem** (or) **Order Property:-**

$$\int_a^b f'(x)\,dx = -\int_b^a f'(x)\,dx$$

(3) **Additivity Property** (or) **Additivity theorem:-**

$$\int_a^c f'(x)\,dx = \int_a^b f'(x)\,dx + \int_b^c f'(x)\,dx$$

EXERCISE : 3·6

Q.1 $\int_1^2 (x^2+1)\,dx$

$= \int_1^2 x^2dx + \int_1^2 dx$

$= \frac{x^3}{3}\Big|_1^2 + x\Big|_1^2$

$= \left(\frac{(2)^3}{3} - \frac{(1)^3}{3}\right) + \left(2-1\right)$

$= \frac{8}{3} - \frac{1}{3} + 1 = \frac{8-1+3}{3} = \frac{10}{3}$

$\int_1^2 (x^2+1)\,dx = 10/3$ Ans.

Q.2 $\int_{-1}^1 (x^{1/3}+1)\,dx$

$= \int_{-1}^1 x^{1/3}dx + \int_{-1}^1 dx$

$= \frac{3}{4}x^{4/3}\Big|_{-1}^1 + x\Big|_{-1}^1$

$= \left[\frac{3}{4}(+1)^{4/3} - \frac{3}{4}(-1)^{4/3}\right] + \left[1-(-1)\right]$

$= \frac{3}{4} - \frac{3}{4} + 1 + 1 = 2$

$\int_{-1}^1 (x^{1/3}+1)\,dx = 2$ Ans.

Q.3 $\int_{-2}^0 \frac{1}{(2x-1)^2}\,dx$

$= \int_{-2}^0 (2x-1)^{-2}\,dx$

$= \frac{1}{2}\int_{-2}^0 (2x-1)^{-2}\cdot 2\,dx$

$= \frac{1}{2}\left[\frac{(2x-1)^{-1}}{-1}\right]_{-2}^0$

$= \frac{1}{2}\left[\frac{-1}{(2x-1)}\right]_{-2}^0$

$= \frac{1}{2}\left(\frac{-1}{0-1} - \frac{-1}{2(-2)-1}\right)$

$= \frac{1}{2}\left(1 - \frac{1}{5}\right) = \frac{1}{2}\left(\frac{5-1}{5}\right) = \frac{4}{2\cdot 5} = \frac{2}{5}$

$\int_{-2}^0 \frac{1}{(2x-1)^2}\,dx = 2/5$ Ans.

Q.4 $\int_{-6}^2 \sqrt{3-x}\,dx = \int_{-6}^2 (3-x)^{1/2}\,dx$

$= -\int_{-6}^2 (3-x)^{1/2}\cdot(-1)\,dx$

$= -\left[(3-x)^{3/2}\frac{2}{3}\right]_{-6}^2$

$= -\left[\frac{2}{3}(3-2)^{3/2} - (3+6)^{3/2}\frac{2}{3}\right]$

$= -\left[\frac{2}{3} - \frac{2\cdot 27}{3}\right] = -\left[\frac{2}{3} - 18\right]$

$\int_{-6}^2 \sqrt{3-x}\,dx = \frac{52}{3}$ Ans.

Q.5 $\int_{1}^{\sqrt{5}} \sqrt{(2t-1)^3}\, dt$

$= \frac{1}{2}\int_{1}^{\sqrt{5}} (2t-1)^{3/2} \cdot 2\, dt$

$= \frac{1}{2}\left[\frac{2}{5}(2t-1)^{5/2}\right]_{1}^{\sqrt{5}}$

$= \frac{1}{5}\left[(2\sqrt{5}-1)^{5/2} - (2-1)^{5/2}\right]$

$= \frac{1}{5}\left[(2\sqrt{5}-1)^{5/2} - 1\right]$ Ans.

Q.6 $\int_{2}^{\sqrt{5}} x\sqrt{x^2-1}\, dx$

$= \frac{1}{2}\int_{2}^{\sqrt{5}} (x^2-1)^{1/2} \cdot 2x\, dx$

$= \frac{1}{2}\left[\frac{2}{3}(x^2-1)^{3/2}\right]_{2}^{\sqrt{5}}$

$= \frac{1}{3}\left[\left((\sqrt{5})^2-1\right)^{3/2} - \left((2)^2-1\right)^{3/2}\right]$

$= \frac{1}{3}\left[(5-1)^{3/2} - (4-1)^{3/2}\right] = \frac{1}{3}\left[4^{3/2} - 3^{3/2}\right]$

$= \frac{1}{3}\left[8 - 3\sqrt{3}\right]$ Ans.

