M.Sc
Mob No:

Exercise 3.5

Evaluate the following integrals:

Q.1 $\int \frac{3x+1}{x^2-x-6}dx = \int \frac{3x+1}{x^2-3x+2x+6}$

$= \int \frac{3x+1\ dx}{(x+2)(x-3)} = \int \frac{A}{x+2}dx + \int \frac{B}{x-3}dx$

$\frac{3x+1}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$

$3x+1 = A(x-3) + B(x+2)$

Putting $x+2=0 \Rightarrow x=-2$

$(-2)3+1 = A(-2-3)+0B \Rightarrow -5=-5A \Rightarrow \boxed{A=1}$

Putting $x-3=0 \Rightarrow x=3$

$3(3)+1 = 0A+(3+2)B \Rightarrow 10=5B \Rightarrow \boxed{B=2}$

$\int \frac{3x+1}{x^2-x-6}dx = \int \frac{1\,dx}{x+2} + \int \frac{2\,dx}{x-3}$

$= \ln|x+2| + 2\ln|x-3| + c$ Ans.

Q.2 $\int \frac{5x+8}{(x+3)(2x-1)}dx = \int \frac{A\,dx}{x+3} + \int \frac{B}{2x-1}dx$

$\frac{5x+8}{(x+3)(2x-1)} = \frac{A}{x+3} + \frac{B}{2x-1}$

$5x+8 = A(2x-1) + B(x+3)$

Putting $x+3=0 \Rightarrow x=-3$

$5(-3)+8 = A(2(-3)-1)+0B \Rightarrow -7=-7A \Rightarrow \boxed{A=1}$

Putting $2x-1=0 \Rightarrow x=1/2$

$5(1/2)+8 = 0A+B(\frac{1}{2}+3) \Rightarrow \frac{21}{2}=\frac{7}{2}B \Rightarrow \boxed{B=3}$

$\int \frac{5x+8\ dx}{(x+3)(2x-1)} = \int \frac{1\,dx}{x+3} + \int \frac{3\,dx}{2x-1}$

$= \ln|x+3| + \frac{3}{2}\int \frac{2\,dx}{2x-1}$

$= \ln|x+3| + \frac{3}{2}\ln|2x-1| + c$ Ans.

Q.3 $\int \frac{x^2+3x-34}{x^2+2x-15}dx$

$x^2+2x-15 \overline{)\,x^2+3x-34\,}(1$
$\underline{\ x^2+2x-15\ }$
$\quad x-19$

$= \int\left(1 + \frac{x-19}{x^2+2x-15}\right)dx$

$\frac{x-19}{x^2+2x-15} = \frac{x-19}{x^2+5x-3x-15} = \frac{x-19}{(x+5)(x-3)}$

$\Rightarrow \frac{x-19}{(x+5)(x-3)} = \frac{A}{x+5} + \frac{B}{x-3}$

$x-19 = A(x-3)+B(x+5)$

Putting $x+5=0 \Rightarrow x=-5$

$-5-19 = A(-5-3)+0B \Rightarrow -24=-8A \Rightarrow \boxed{A=3}$

Putting $x-3=0 \Rightarrow x=3$

$3-19 = 0A + B(3+5) \Rightarrow -16=8B \Rightarrow \boxed{B=-2}$

$\int \frac{x^2+3x-34}{x^2+2x-15}dx = \int\left(1 + \frac{3}{x+5} + \frac{-2}{x-3}\right)dx$

$= \int 1dx + \int \frac{3}{x+5}dx - \int \frac{2}{x-3}dx$

$= x + 3\int \frac{dx}{x+5} - 2\int \frac{dx}{x-3}$

$= x + 3\ln|x+5| - 2\ln|x-3| + c$ Ans.

Q.4 $\int \frac{(a-b)x}{(x-a)(x-b)} = \int \frac{A}{x-a}dx + \int \frac{B}{x-b}dx$

$\frac{(a-b)x}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$

$(a-b)x = A(x-b) + B(x-a)$

Putting $x-a=0 \Rightarrow x=a$

$(a-b)a = A(a-b)+0B \Rightarrow \boxed{A=a}$

Putting $x-b=0 \Rightarrow x=b$

$(a-b)b = 0A + (b-a) \Rightarrow \boxed{B=-b}$

$\int \frac{(a-b)x\,dx}{(x-a)(x-b)} = \int \frac{a}{x-a}dx + \int \frac{-b}{x-b}dx$

$= a\ln|x-a| - b\ln|x-b| + c$ Ans.

Q.5 $\int \frac{3-x}{1-x-6x^2}\,dx = \int \frac{3-x}{1-3x+2x-6x^2}$

$= \int \frac{3-x}{(1-3x)(1+2x)} = \int \frac{A}{1-3x}\,dx + \int \frac{B\,dx}{1+2x}$

$\frac{3-x}{(1-3x)(1+2x)} = \frac{A}{1-3x} + \frac{B}{1+2x}$

$3-x = A(1+2x) + B(1-3x)$

Putting $1-3x=0 \Rightarrow x = 1/3$

$3-\frac{1}{3} = A\left(1+\frac{2}{3}\right) + 0B \Rightarrow \frac{5}{3}A = \frac{8}{3} \Rightarrow \boxed{A = 8/5}$

Puti [illegible] $0 \Rightarrow x = -1/2$

$3-\left(-\frac{1}{2}\right) = 0A + B\left[1-3\left(-\frac{1}{2}\right)\right] \Rightarrow \frac{7}{2} = \frac{5}{2}B \Rightarrow \boxed{B = 7/5}$

$\int \frac{3-x}{1-x-6x^2}\,dx = \int \frac{8/5\,dx}{1-3x} + \int \frac{7/5\,dx}{1+2x}$

$= \frac{8}{5}\int \frac{dx}{1-3x} + \frac{7}{5}\int \frac{dx}{1+2x}$

$= -\frac{8}{3}\int \frac{-3\,dx}{1-3x} + \frac{7}{10}\int \frac{2\,dx}{1+2x}$

$= -\frac{8}{3}\ln|1-3x| + \frac{7}{10}\ln|1+2x| + C$ Ans

Q.6 $\int \frac{2x}{x^2-a^2}\,dx = \int \frac{2x\,dx}{(x-a)(x+a)}$

$\Rightarrow \int \frac{2x\,dx}{x^2-a^2} = \int \frac{A}{x-a}\,dx + \int \frac{B}{x+a}\,dx$