Q.7 $\int_{1}^{2} \frac{x}{x^2+2}\, dx$

$= \frac{1}{2}\int_{1}^{2} \frac{2x}{x^2+2}\, dx$

$= \frac{1}{2}\left[\ln|x^2+2|\right]_{1}^{2}$

$= \frac{1}{2}\left[\ln(2^2+2) - \ln(1^2+2)\right] = \frac{1}{2}\left[\ln 6 - \ln 3\right]$

$= \frac{1}{2}\ln\left|\frac{6}{3}\right| = \frac{1}{2}\ln 2 = \ln\sqrt{2}$ Ans.

Q.8 $\int_{2}^{3} \left(x - \frac{1}{x}\right)^2 dx = \int_{2}^{3} \left(x^2 + \frac{1}{x^2} - 2\right) dx$

$= \int_{2}^{3} (x^2 + x^{-2} - 2)\, dx$

$= \left[\frac{x^3}{3} + \frac{x^{-1}}{-1} - 2x\right]_{2}^{3}$

$= \left[\frac{3^3}{3} + \frac{(3)^{-1}}{-1} - 2(3) - \frac{2^3}{3} - \frac{(2)^{-1}}{-1} + 2(2)\right]$

$= 9 - \frac{1}{3} - 6 - \frac{8}{3} + \frac{1}{2} + 4 = 7 - \frac{1}{3} - \frac{8}{3} + \frac{1}{2}$

$= \frac{42 - 2 - 16 + 3}{6} = \frac{27}{6} = \frac{9}{2}$

$\int_{2}^{3} \left(x - \frac{1}{x}\right)^2 dx = 9/2$ Ans.

Q.9 $\int_{-1}^{1} \sqrt{x^2+x+1}\, \left(x + \frac{1}{2}\right) dx$

$= \int_{-1}^{1} (x^2+x+1)^{1/2} \left(\frac{2x+1}{2}\right) dx$

$= \frac{1}{2}\int_{-1}^{1} (x^2+x+1)^{1/2} (2x+1)\, dx$

$= \frac{1}{2}\left[\frac{2}{3}(x^2+x+1)^{3/2}\right]_{-1}^{1}$

$= \frac{1}{3}\left[(x^2+x+1)^{3/2}\right]_{-1}^{1}$

$= \frac{1}{3}\left[(1+1+1)^{3/2} - (1-1+1)^{3/2}\right]$

$= \frac{1}{3}\left(3^{3/2} - 1^{3/2}\right) = \frac{1}{3}\left(3\sqrt{3} - 1\right)$

$= \frac{3\sqrt{3}}{3} - \frac{1}{3} = \sqrt{3} - 1/3$

Thus $\int_{-1}^{1} \sqrt{x^2+x+1}\, \left(x + \frac{1}{2}\right) dx = \sqrt{3} - 1/3$ Ans.

Q.10 $\int_{0}^{3} \frac{dx}{x^2+9}$

$= \int_{0}^{3} \frac{dx}{x^2+(3)^2}$

$= \frac{1}{3}\left[\tan^{-1}\left(\frac{x}{3}\right)\right]_{0}^{3}$

$= \frac{1}{3}\left[\tan^{-1}\left(\frac{3}{3}\right) - \tan^{-1}\left(\frac{0}{3}\right)\right]$

$= \frac{1}{3}\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$

$= \frac{1}{3}\left[\frac{\pi}{4} - 0\right] = \pi/12$ Ans.

Q.11 $\int_{1}^{2} a^x\, dx = \left.\frac{a^x}{\ln a}\right|_{1}^{2}$

$= \frac{1}{\ln a}\left[a^x\right]_{1}^{2}$

$= \frac{1}{\ln a}\left[a^2 - a^1\right]$

$= \frac{a^2 - a}{\ln a}$ Ans.

Q.12 $\int_{\pi/6}^{\pi/3} \cos t\, dt$

$= \left[\sin t\right]_{\pi/6}^{\pi/3}$

$= \sin \pi/3 - \sin \pi/6$

$= \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{1}{2}(\sqrt{3}-1)$ Ans.

Q.13 $\int_1^2 (x+1/x)^{1/2}(1-1/x^2)\,dx$

$= \frac{2}{3}(x+1/x)^{3/2}\Big|_1^2$

$= \frac{2}{3}\left[(2+\frac{1}{2})^{3/2} - (1+\frac{1}{1})^{3/2}\right]$

$= \frac{2}{3}\left[(\frac{5}{2})^{3/2} - (2)^{3/2}\right] = \frac{2}{3}\left[\frac{5\sqrt{5}}{2\sqrt{2}} - 2\sqrt{2}\right]$

$= \frac{2}{3}\left(\frac{5\sqrt{5}-(2\sqrt{2})^2}{2\sqrt{2}}\right) = \frac{2}{3}\left(\frac{5\sqrt{5}-8}{2\sqrt{2}}\right)$

$= \frac{1}{3\sqrt{2}}(5\sqrt{5}-8)$ Ans.

Q.14 $\int_4^{2\sqrt{2}} 3x\sqrt{17-x^2}\,dx$

$= -\frac{3}{2}\int_4^{2\sqrt{2}} (17-x^2)^{1/2}\cdot(-2x)\,dx$

$= -\frac{3}{2}\left[\frac{2}{3}(17-x^2)^{3/2}\right]_4^{2\sqrt{2}}$

$= -\left[\left[17-(2\sqrt{2})^2\right]^{3/2} - (17-4^2)^{3/2}\right]$

$= \left[(17-16)^{3/2} - (17-8)^{3/2}\right]$

$= (1)^{3/2} - 9^{3/2} \Rightarrow 1-27 = -26$

$\int_4^{2\sqrt{2}} 3x\sqrt{17-x^2}\,dx = -26$ Ans.

Q.15 $\int_1^2 \ln x\, dx$

$= \int_1^2 \underset{II}{1}\cdot \underset{I}{\ln x}\, dx$

$x\ln x\Big|_1^2 - \int_1^2 x\cdot\frac{1}{x}\,dx$

$= x\ln x\Big|_1^2 - x\Big|_1^2$

$= 2\ln 2 - \ln 1 - (2-1)$

$= 2\ln 2 - 0 - 1$ $\quad \because \ln 1 = 0$

$= 2\ln 2 - 1$ Ans.

Q.16 $\int_0^2 \left[e^{x/2} - e^{-x/2}\right] dx$

$= \left[2e^{x/2} + 2e^{-x/2}\right]_0^2$

$= 2\left[e^{2/2} + e^{-2/2} - e^{0} - e^{-0}\right]$

$= 2\left[e^1 + e^{-1} - 1 - 1\right]$

$= 2\left[e + \frac{1}{e} - 2\right]$

$= 2\left[\frac{e^2+1-2e}{e}\right]$

$= \frac{2}{e}(e-1)^2$ Ans.

Q.17 $\int_0^{\pi/4} \frac{\cos\theta + \sin\theta}{\cos 2\theta + 1}\, d\theta$

$\because \cos 2\theta = 2\cos^2\theta - 1$
$1+\cos 2\theta = 2\cos^2\theta$

$= \int_0^{\pi/4} \frac{\cos\theta + \sin\theta}{2\cos^2\theta}$

$= \int_0^{\pi/4} \left(\frac{\cos\theta}{2\cos^2\theta} + \frac{\sin\theta}{2\cos^2\theta}\right) d\theta$

$= \frac{1}{2}\int_0^{\pi/4} \sec\theta\, d\theta + \frac{1}{2}\int_0^{\pi/4} \tan\theta \sec\theta\, d\theta$

$= \frac{1}{2}\ln|\sec\theta + \tan\theta|\Big]_0^{\pi/4} + \frac{1}{2}\sec\theta\Big|_0^{\pi/4}$

$= \frac{1}{2}\left[\ln(\sec\frac{\pi}{4} + \tan\frac{\pi}{4}) + \ln(\sec 0 + \tan 0)\right]$
$- \frac{1}{2}(\sec \pi/4 - \sec 0)$

$= \frac{1}{2}\ln(\sqrt{2}+1) - \ln(1+0) + \frac{1}{2}(\sqrt{2}-1)$

$= \frac{1}{2}\left[\ln(\sqrt{2}+1) + \sqrt{2} - 1\right]$ Ans.