$\frac{2x}{x^2-a^2} = \frac{A}{x-a} + \frac{B}{x+a}$

$2x = A(x+a) + B(x-a)$

Putting $x-a=0 \Rightarrow x = a$

$2a = A(a+a) + 0B \Rightarrow 2aA = 2a \Rightarrow \boxed{A=1}$

Putting $x+a=0 \Rightarrow x = -a$

$-2a = 0A + B(-a-a) \Rightarrow -2a = -2aB \Rightarrow \boxed{B=1}$

$\int \frac{2x\,dx}{x^2-a^2} = \int \frac{1\,dx}{x-a} + \int \frac{1\,dx}{x+a}$

$= \ln|x-a| + \ln|x+a| + c$ Ans

Q.7 $\int \frac{dx}{6x^2+5x-4} = \int \frac{dx}{6x^2+8x-3x-4}$

$\int \frac{dx}{(2x-1)(3x+4)} = \int \frac{A}{2x-1}\,dx + \int \frac{B}{3x+4}\,dx$

$\frac{1}{(2x-1)(3x+4)} = \frac{A}{2x-1} + \frac{B}{3x+4}$

$1 = A(3x+4) + B(2x-1)$

Putting $2x-1=0 \Rightarrow x = 1/2$

$1 = A\left(\frac{3}{2}+4\right) + 0B \Rightarrow \frac{11}{2}A = 1 \Rightarrow \boxed{A = \frac{2}{11}}$

Putting $3x+4=0 \Rightarrow x = -4/3$

$1 = 0A + B\left(-\frac{8}{3}-1\right) \Rightarrow 1 = -\frac{11}{3}B \Rightarrow \boxed{B = -\frac{3}{11}}$

$\int \frac{dx}{6x^2+5x-4} = \int \frac{2/11}{2x-1}\,dx + \int \frac{-3/11}{3x+4}\,dx$

$= \frac{1}{11}\int \frac{2}{2x-1}\,dx - \frac{1}{11}\int \frac{3}{3x+4}\,dx$

$= \frac{1}{11}\ln|2x-1| - \frac{1}{11}\ln|3x+4| + C$

$= \frac{1}{11}\ln\left|\frac{2x-1}{3x+4}\right| + C$ Ans.

Q.8 $\int \frac{2x^3-3x^2-x-7}{2x^2-3x-2}\,dx$

$$\begin{array}{r|l} & x \\ 2x^2-3x-2 & 2x^3-3x^2-x-7 \\ & \mp 2x^3 \pm 3x^2 \pm 2x \\ & x-7 \end{array}$$

$= \int \left(x + \frac{x-7}{2x^2-3x-2}\right)dx$

$= \int \left(x + \frac{x-7}{(2x^2-4x+x-2)}\right)dx$

$= \int \left(x + \frac{x-7}{(2x+1)(x-2)}\right)dx$

$= \int x\,dx + \int \frac{A}{2x+1}\,dx + \int \frac{B}{x-2}\,dx$

$\frac{x-7}{(2x+1)(x-2)} = \frac{A}{2x+1} + \frac{B}{x-2}$

$x-7 = A(x-2) + B(2x+1)$

Putting $2x+1=0 \Rightarrow x = -1/2$

$-\frac{1}{2}-7 = A\left(-\frac{1}{2}-2\right) + 0B \Rightarrow -\frac{15}{2} = -\frac{5}{2}A \Rightarrow \boxed{A=3}$

Putting $x-2=0 \Rightarrow x = 2$

$2-7 = 0A + B(2(2)+1) \Rightarrow -5+5B \Rightarrow \boxed{B=-1}$

$\int \frac{2x^3-3x^2-x-7}{2x^2-3x-2}\,dx = \int x\,dx + \int \frac{3\,dx}{2x+1} + \int \frac{-1}{x-2}\,dx$

$$= \int x\,dx + 3\int \frac{dx}{2x+1} - \int \frac{dx}{x-2}$$

$$= \int x\,dx + \frac{3}{2}\int \frac{2\,dx}{2x+1} - \int \frac{dx}{x-2}$$

$$= \frac{x^2}{2} + \frac{3}{2}\ln|2x+1| - \ln|x-2| + c \quad Ans.$$

Q.9 $\int \frac{3x^2-12x+11\,dx}{(x-1)(x-2)(x-3)} = \int \frac{A\,dx}{x-1} + \int \frac{B\,dx}{x-2} + \int \frac{C\,dx}{x-3}$

$$\frac{3x^2-12x+11}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$3x^2-12x+11 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

Putting $x-1=0 \Rightarrow x=1$

$3-12+11 = A(1-2)(1-3) + 0B + 0C \Rightarrow 2 = 2A \Rightarrow \boxed{A=1}$

Putting $x-2=0 \Rightarrow x=2$

$3(2)^2-12(2)+11 = 0A + B(2-1)(2-3) + 0C \Rightarrow -1 = -B \Rightarrow \boxed{B=1}$

Putting $x-3=0 \Rightarrow x=3$

$3(3)^2-12(3)+11 = 0A + 0B + C(3-1)(3-2) \Rightarrow 2 = 2C \Rightarrow \boxed{C=1}$

$$\int \frac{3x^2-12x+11\,dx}{(x-1)(x-2)(x-3)} = \int \frac{1\,dx}{x-1} + \int \frac{1\,dx}{x-2} + \int \frac{1\,dx}{x-3}$$

$$= \ln|x-1| + \ln|x-2| + \ln|x-3| + c \quad Ans.$$

Q.10 $\int \frac{2x-1\,dx}{x(x-1)(x-3)} = \int \frac{A}{x}dx + \int \frac{B}{x-1}dx + \int \frac{C\,dx}{x-3}$

$$\frac{2x-1}{x(x-1)(x-3)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-3}$$

$$2x-1 = A(x-1)(x-3) + B(x-3)x + C(x-1)x$$

Putting $x=0$

$-1 = A(-1)(-3) + 0B + 0C \Rightarrow -1 = 3A \Rightarrow \boxed{A=-\frac{1}{3}}$

Putting $x-1=0 \Rightarrow x=1$

$2-1 = 0A + B(1-3)(1) + 0C \Rightarrow 1 = -2B \Rightarrow \boxed{B=-\frac{1}{2}}$

Putting $x-3=0 \Rightarrow x=3$

$2(3)-1 = 0A + 0B + C(3)(3-1) \Rightarrow 5 = 6C \Rightarrow \boxed{C=\frac{5}{6}}$

$$\int \frac{2x-1\,dx}{(x-1)(x-3)x} = \int \frac{-\frac{1}{3}}{x}dx + \int \frac{-\frac{1}{2}}{x-1}dx + \int \frac{\frac{5}{6}}{x-3}dx$$

$$= -\frac{1}{3}\ln|x| - \frac{1}{2}\ln|x-1| + \frac{5}{6}\ln|x-3| + c \quad Ans.$$