Q.18 $\int_0^{\pi/6} \cos^3\theta \, d\theta$

$= \int_0^{\pi/6} \cos^2\theta \cos\theta \, d\theta$

$= \int_0^{\pi/6} (1-\sin^2\theta)\cos\theta \, d\theta$

$= \int_0^{\pi/6} (\cos\theta - \sin^2\theta\cos\theta) \, d\theta$

$= \int_0^{\pi/6} \cos\theta d\theta - \int_0^{\pi/6} \sin^2\theta \cos\theta \, d\theta$

$= \left[\sin\theta\right]_0^{\pi/6} - \left[\frac{1}{3}\sin^3\theta\right]_0^{\pi/6}$

$= (\sin\pi/6 - \sin 0) - \frac{1}{3}\left[(\sin\pi/6)^3 - (\sin 0)^3\right]$

$= (\frac{1}{2} - 0) - \frac{1}{3}((\frac{1}{2})^3 - 0^3)$

$= \frac{1}{2} - \frac{1}{3}(\frac{1}{8}) = \frac{1}{2} - \frac{1}{24} = \frac{12-1}{24}$

$\int_0^{\pi/6} \cos^3\theta d\theta = 11/24$ Ans.

Q.19 $\int_{\pi/6}^{\pi/4} \cos^2\theta \cot^2\theta \, d\theta$

$= \int_{\pi/6}^{\pi/4} \cos^2\theta \cdot \frac{\cos^2\theta}{\sin^2\theta} d\theta$

$= \int_{\pi/6}^{\pi/4} \frac{(\cos^2\theta)^2}{\sin^2\theta} d\theta$

$= \int_{\pi/6}^{\pi/4} \frac{(1-\sin^2\theta)^2}{\sin^2\theta} d\theta$

$= \int_{\pi/6}^{\pi/4} \left(\frac{1+\sin^4\theta - 2\sin^2\theta}{\sin^2\theta}\right) d\theta$

$= \int_{\pi/6}^{\pi/4} \frac{1}{\sin^2\theta} d\theta + \int_{\pi/6}^{\pi/4} \frac{\sin^4\theta}{\sin^2\theta} d\theta - 2\int_{\pi/6}^{\pi/4} \frac{\sin^2\theta}{\sin^2\theta} d\theta$

$= \int_{\pi/6}^{\pi/4} \operatorname{cosec}^2\theta d\theta + \int_{\pi/6}^{\pi/4} \sin^2\theta d\theta - 2\int_{\pi/6}^{\pi/4} d\theta$

$= (-\cot\theta)_{\pi/6}^{\pi/4} + \frac{1}{2}\int_{\pi/6}^{\pi/4} (1-\cos 2\theta) d\theta - 2(\theta)_{\pi/6}^{\pi/4}$

$= -(\cot\pi/4 - \cot\pi/6) + \frac{1}{2}\int_{\pi/6}^{\pi/4} d\theta - \frac{1}{2}\int_{\pi/6}^{\pi/4} \cos 2\theta d\theta - 2(\frac{\pi}{4} - \frac{\pi}{6})$

$= -(1-\sqrt{3}) + \frac{\theta}{2}\Big|_{\pi/6}^{\pi/4} - \frac{1}{4}\sin 2\theta\Big|_{\pi/6}^{\pi/4} - 2\left(\frac{6\pi - 4\pi}{24}\right)$

$= (\sqrt{3}-1) + \frac{1}{2}(\frac{\pi}{4} - \pi/6) - \frac{1}{4}(\sin\frac{\pi}{2} - \sin\frac{\pi}{3}) - \frac{4\pi}{24}$

$= (\sqrt{3}-1) + \frac{1}{2}(\frac{2\pi}{24}) - \frac{1}{4}(1 - \frac{\sqrt{3}}{2}) - \frac{\pi}{6}$

$= (\sqrt{3}-1) + \frac{\pi}{24} - \frac{\pi}{6} - \frac{1}{4} + \frac{\sqrt{3}}{8}$

$= \sqrt{3} - 1 + \frac{\pi - 4\pi}{24} - \frac{1}{4} + \frac{\sqrt{3}}{8}$

$= \sqrt{3} + \frac{\sqrt{3}}{8} - 1 - \frac{1}{4} - \frac{3\pi}{24}$

$= \left(\frac{8\sqrt{3}+\sqrt{3}}{8}\right) - \frac{5}{4} - \frac{\pi}{8}$

$= \frac{\sqrt{3}}{8}(8+1) - \frac{5}{4} - \pi/8$

$= \frac{9\sqrt{3}}{8} - \frac{5}{4} - \pi/8$ Ans.