Q.11 $\int \frac{5x^2+9x+6}{(x^2-1)(2x+3)}dx = \int \frac{5x^2+9x+6\,dx}{(x-1)(x+1)(2x+3)}$

$$= \int \frac{A}{x-1}dx + \int \frac{B}{x+1}dx + \int \frac{C\,dx}{2x+3}$$

$$\frac{5x^2+9x+6}{(x^2-1)(2x+3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{2x+3}$$

$$5x^2+9x+6 = A(x+1)(2x+3) + B(x-1)(2x+3) + C(x^2-1)$$

Putting $x-1=0 \Rightarrow x=1$

$5+9+6 = A(1+1)(2+3) + 0B + 0C \Rightarrow 20 = 10A \Rightarrow \boxed{A=2}$

Putting $x+1=0 \Rightarrow x=-1$

$5-9+6 = 0A + B(-2)(+1) \Rightarrow 2 = -2B \Rightarrow \boxed{B=-1}$

Putting $2x+3=0 \Rightarrow x=-\frac{3}{2}$

$5\left(\frac{-3}{2}\right)^2 + 9\left(\frac{-3}{2}\right) + 6 = 0A + 0B + C\left(\left(\frac{-3}{2}\right)^2 - 1\right)$

$\frac{45}{4} - \frac{27}{2} + 6 = C\left(\frac{9}{4}-1\right) \Rightarrow \frac{15}{4} = \frac{5}{4}C \Rightarrow \boxed{C=3}$

$$\int \frac{5x^2+9x+6\,dx}{(x^2-1)(2x+3)} = \int \frac{2\,dx}{x-1} + \int \frac{-1\cdot dx}{x+1} + \int \frac{3\,dx}{2x+3}$$

$$= 2\ln|x-1| - \ln|x+1| + \frac{3}{2}\int \frac{2\,dx}{2x+3}$$

$$= 2\ln(x-1) - \ln(x+1) + \frac{3}{2}\ln(2x+3) + c$$

Q.12 $\int \frac{4+7x\,dx}{(1+x)^2(2+3x)} = \int \frac{A\,dx}{x+1} + \int \frac{B\,dx}{(1+x)^2} + \int \frac{C\,dx}{(2+3x)}$

$$\frac{4+7x}{(1+x)^2(2+3x)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(2+3x)}$$

$$4+7x = A(x+1)(2+3x) + B(2+3x) + C(x+1)^2$$

Putting $x+1=0 \Rightarrow x=-1$

$4+7(-1) = 0A + B(2+3(-1)) + 0C \Rightarrow -3 = -B \Rightarrow \boxed{B=3}$

Putting $2+3x=0 \Rightarrow x=-\frac{2}{3}$

$4+7\left(-\frac{2}{3}\right) = 0A+0B+C\left(-\frac{2}{3}+1\right)^2 \Rightarrow \frac{-2}{3} = C\cdot\frac{1}{9}$

$\Rightarrow \boxed{C = -6}$

Equating the Coefficients of x^2

$0 = 3A + C \Rightarrow A = \frac{-1}{3}C \Rightarrow \boxed{A = 2}$

$\int \frac{4+7x\,dx}{(x+1)^2(2+3x)} = \int \frac{2\,dx}{x+1} + \int \frac{3\,dx}{(x+1)^2} + \int \frac{-6\,dx}{(2+3x)}$

$= 2\int \frac{dx}{x+1} + 3\int (x+1)^{-2}\cdot 1\,dx - 2\int \frac{3\,dx}{(2+3x)}$

$= 2\ln(x+1) + \frac{3(x+1)^{-1}}{-1} - 2\ln|2+3x| + c$

$= 2\ln(x+1) - \frac{3}{(x+1)} + 2\ln(2+3x) + c$ Ans.

Q.13 $\int \frac{2x^2\,dx}{(x-1)^2(x+1)} = \int \frac{A\,dx}{x-1} + \int \frac{B\,dx}{(x-1)^2} + \int \frac{c\,dx}{(x+1)}$

$\frac{2x^2}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{c}{x+1}$

$2x^2 = A(x-1)(x+1) + B(x+1) + c(x-1)^2$

Putting $x-1=0 \Rightarrow x=1$

$2 = 0A + 2B \Rightarrow \boxed{B=1}$

Putting $x+1=0 \Rightarrow x=-1$

$2 = 0A+0B+C(-1-1)^2 \Rightarrow 2=4c \Rightarrow \boxed{C=\frac{1}{2}}$

Comparing the Coefficients of x^2

$2 = A + c \Rightarrow A = 2 - c = 2 - \frac{1}{2} \Rightarrow \boxed{\frac{3}{2} = A}$

$\int \frac{2x^2\,dx}{(x-1)^2(x+1)} = \int \frac{3/2\,dx}{x-1} + \int \frac{1\,dx}{(x-1)^2} + \int \frac{1/2\,dx}{x+1}$

$= \frac{3}{2}\int \frac{dx}{x-1} + \int (x-1)^{-2}dx + \frac{1}{2}\int \frac{dx}{x+1}$

$= \frac{3}{2}\ln|x-1| + \frac{(x-1)^{-1}}{-1} + \frac{1}{2}\ln(x+1) + c$

$= \frac{3}{2}\ln(x-1) - \frac{1}{(x-1)} + \frac{1}{2}\ln(x+1) + c$ Ans.

Q.14 $\int \frac{1\,dx}{(x-1)(x+1)^2} = \int \frac{A\,dx}{x-1} + \int \frac{dx(B)}{x+1} + \int \frac{c\,dx}{(x+1)^2}$

$\frac{1}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{c}{(x+1)^2}$

$1 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$

Putting $x-1=0 \Rightarrow x=1$

$1 = A(1+1)^2 \Rightarrow \boxed{A = 1/4}$

Putting $x+1=0 \Rightarrow x=-1$

$1 = 0A+0B+C(-1-1) \Rightarrow \boxed{C = \frac{-1}{2}}$

Comparing the Coefficients of x^2

$0 = A+B \Rightarrow B = -A \Rightarrow \boxed{B = -1/4}$

$\int \frac{1\,dx}{(x-1)(x+1)^2} = \int \frac{1/4\,dx}{x-1} + \int \frac{-1/4\,dx}{x+1} + \int \frac{-1/2\,dx}{(x+1)^2}$

$= \frac{1}{4}\int \frac{dx}{x-1} - \frac{1}{4}\int \frac{dx}{x+1} - \frac{1}{2}\int (x+1)^{-2}\cdot 1\,dx$

$= \frac{1}{4}\ln(x-1) - \frac{1}{4}\ln(x+1) - \frac{1}{2}\frac{(x+1)^{-1}}{-1} + c$

$= \frac{1}{4}\ln\left(\frac{x-1}{x+1}\right) - \frac{1}{2(x+1)} + c$ Ans.

Q.15 $\int \frac{x+4\,dx}{x^3-3x^2+4}$

$x=-1$

	1	−3	0	4	$\Rightarrow x^3-3x^2+4=0$
−1		−1	4	−4	
	1	−4	+4	0	

$(x+1)(x^2-4x+4) = x^3-3x^2+4$

$x^3-3x^2+4 = (x+1)(x-2)^2$

$\Rightarrow \int \frac{(x+4)dx}{x^3-3x^2+4} = \int \frac{x+4\,dx}{(x+1)(x-2)^2}$

$= \int \frac{A}{x+1}dx + \int \frac{B}{x-2}dx + \int \frac{c}{(x-2)^2}dx$

$$\frac{x+4}{x^3-3x^2+4} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$x+4 = A(x-2)^2 + B(x+1)(x-2) + C(x+1)$$

Putting $x+1=0 \Rightarrow x=-1$

$$-1+4 = A(-1-2)^2 + 0B + 0C \Rightarrow 3 = 9A \Rightarrow \boxed{A=1/3}$$

Putting $x-2=0 \Rightarrow x=2$

$$2+4 = 0A + 0B + C(2+1) \Rightarrow 6 = 3C \Rightarrow \boxed{C=2}$$

Comparing the Coefficients of x^2

$$0 = A + B \Rightarrow B = -A \Rightarrow \boxed{B=-1/3}$$

$$\int \frac{x+4\,dx}{x^3-3x^2+4} = \int \frac{1/3}{x+1}dx + \int \frac{-1/3\,dx}{x-2} + \int \frac{2}{(x-2)^2}dx$$

$$= \frac{1}{3}\int \frac{dx}{x+1} - \frac{1}{3}\int \frac{dx}{x-2} + 2\int (x-2)^{-2}.1\,dx$$

$$= \frac{1}{3}\ln(x+1) - \frac{1}{3}\ln(x-2) + 2\frac{(x-2)^{-1}}{-1} + C$$

$$= \frac{1}{3}\ln\left(\frac{x+1}{x-2}\right) - \frac{2}{x-2} + C \quad \text{Ans.}$$

Q.16 $$\int \frac{x^3-6x^2+25\,dx}{(x+1)^2(x-2)^2} = \int \frac{A\,dx}{x+1} + \int \frac{B\,dx}{(x+1)^2} + \int \frac{C\,dx}{(x-2)} + \int \frac{D\,dx}{(x-2)^2}$$

$$\frac{x^3-6x^2+25}{(x+1)^2(x-2)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-2} + \frac{D}{(x-2)^2}$$

$$x^3-6x^2+25 = A(x+1)(x-2)^2 + B(x-2)^2 + C(x-2)(x+1)^2 + D(x+1)^2$$

Putting $x+1=0 \Rightarrow x=-1$

$$-1-6+25 = 0A + B(-1-2)^2 + 0C + 0D \Rightarrow 18 = 9B \Rightarrow \boxed{B=2}$$

Putting $x-2=0 \Rightarrow x=2$

$$8-24+25 = 0A + 0B + 0C + D(2+1)^2 \Rightarrow 9 = 9D \Rightarrow \boxed{D=1}$$

Comparing the Coefficients of

$$x^3 \Rightarrow 1 = A + C \text{ —— (1)}$$

$$x^2 \Rightarrow -6 = -3A + B - 0C + D \text{ —— (2)}$$

$$-6 = -3A + (2) - 0 + 1 \Rightarrow \boxed{A=3}$$

$$A + C = 1 \Rightarrow C = 1 - A = 1 - 3 \Rightarrow \boxed{C=-2}$$

$$\Rightarrow \int \frac{x^3-6x^2+25\,dx}{(x+1)^2(x-2)^2} = \int \frac{3\,dx}{x+1} + \int \frac{2\,dx}{(x+1)^2} + \int \frac{-2\,dx}{x-2} + \int \frac{1\,dx}{(x-2)^2}$$

$$= 3\int \frac{dx}{x+1} + 2\int (x+1)^{-2}.1\,dx - 2\int \frac{dx}{x-2} + \int (x-2)^{-2}.1\,dx$$

$$= 3\ln|x+1| + \frac{2(x+1)^{-1}}{-1} - 2\ln(x-2) + \frac{(x-2)^{-1}}{-1} + C$$

$$= 3\ln(x+1) - \frac{2}{(x+1)} - 2\ln(x-2) - \frac{1}{(x-2)} + C \quad \text{Ans.}$$

Q.17 $$\int \frac{x^3+22x^2+14x-17}{(x-3)(x+2)^3}dx$$

$$= \int \frac{A}{x-3}dx + \int \frac{B\,dx}{x+2} + \int \frac{C\,dx}{(x+2)^2} + \int \frac{D\,dx}{(x+2)^3}$$

$$\frac{x^3+22x^2+14x-17}{(x-3)(x+2)^3} = \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2} + \frac{D}{(x+2)^3}$$

$$x^3+22x^2+14x-17 = A(x+2)^3 + B(x-3)(x+2)^2 + C(x+2)(x-3) + D(x-3)$$

Putting $x-3=0 \Rightarrow x=3$

$$27+198+42-17 = A(125) \Rightarrow 250 = 125A \Rightarrow \boxed{A=2}$$

Putting $x+2=0 \Rightarrow x=-2$

$$-8+88-28-17 = 0A + 0B + 0C + (-5)D \Rightarrow \boxed{D=-7}$$

Comparing the Coefficients of

$$x^3 \Rightarrow 1 = A + B \Rightarrow B = 1 - A \Rightarrow \boxed{B=-1}$$

$$x^2 \Rightarrow 22 = 6A + B + C \Rightarrow C = 22 - 6A - B$$

$$C = 22 - 12 + 1 \Rightarrow \boxed{C=11}$$

$$\Rightarrow \int \frac{2}{x-3}dx + \int \frac{-1\,dx}{x+2} + \int \frac{11\,dx}{(x+2)^2} + \int \frac{-7\,dx}{(x+2)^3}$$

$$= 2\int \frac{dx}{x-3} - \int \frac{dx}{x+2} + 11\int (x+2)^{-2}dx - 7\int (x+2)^{-3}.1\,dx$$

$$= 2\ln(x-3) - \ln(x+2) + 11\frac{(x+2)^{-1}}{-1} - 7\frac{(x+2)^{-2}}{-2} + C$$

$$= 2\ln(x-3) - \ln(x+2) - \frac{11}{(x+2)} + \frac{7}{2(x+2)^2} + C \quad \text{Ans.}$$