Q.20 $\int_0^{\pi/4} \cos^4 t \, dt$ $\quad \because \cos 2\theta = 2\cos^2\theta - 1$

$= \int_0^{\pi/4} [\cos^2 t]^2 dt$

$= \int_0^{\pi/4} \left(\frac{1+\cos 2t}{2}\right)^2 dt$

$= \int_0^{\pi/4} \left(\frac{1+\cos^2 2t + 2\cos 2t}{4}\right) dt$

$= \frac{1}{4}\int_0^{\pi/4} dt + \frac{1}{4}\int_0^{\pi/4} \cos^2 2t \, dt + \frac{2}{4}\int_0^{\pi/4} \cos 2t \, dt$

$= \frac{1}{4} t \Big|_0^{\pi/4} + \frac{1}{4}\int_0^{\pi/4} \frac{1+\cos 8t}{2} dt + \frac{1}{2}\int_0^{\pi/4} \cos 2t \, dt$

$= \frac{1}{4}(\pi/4 - 0) + \frac{1}{8}\int_0^{\pi/4} (1+\cos 8t) dt + \frac{1}{2}\frac{\sin 2t}{2}\Big|_0^{\pi/4}$

$= \frac{\pi}{16} + \frac{1}{8}\left(t + \frac{\sin 8t}{8}\right)_0^{\pi/4} + \frac{1}{4}(\sin\frac{\pi}{2} - \sin 0)$

$= \frac{\pi}{16} + \frac{1}{8}\left(\frac{\pi}{4} - 0 + \frac{\sin 2\pi}{8} - \frac{0}{8}\right) + \frac{1}{4}(1-0)$

$= \frac{\pi}{16} + \frac{1}{8}\left(\frac{\pi}{4} - 0 + 0 + 0\right) + \frac{1}{4}$

$\frac{\pi}{16} + \frac{\pi}{32} + 1/4 = \frac{2\pi + \pi}{32} + 1/4$

$$= \frac{3\pi}{32} + \frac{1}{4} = \frac{3\pi + 8}{32} \quad \text{Ans.}$$

Q.21 $\int_0^{\pi/3} \cos^2\theta \sin\theta \, d\theta$

$$= -\int_0^{\pi/3} \cos^2\theta \, (-\sin\theta) \, d\theta$$

$$= -\frac{1}{3}\left[\cos^3\theta\right]_0^{\pi/3}$$

$$= -\frac{1}{3}\left[\left(\cos\frac{\pi}{3}\right)^3 - (\cos 0)^3\right]$$

$$= -\frac{1}{3}\left[\left(\frac{1}{2}\right)^3 - (1)^3\right] = -\frac{1}{3}\left(\frac{1}{8} - 1\right)$$

$$= -\frac{1}{3}\left(\frac{1-8}{8}\right) = -\frac{1}{3}\left(\frac{-7}{8}\right) = \frac{7}{24} \quad \text{Ans.}$$

Q.22 $\int_0^{\pi/4} (1+\cos^2\theta) \tan^2\theta \, d\theta$

$$= \int_0^{\pi/4} (1+\cos^2\theta) \frac{\sin^2\theta}{\cos^2\theta} \, d\theta$$

$$= \int_0^{\pi/4} \left(\frac{\sin^2\theta}{\cos^2\theta} + \frac{\sin^2\theta \cos^2\theta}{\cos^2\theta}\right) d\theta$$

$$= \int_0^{\pi/4} (\tan^2\theta + \sin^2\theta) \, d\theta$$

$$= \int_0^{\pi/4} \tan^2\theta \, d\theta + \int_0^{\pi/4} \sin^2\theta \, d\theta$$

$$= \int_0^{\pi/4} (\sec^2\theta - 1) \, d\theta + \int_0^{\pi/4} \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \int_0^{\pi/4} \sec^2\theta \, d\theta - \int_0^{\pi/4} d\theta + \frac{1}{2}\int_0^{\pi/4} d\theta - \frac{1}{2}\int_0^{\pi/4} \cos 2\theta \, d\theta$$

$$= \tan\theta\Big|_0^{\pi/4} - \theta\Big|_0^{\pi/4} + \frac{\theta}{2}\Big|_0^{\pi/4} - \frac{1}{2}\left(\frac{\sin 2\theta}{2}\right)_0^{\pi/4}$$