Q.18 $\int \frac{x-2\,dx}{(x+1)(x^2+1)} = \int \frac{A\,dx}{x+1} + \int \frac{Bx+C}{x^2+1}\,dx$

$\frac{x-2}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

$x-2 = A(x^2+1) + B(x^2+x) + C(x+1)$

Putting $x+1=0 \Rightarrow x=-1$

$-1-2 = A(1+1)+0+0 \Rightarrow \boxed{A=-3/2}$

Comparing the Coefficients of

$x^2 \Rightarrow 0 = A+B \Rightarrow B=-A \Rightarrow \boxed{B=3/2}$

$x \Rightarrow 1 = B+C \Rightarrow C = 1-B = 1-\frac{3}{2} \Rightarrow \boxed{C=-1/2}$

$\int \frac{x-2\,dx}{(x+1)(x^2+1)} = \int \frac{-3/2}{x+1}\,dx + \int \frac{\frac{3}{2}x-\frac{1}{2}}{x^2+1}\,dx$

$= -\frac{3}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{3x-1}{x^2+1}\,dx$

$= \frac{-3}{2}\ln(x+1) + \frac{1}{2}\int \frac{3x}{x^2+1}\,dx - \frac{1}{2}\int \frac{1\,dx}{1+x^2}$

$= \frac{-3}{2}\ln(x+1) + \frac{3}{4}\int \frac{2\,dx}{x^2+1} - \frac{1}{2}\int \frac{dx}{1+x^2}$

$= \frac{-3}{2}\ln(x+1) + \frac{3}{4}\ln(x^2+1) - \frac{1}{2}\tan^{-1}x + c$ Ans.

Q.19 $\int \frac{x\,dx}{(x-1)(x^2+1)} = \int \frac{A}{x-1}\,dx + \int \frac{Bx+C}{x^2+1}\,dx$

$\frac{x}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

$x = A(x^2+1) + B(x^2-x) + C(x-1)$

Putting $x-1=0 \Rightarrow x=1$

$1 = 2A+0B+0C \Rightarrow \boxed{A=1/2}$

Comparing the Coefficients of

$x^2 \Rightarrow 0 = A+B \Rightarrow B=-A \Rightarrow \boxed{B=-1/2}$

$x \Rightarrow 1 = -B+C \Rightarrow C = 1+B = 1-\frac{1}{2} \Rightarrow \boxed{C=1/2}$

$\int \frac{x\,dx}{(x-1)(x^2+1)} = \int \frac{1/2\,dx}{x-1} + \int \frac{-\frac{1}{2}x+\frac{1}{2}}{x^2+1}\,dx$

$= \frac{1}{2}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{x-1}{x^2+1}\,dx$

$= \frac{1}{2}\ln(x-1) - \frac{1}{4}\int \frac{2x}{x^2+1}\,dx + \frac{1}{2}\int \frac{1}{x^2+1}\,dx$

$= \frac{1}{2}\ln(x-1) - \frac{1}{4}\ln(x^2+1) + \frac{1}{2}\tan^{-1}x + c$ Ans.

Q.20 $\int \frac{9x-7\,dx}{(x+3)(x^2+1)} = \int \frac{A}{x+3}\,dx + \int \frac{Bx+C}{x^2+1}\,dx$

$\frac{9x-7}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$

$9x-7 = A(x^2+1) + B(x^2+3x) + C(x+3)$

Putting $x+3=0 \Rightarrow x=-3 \quad \Rightarrow \boxed{A=-17/5}$

$-27-7 = A(9+1) + 0B + 0C \Rightarrow 10A = -34$

Comparing the coefficients of

$x^2 \Rightarrow 0 = A+B \Rightarrow B=-A \Rightarrow \boxed{B=17/5}$

$x \Rightarrow 9 = 3B+C \Rightarrow C = 9-3\left(\frac{17}{5}\right)$

$C = 9-\frac{51}{5} = -6/5 \quad \boxed{C=-6/5}$

$\int \frac{9x-7\,dx}{(x+3)(x^2+1)} = \int \frac{-17/5\,dx}{x+3} + \int \frac{\frac{17}{5}x-\frac{6}{5}}{x^2+1}$

$= \frac{-17}{5}\int \frac{dx}{x+3} + \frac{1}{5}\int \frac{17x-6}{x^2+1}\,dx$

$= \frac{-17}{5}\ln(x+3) + \frac{1}{5}\int \frac{17x}{x^2+1}\,dx - \frac{6}{5}\int \frac{dx}{1+x^2}$

$= \frac{-17}{5}\ln(x+3) + \frac{17}{10}\int \frac{2\,dx}{x^2+1} - \frac{6}{5}\tan^{-1}x + c$

$= \frac{-17}{5}\ln(x+3) + \frac{17}{10}\ln(x^2+1) - \frac{6}{5}\tan^{-1}x + c$ Ans.

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Q.21 $\int \frac{(1+4x)\,dx}{(x-3)(x^2+4)} = \int \frac{A\,dx}{x-3} + \int \frac{Bx+C}{x^2+4}dx$

$\frac{1+4x}{(x-3)(x^2+4)} = \frac{A}{x-3} + \frac{Bx+C}{x^2+4}$

$1+4x = A(x^2+4) + B(x^2-3x) + C(x-3)$

Putting $x-3=0 \Rightarrow x=3$

$1+4(3) = A(3^2+4) + 0B + 0C \Rightarrow 13 = 13A \Rightarrow \boxed{A=1}$

Comparing the Coefficients of

$x^2 \Rightarrow 0 = A+B \Rightarrow B=-A \Rightarrow \boxed{B=-1}$

$x \Rightarrow 4 = -3B + C \Rightarrow C = 4+3(-1) \Rightarrow \boxed{C=1}$

$\int \frac{1+4x\,dx}{(x-3)(x^2+4)} = \int \frac{1\,dx}{x-3} + \int \frac{-x+1}{x^2+1}dx$

$= \ln(x-3) - \int \frac{x-1}{x^2+1}dx$

$= \ln(x-3) - \frac{1}{2}\int \frac{2x\,dx}{x^2+1} + \int \frac{dx}{x^2+1}$

$= \ln(x-3) - \frac{1}{2}\ln(x^2+1) + \tan^{-1}(x) + c$ Ans.

Q.22 $\int \frac{12\,dx}{x^3+8} = \int \frac{12\,dx}{(x+2)(x^2-2x+4)}$

$\int \frac{12\,dx}{x^3+8} = \int \frac{A\,dx}{x+2} + \int \frac{Bx+C}{x^2-2x+4}dx$

$\frac{12}{x^3+8} = \frac{A}{x+2} + \frac{Bx+C}{x^2-2x+4}$

$12 = A(x^2-2x+4) + B(x^2+2x) + C(x+2)$

Let $x+2=0 \Rightarrow x=-2$

$12 = A(4+4+4) \Rightarrow 12A = 12 \Rightarrow \boxed{A=1}$

Comparing the Coefficients of

$x^2 \Rightarrow 0 = A+B \Rightarrow B=-A \Rightarrow \boxed{B=-1}$

$x \Rightarrow 0 = -2A+2B+C \Rightarrow C = 2A-2B$

$C = 2+2 \Rightarrow \boxed{C=4}$

$\int \frac{12\,dx}{x^3+8} = \int \frac{1\,dx}{x+2} + \int \frac{-x+4}{x^2-2x+4}dx$

$= \int \frac{dx}{x+2} - \int \frac{x-4}{x^2-2x+4}dx$

$= \ln(x+2) - \frac{1}{2}\int \frac{2x-8}{x^2-2x+4}dx$

$= \ln(x+2) - \frac{1}{2}\int \frac{2x-2-6}{x^2-2x+4}dx$

$= \ln(x+2) - \frac{1}{2}\int \frac{2x-2}{x^2-2x+4} + \frac{6}{2}\int \frac{dx}{x^2-2x+4}$

$= \ln(x+2) - \frac{1}{2}\ln(x^2-2x+4) + 3\int \frac{dx}{x^2-2x+1+3}$

$= \ln(x+2) - \frac{1}{2}\ln(x^2-2x+4) + 3\int \frac{dx}{(x-1)^2+(\sqrt{3})^2}$

$= \ln(x+2) - \frac{1}{2}\ln(x^2-2x+4) + \frac{3}{\sqrt{3}}\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + c$

$= \ln(x+2) - \frac{1}{2}\ln(x^2-2x+4) + \sqrt{3}\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + c$ Ans.

Q.23 $\int \frac{9x+6\,dx}{x^3-8} = \int \frac{9x+6}{(x-2)(x^2+2x+4)}dx$

$\int \frac{9x+6}{x^3-8}dx = \int \frac{A\,dx}{x-2} + \int \frac{Bx+C}{x^2+2x+4}dx$

$\frac{9x+6}{x^3-8} = \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+4}$

$9x+6 = A(x^2+2x+4) + B(x^2-2x) + C(x-2)$

Putting $x-2=0 \Rightarrow x=2$

$9(2)+6 = A(4+4+4) + 0B + 0C \Rightarrow 24 = 12A \Rightarrow \boxed{A=2}$

Comparing the Coefficients of

$x^2 \Rightarrow 0 = A+B \Rightarrow B=-A \Rightarrow \boxed{B=-2}$

$x \Rightarrow 9 = 2A-2B+C \Rightarrow C = 9-2A+2B$

$C = 9-4-4 \Rightarrow \boxed{C=1}$

$\int \frac{9x+6\,dx}{x^3-8} = \int \frac{2\,dx}{x-2} + \int \frac{-2x+1}{x^2+2x+4}$

$$= 2\int\frac{dx}{x-2} - \int\frac{2x+2-3}{x^2+2x+4}\,dx$$

$$= 2\ln(x-2) - \int\frac{2x+2\,dx}{x^2+2x+4} + 3\int\frac{dx}{x^2+2x+1+3}$$

$$= 2\ln(x-2) - \ln|x^2+2x+4| + 3\int\frac{dx}{(x+1)^2+(\sqrt{3})^2}$$

$$= 2\ln(x-2) - \ln(x^2+2x+4) + \frac{3}{\sqrt{3}}\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right) + c \quad \text{Ans}$$

Q.24 $\int\frac{2x^2+5x+3\,dx}{(x-1)^2(x^2+4)}$

$$\frac{2x^2+5x+3}{(x-1)^2(x^2+4)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+4}$$

$$2x^2+5x+3 = A(x-1)(x^2+4) + B(x^2+4) + Cx(x-1)^2 + D(x-1)^2$$

Putting $x-1=0 \Rightarrow x=1$

$2+5+3 = 0A + B(1+4) + 0C + 0D \Rightarrow 10 = 5B \Rightarrow \boxed{B=2}$

Comparing the Coefficients of

$x^3 \Rightarrow 0 = A + C$ —— ①

$x^2 \Rightarrow 2 = -A + B - 2C + D$ —— ②

$x \Rightarrow 5 = 4A + C - 2D$ —— ③

From ② $\Rightarrow 2 = -A + 2 - 2C + D$

$-A - 2C + D = 0$

Multiplying by 2, we have

Adding $-2A - 4C + 2D = 0$ —— ④

$4A + C - 2D = 5$ —— ③

$2A - 3C = 5$ —— ⑤

$A + C = 0 \Rightarrow -A = C$

⑤ $\Rightarrow 2A - 3[-A] = 5$

$2A + 3A = 5 \Rightarrow 5A = 5 \Rightarrow \boxed{A=1}$

$C = -A \Rightarrow \boxed{C=-1}$

Putting A, B, C in ③, we have

$5 = 4 - 1 - 2D \Rightarrow 2D = 4 - 1 - 5$

$2D = -2 \Rightarrow \boxed{D=-1}$

$$= \int\frac{1\,dx}{x-1} + \int\frac{2\,dx}{(x-1)^2} + \int\frac{-x-1}{x^2+4}\,dx$$

$$= \int\frac{dx}{x-1} + 2\int(x-1)^{-2}dx - \frac{1}{2}\int\frac{2x\,dx}{x^2+4} - \int\frac{dx}{x^2+(2)^2}$$

$$= \ln(x-1) + \frac{2(x-1)^{-1}}{-1} - \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

$$= \ln(x-1) - \frac{2}{x-1} - \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c \quad \text{Ans}$$

Q.25 $\int\frac{2x^2-x-7\,dx}{(x+2)^2(x^2+x+1)}$

$$\frac{2x^2-x-7}{(x+2)^2(x^2+x+1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{Cx+D}{x^2+x+1}$$

$$2x^2-x-7 = A(x+2)(x^2+x+1) + B(x^2+x+1) + Cx(x+2)^2 + D(x+2)^2$$

Putting $x+2=0 \Rightarrow x=-2$

$8+2-7 = A(0) + B(4-2+1) + 0C + 0D \Rightarrow 3 = 3B$

$\boxed{B=1}$

Comparing the Coefficients of

$x^3 \Rightarrow 0 = A + C$ —— ①

$\boxed{\because B=1}$

$x^2 \Rightarrow 2 = 3A + B + 4C + D \Rightarrow 3A + 4C + D = 2 - 1 = 1$ —— ②

$x \Rightarrow -1 = 3A + B + 4C + 4D \Rightarrow 3A + 4C + 4D = -1 - 1 = -2$ —— ③