$$= \left(\tan\frac{\pi}{4} - \tan 0\right) - \left(\frac{\pi}{4} - 0\right) + \left(\frac{\pi}{8} - 0\right) - \frac{1}{4}\left(\sin\frac{\pi}{2} - 0\right)$$

$$= (1-0) - \left(\frac{\pi}{4}\right) + \frac{\pi}{8} - \frac{1}{4}(1-0)$$

$$= 1 - \frac{\pi}{4} + \pi/8 - 1/4 = \frac{8 - 2\pi + \pi - 2}{8}$$

$$= \frac{6-\pi}{8} \quad \text{Ans.}$$

Q.23 $\int_0^{\pi/4} \frac{\sec\theta}{(\sin\theta + \cos\theta)} \, d\theta$

$$= \int_0^{\pi/4} \frac{\sec\theta}{\cos\theta\left(\frac{\sin\theta + \cos\theta}{\cos\theta}\right)} \, d\theta$$

$$= \int_0^{\pi/4} \frac{\sec\theta . \sec\theta}{(\tan\theta + 1)} \, d\theta$$

$$= \int_0^{\pi/4} \frac{\sec^2\theta}{1+\tan\theta} \, d\theta$$

$$= \ln\left[1+\tan\theta\right]\Big|_0^{\pi/4}$$

$$= \ln(1+\tan\pi/4) - \ln(1+\tan 0)$$

$$= \ln(1+1) - \ln(1+0) = \ln 2 - \ln 1$$

$\because \ln 1 = 0$

$$= \ln 2 \quad \text{Ans}$$

Q.24 $\int_{-1}^{5} |x-3| \, dx$

$$= \int_{-1}^{3} |x-3| \, dx + \int_{3}^{5} |x-3| \, dx$$

$$= \int_{-1}^{3} -(x-3) \, dx + \int_{3}^{5} (x-3) \, dx$$

$$= \int_{-1}^{3} (-x+3) \, dx + \int_{3}^{5} (x-3) \, dx$$

$$= \left[-\frac{x^2}{2} + 3x\right]_{-1}^{3} + \left[\frac{x^2}{2} - 3x\right]_{3}^{5}$$

$$= \left[-\frac{(3)^2}{2} + 3(3) + \frac{(-1)^2}{2} - 3(-1)\right] + \left[\frac{5^2}{2} - 3(5) - \frac{(3)^2}{2} + 3(3)\right]$$

$$= \left[-\frac{9}{2} + 9 + \frac{1}{2} + 3\right] + \left[\frac{25}{2} - 15 - \frac{9}{2} + 9\right]$$

$$= -\frac{9}{2} + 9 + \frac{1}{2} + 3 + \frac{25}{2} - 15 - 9/2 + 9$$

$$= \frac{-9 + 18 + 1 + 6 + 25 - 30 - 9 + 18}{2}$$

$$= \frac{20}{2} = 10$$

$$\int_{-1}^{5} |x-3| \, dx = 10 \quad \text{Ans.}$$

Q.25 $\int_{1/8}^{1} \frac{(x^{1/3}+2)^2}{x^{2/3}}\,dx$

$= \int_{1/8}^{1} \frac{(x^{2/3}+4+4x^{1/3})}{x^{2/3}}\,dx$

$= \int_{1/8}^{1} \left(\frac{x^{2/3}}{x^{2/3}} + \frac{4}{x^{2/3}} + \frac{4x^{1/3}}{x^{2/3}}\right)dx$

$= \int_{1/8}^{1} (1 + 4x^{-2/3} + 4x^{-1/3})\,dx$

$= \left[x + \frac{4x^{1/3}}{1/3} + \frac{4x^{2/3}}{2/3}\right]_{1/8}^{1}$

$= (x + 12x^{1/3} + 6x^{2/3})_{1/8}^{1}$

$= \left[1 + 12(1)^{1/3} + 6(1)^{2/3}\right] - \left[(1/8) + 12(\frac{1}{8})^{1/3} + 6(1/8)^{2/3}\right]$

$= (1+12+6) - (\frac{1}{8} + 12(1/2) + 6(1/4))$

$= 19 - (\frac{1}{8} + 6 + \frac{3}{2}) = 19 - \frac{1+48+12}{8}$

$= 19 - \frac{61}{8} = \frac{152-61}{8} = \frac{91}{8}$ Ans.