Subtracting ② & ③, we have

$\cancel{3A} + \cancel{4C} + D = 1$

$\cancel{3A} + \cancel{4C} + 4D = -2$

$-3D = 3 \Rightarrow \boxed{D=-1}$

Putting in ②, we have

$3A + 4C + (-1) = 1 \Rightarrow 3A + 4C = 2$ —— ④

From ① $\Rightarrow A = -C$ Putting in ④

$3(-C) + 4C = 2 \Rightarrow \boxed{C=2}$

$A = -C \Rightarrow \boxed{A=-2}$

$$= \int\frac{-2\,dx}{x+2} + \int\frac{1\,dx}{(x+2)^2} + \int\frac{2x+(-1)\,dx}{x^2+x+1}$$

$$= \int \frac{-2dx}{x+2} + \int (x+2)^{-2} \cdot 1 dx + \int \frac{2x+1-2}{x^2+x+1} dx$$

$$= -2\int \frac{dx}{x+2} + \int (x+2)^{-2} \cdot 1 dx + \int \frac{2x+1}{x^2+x+1} dx - 2\int \frac{dx}{x^2+x+\frac{1}{4}+\frac{3}{4}}$$

$$= -2\ln(x+2) + \frac{(x+2)^{-1}}{-1} + \ln(x^2+x+1) - 2\int \frac{dx}{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

$$= -2\ln(x+2) - \frac{1}{(x+2)} + \ln(x^2+x+1) - \frac{2}{(\sqrt{3}/2)} \tan^{-1}\left(\frac{x+1/2}{\sqrt{3}/2}\right) + c$$

$$= -2\ln(x+2) - \frac{1}{(x+2)} + \ln(x^2+x+1) - \frac{4}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c \quad \text{Ans.}$$

Q.26 $\int \frac{3x+1}{(4x^2+1)(x^2-x+1)} dx = \int \frac{Ax+B}{4x^2+1} dx + \int \frac{Cx+D}{x^2-x+1} dx$

$$\Rightarrow \frac{3x+1}{(4x^2+1)(x^2-x+1)} = \frac{Ax+B}{4x^2+1} + \frac{Cx+D}{x^2-x+1}$$

$$3x+1 = A(x^3-x^2+x) + B(x^2-x+1) + C(4x^3+x) + D(4x^2+1).$$

Comparing the Coefficients of

$x^3 \Rightarrow \quad 0 = A+4C$ —— ① $\qquad x^2 \Rightarrow \quad 0 = -A+B+4D$ —— ②

$x \Rightarrow \quad 3 = A-B+C$ —— ③ $\qquad$ Constant $\Rightarrow \quad 1 = B+D$ —— ④

From ① and ④ $\qquad A = -4C \qquad$ and $\qquad B = 1-D$

Putting in ② and ③ $\qquad 3 = (-4C)-(1-D)+C \Rightarrow -4C+D+C = 4$

$\Rightarrow \quad -3C+D = 4$ —— ⑤ $\qquad$ Also $\quad 0 = -(-4C)+(1-D)+4D$

$\Rightarrow \quad 4C+1-D+4D = 0 \quad \Rightarrow \quad 4C+3D = -1$ —— ⑥

Eq ⑥ $- 3\times$ Eq ⑤

$$\begin{aligned} 4C+3D &= -1 \\ -9C+3D &= 12 \\ \hline 13C &= -13 \end{aligned} \Rightarrow \boxed{C=-1}$$

Also $\quad A = -4C \Rightarrow A = -4(-1) \Rightarrow \boxed{A=4}$

Putting in ⑤ again $\quad -3(-1)+D = 4 \Rightarrow D = 4-3 \Rightarrow \boxed{D=1}$

Now $\quad B = 1-D \Rightarrow B = 1-1 \Rightarrow \boxed{B=0}$

Thus $\int \frac{3x+1}{(4x^2+1)(x^2-x+1)} dx = \int \frac{4x+0}{4x^2+1} dx + \int \frac{-x+1}{x^2-x+1} dx$

$$= \frac{1}{2}\int \frac{8x}{4x^2+1} dx - \frac{1}{2}\int \frac{2x-2}{x^2-x+1} dx$$

$$= \frac{1}{2}\ln(4x^2+1) - \frac{1}{2}\int \frac{2x-1}{x^2-x+1} + \frac{1}{2}\int \frac{dx}{x^2-x+\frac{1}{4}+\frac{3}{4}}$$

$$= \frac{1}{2}\ln(4x^2+1) - \frac{1}{2}\ln(x^2-x+1) + \frac{1}{2}\int \frac{dx}{(x-\frac{1}{2})^2+(\sqrt{3}/2)^2}$$

$$\Rightarrow \frac{1}{2}\ln\left(\frac{4x^2+1}{x^2-x+1}\right) + \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x-1/2}{\sqrt{3}/2}\right) + c$$

$$= \frac{1}{2}\ln\left(\frac{4x^2+1}{x^2-x+1}\right) + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c \quad \text{Ans.}$$

Q.27 $\displaystyle\int \frac{4x+1 \quad dx}{(x^2+4)(x^2+4x+5)} = \int \frac{Ax+B}{x^2+4}dx + \int \frac{Cx+D}{x^2+4x+5}$

$$\frac{4x+1}{(x^2+4)(x^2+4x+5)} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{x^2+4x+5}$$

$$4x+1 = A(x^3+4x^2+5x) + B(x^2+4x+5) + C(x^3+4x) + D(x^2+4)$$

Comparing the Coefficients of

$x^3 \Rightarrow 0 = A+C$ —— ① $x^2 \Rightarrow 0 = 4A+B+D$ —— ②

$x \Rightarrow 4 = 5A+4B+4C$ —— ③   Constants $\Rightarrow 1 = 5B+4D$ —— ④

From ① and ④ $\quad A = -C \quad$ and $\quad B = \frac{1-4D}{5}$

Putting in ② and ③

② $\Rightarrow \quad 0 = 4(-C) + \left(\frac{1-4D}{5}\right) + D$

$0 = -4C + \frac{1-4D}{5} + D$

$0 = -20C + 1 - 4D + 5D$

$20C + 4D - 5D = 1$

$20C - D = 1$ —— ⑤

③ $\Rightarrow \quad 4 = 5(-C) + 4\left(\frac{1-4D}{5}\right) + 4C$

$4 = -5C + \frac{4-16D}{5} + 4C$

$4 = -C + \frac{4-16D}{5}$

$20 = -5C + 4 - 16D$

$5C + 16D = 4 - 20$

$5C + 16D = -16$ —— ⑥

By Eq ⑤ $- 4\times$ Eq ⑥ we have

$$\begin{array}{r} 20C - D = 1 \\ \underline{-20C \pm 64D = \mp 64} \\ -65D = 65 \end{array} \Rightarrow \boxed{D = -1}$$