Q.26 $\int_{1}^{3} \frac{x^2-2}{x+1}\,dx$

$$\begin{array}{r|l} & x-1 \\ \hline x+1 & \cancel{x^2}-2 \\ & \underline{\overset{-}{x^2}\quad \overset{-}{+x}} \\ & \cancel{-x}-2 \\ & \underline{\overset{+}{-x}\ \overset{+}{-1}} \\ & -1 \end{array}$$

$= \int_{1}^{3} \left(x - 1 - \frac{1}{x+1}\right)dx$

$= \left[\frac{x^2}{2} - x - \ln(x+1)\right]_1^3$

$= \left[\frac{(3)^2}{2} - 3 - \ln(3+1)\right] - \left[\frac{(1)^2}{2} - 1 - \ln(1+1)\right]$

$= \frac{9}{2} - 3 - \ln 4 - \frac{1}{2} + 1 + \ln 2$

$= \frac{9}{2} - 3 + 1 - \frac{1}{2} - (\ln 4 - \ln 2)$

$= \frac{9-6+2-1}{2} - \ln\left[\frac{4}{2}\right]$

$= \frac{4}{2} - \ln|2| = 2 - \ln[2]$

Thus $\int_{1}^{3} \frac{x^2-2}{x+1}\,dx = 2 - \ln[2]$

Ans.

Q.27 $\int_{2}^{3} \frac{3x^2-2x+1}{(x-1)(x^2+1)}\,dx$

$\frac{3x^2-2x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

$3x^2-2x+1 = A(x^2+1) + B(x^2-x) + C(x-1)$

Putting $x-1=0 \Rightarrow x=1$

$3-2+1 = A(1+1) + 0B + 0C \Rightarrow 2A = 2 \Rightarrow \boxed{A=1}$

Comparing the Coefficients of

$x^2 \Rightarrow 3 = A + B \Rightarrow B = 3-1 \Rightarrow \boxed{B=2}$

$x \Rightarrow -2 = -B + C \Rightarrow C = -2+2 = 0$

$\int_{2}^{3} \frac{3x^2-2x+1\,dx}{(x-1)(x^2+1)} = \int_{2}^{3}\left(\frac{1}{x-1} + \frac{2x+0}{x^2+1}\right)dx$

$= \int_{2}^{3} \frac{dx}{x-1} + \int_{2}^{3} \frac{2x}{x^2+1}\,dx$

$= \ln(x-1)\Big|_2^3 + \ln(x^2+1)\Big|_2^3$

$= \left[\ln(3-1) - \ln(2-1)\right] + \left[\ln(3^2+1) - \ln(2^2+1)\right]$

$= \ln 2 - \ln 1 + \ln 10 - \ln(5)$

$= \ln\left(\frac{2\times 10}{5}\right) \Rightarrow \ln[4]$ Ans.

Q.28 $\int_{0}^{\pi/4} \frac{\sin x - 1}{\cos^2 x}\,dx$

$= \int_{0}^{\pi/4} \frac{\sin x}{\cos^2 x}\,dx - \int_{0}^{\pi/4} \frac{1}{\cos^2 x}\,dx$

$= \int_{0}^{\pi/4} \tan x \cdot \sec x\,dx - \int_{0}^{\pi/4} \sec^2 x\,dx$

$= \sec x\Big|_0^{\pi/4} - \tan x\Big|_0^{\pi/4}$

$= (\sec\frac{\pi}{4} - \sec 0) - (\tan\frac{\pi}{4} - \tan 0)$

$= (\sqrt{2} - 1) - (1 - 0)$

$= \sqrt{2} - 1 - 1 = \sqrt{2} - 2$

$\int_{0}^{\pi/4} \frac{\sin x - 1}{\cos^2 x}\,dx = \sqrt{2} - 2$

Ans.

Q.29 $\int_0^{\pi/4} \frac{1}{1+\sin x}\,dx$

$= \int_0^{\pi/4} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}\,dx$

$= \int_0^{\pi/4} \frac{1-\sin x}{\cos^2 x}\,dx$

$= \int_0^{\pi/4} \frac{1}{\cos^2 x}\,dx - \int_0^{\pi/4} \frac{\sin x}{\cos^2 x}\,dx$

$= \int_0^{\pi/4} \sec^2 x\,dx - \int_0^{\pi/4} \tan x \cdot \sec x\,dx$

$= \tan x \Big|_0^{\pi/4} - \sec x \Big|_0^{\pi/4}$

$= (\tan\frac{\pi}{4} - \tan 0) - (\sec\frac{\pi}{4} - \sec 0)$

$= (1-0) - (\sqrt{2}-1) = 1-\sqrt{2}+1$

$= 2-\sqrt{2}$ Ans

Q.30 $\int_0^1 \frac{3x}{\sqrt{4-3x}}\,dx$

$= -\int_0^1 \frac{4-3x-4}{\sqrt{4-3x}}\,dx$

$= -\int_0^1 \frac{4-3x}{\sqrt{4-3x}} + \int_0^1 \frac{4}{\sqrt{4-3x}}\,dx$

$= -\int_0^1 (4-3x)^{1/2} + 4\int_0^1 (4-3x)^{-1/2}\,dx$

$= +\frac{1}{3}\int_0^1 (4-3x)^{1/2}\cdot(-3)\,dx - \frac{4}{3}\int_0^1 (4-3x)^{-1/2}\cdot(-3)\,dx$

$= \left[\frac{1}{3}\frac{(4-3x)^{3/2}}{3/2} - \frac{4}{3}\frac{(4-3x)^{1/2}}{1/2}\right]_0^1$

$= \frac{2}{9}(4-3x)^{3/2}\Big|_0^1 - \frac{8}{3}(4-3x)^{1/2}\Big|_0^1$

$= \frac{2}{9}\left[(4-3)^{3/2} - (4-0)^{3/2}\right] - \frac{8}{3}\left[(4-3)^{1/2} - (4-0)^{1/2}\right]$

$= \frac{2}{9}(1-8) - \frac{8}{3}(1-2)$

$= -\frac{14}{9} + \frac{8}{3} = \frac{-14+24}{9}$

$= \frac{10}{9}$ Ans.

Q.31 $\int_{\pi/6}^{\pi/2} \frac{dx(\cos x)}{\sin x(2+\sin x)}$

Let $\sin x = Z$ $\cos x\,dx = dZ$

if $x = \frac{\pi}{6} \Rightarrow Z = \frac{1}{2}$ ∧ if $x = \frac{\pi}{2} \Rightarrow Z = 1$

$\int_{1/2}^1 \frac{dZ}{Z(2+Z)} = \int_{1/2}^1 \left(\frac{A}{Z} + \frac{B}{2+Z}\right) dZ$

$\frac{1}{Z(2+Z)} = \frac{A}{Z} + \frac{B}{2+Z}$

$1 = A(2+Z) + BZ$

Putting $Z=0$ and $Z+2=0 \Rightarrow Z=-2$

$1 = 2A$ $1 = 0A - 2B$

$\boxed{A = 1/2}$ $\boxed{B = -1/2}$

$\int_{1/2}^1 \frac{dZ}{Z(2+Z)} = \int_{1/2}^1 \frac{1/2\,dZ}{Z} + \int_{1/2}^1 \frac{-1/2}{Z+2}\,dZ$

$= \frac{1}{2}\ln|Z|\Big|_{1/2}^1 - \frac{1}{2}\ln|Z+2|\Big|_{1/2}^1$

$= \frac{1}{2}\ln\left(\frac{Z}{Z+2}\right)\Big|_{1/2}^1$

$= \frac{1}{2}\ln\left(\frac{1}{1+2}\right) - \frac{1}{2}\ln\left[\frac{1/2}{1/2+2}\right]$

$= \frac{1}{2}\ln\left[\frac{1}{3}\right] - \frac{1}{2}\ln\left[1/5\right]$

$= \frac{1}{2}\ln\left(\frac{1/3}{1/5}\right) \Rightarrow \frac{1}{2}\ln\left[\frac{5}{3}\right]$ Ans

Q.32 $\int_0^{\pi/2} \frac{\sin x\,dx}{(1+\cos x)(2+\cos x)}$

Let $Z = \cos x \Rightarrow dZ = -\sin x\,dx$

if $x = 0 \Rightarrow Z = 1$ ∧ if $x = \frac{\pi}{2} \Rightarrow Z = 0$

$\int_1^0 \frac{-dZ}{(2+Z)(1+Z)} = \int_0^1 \frac{dZ}{(1+Z)(2+Z)}$

$\because \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$