Also $B = \frac{1-4D}{5} \Rightarrow B = \frac{1-4(-1)}{5} \Rightarrow B = \frac{1+4}{5} = \frac{5}{5} \Rightarrow \boxed{B = 1}$

Putting in ⑤ $\quad 20C + 1 = 1 \Rightarrow 20C = 1-1 \Rightarrow 20C = 0 \Rightarrow \boxed{C = 0}$

As $C = -A$ or $A = -C \Rightarrow \boxed{A = 0}$

$$\int \frac{4x+1 \quad dx}{(x^2+4)(x^2+4x+5)} = \int \frac{0x+1}{x^2+4}dx + \int \frac{0x+(-1)}{x^2+4x+5}dx$$

$$= \int \frac{dx}{(x)^2+(2)^2} + \int \frac{-1\, dx}{x^2+4x+4+1}$$

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$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) - 1\int \frac{dx}{(x+2)^2+(1)^2}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) - \tan^{-1}\left(\frac{x+2}{1}\right) + C$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) - \tan^{-1}(x+2) + C \quad \text{Ans.}$$

Q.28 $\int \frac{6a^2\,dx}{(x^2+a^2)(x^2+4a^2)} = \int \frac{Ax+B}{x^2+a^2}dx + \int \frac{Cx+D}{x^2+4a^2}dx$

$$\frac{6a^2}{(x^2+a^2)(x^2+4a^2)} = \frac{Ax+B}{x^2+a^2} + \frac{Cx+D}{x^2+4a^2}$$

$$6a^2 = A(x^3+4a^2x)+B(x^2+4a^2)+C(x^3+a^2x)+D(x^2+a^2)$$

Comparing the Coefficients of

$x^3 \Rightarrow 0 = A+C$ —① $\quad x^2 \Rightarrow 0 = B+D$ —②

$x \Rightarrow 0 = 4a^2A + a^2C$ —③ $\quad$ Constant $\Rightarrow 6a^2 = 4a^2B + a^2D$ —④

From ① and ② $\quad A = -C \quad$ and $\quad B = -D$

Putting in ③ and ④

③ $\Rightarrow 0 = 4a^2(-C) + a^2C$

$0 = -4a^2C + a^2C$

$-3a^2C = 0 \Rightarrow \boxed{C=0}$

also $A = -C \Rightarrow \boxed{A=0}$

④ $\Rightarrow 6a^2 = 4a^2(-D)+a^2D$

$6a^2 = -4a^2D + a^2D$

$6a^2 = -3a^2D \Rightarrow D = \frac{6a^2}{-3a^2}$

$\boxed{D = -2}$

Also $B = -D \Rightarrow \boxed{B=2}$

Thus $\int \frac{6a^2\,dx}{(x^2+a^2)(x^2+4a^2)} = \int \frac{0x+2}{x^2+a^2}dx + \int \frac{0x-2}{x^2+4a^2}dx$

$$= 2\int \frac{dx}{x^2+a^2} - 2\int \frac{dx}{x^2+(2a)^2}$$

$$= \frac{2}{a}\tan^{-1}\left(\frac{x}{a}\right) - \frac{2}{2a}\tan^{-1}\left(\frac{x}{2a}\right) + c$$

$$= \frac{2}{a}\tan^{-1}(x/a) - \frac{1}{a}\tan^{-1}\left(\frac{x}{2a}\right) + c \quad \text{Ans.}$$

Q.29 $\int \frac{(2x^2-2)\,dx}{x^4+x^2+1} = \int \frac{2x^2-2\,dx}{x^4+2x^2+1-x^2} = \int \frac{2x^2-2\,dx}{(x^2+1)^2-(x)^2}$

$$= \int \frac{2x^2-2\,dx}{(x^2+1)^2-x^2} = \int \frac{2x^2-2\,dx}{(x^2-x+1)(x^2+x+1)}$$

$$\frac{2x^2-2}{(x^2-x+1)(x^2+x+1)} = \frac{Ax+B}{x^2-x+1} + \frac{Cx+D}{x^2+x+1}$$

$$2x^2-2 = A(x^3+x^2+x)+B(x^2+x+1)+C(x^3-x^2+x)+D(x^2-x+1)$$

Comparing the Coefficients of

$x^3 \Rightarrow 0 = A + C$ —— ① $\quad x^2 \Rightarrow 2 = A+B-C+D$ —— ②

$x \Rightarrow 0 = A+B+C-D$ —— ③ $\quad$ Constant $\Rightarrow -2 = B+D$ —— ④

③ $\Rightarrow (A+C)+B-D = 0$

$(0) + B - D = 0 \quad \because A+C=0$

$B - D = 0$ —— ⑤

Adding ① and ⑥, we have

$$\begin{aligned} A+\cancel{C} &= 0 \\ A-\cancel{C} &= 4 \\ \hline 2A &= 4 \Rightarrow \boxed{A=2} \end{aligned}$$

② $\Rightarrow A - C + (B+D) = 2$

$A - C + (-2) = 2$

$A - C = 2 + 2 = 4$

$A - C = 4$ —— ⑥

Adding ⑤ and ④

$$\begin{aligned} B+\cancel{D} &= -2 \\ B-\cancel{D} &= 0 \\ \hline 2B &= -2 \Rightarrow \boxed{B=-1} \end{aligned}$$

Putting $A=2$ and $B=-1$ in ⑤ and ⑥

$B - D = 0$

$-1 - D = 0 \Rightarrow \boxed{D = -1}$

$A - C = 4$

$2 - C = 4 \Rightarrow C = 2-4$

$\boxed{C = -2}$

Thus $\displaystyle\int \frac{2x^2-2\,dx}{x^4+x^2+1} = \int \frac{2x-1}{x^2-x+1}dx + \int \frac{-2x-1}{x^2+x+1}dx$

$$= \int \frac{2x-1}{x^2-x+1}dx - \int \frac{2x+1}{x^2+x+1}dx$$

$$= \ln(x^2-x+1) - \ln(x^2+x+c) + C \quad \text{Ans}$$

$$= \ln\left(\frac{x^2-x+1}{x^2+x+1}\right) + c \quad \text{Ans.}$$

Q:30 $\displaystyle\int \frac{3x-8\ \ dx}{(x^2-x+2)(x^2+x+2)} = \int \frac{Ax+B\ \ dx}{x^2-x+2} + \int \frac{Cx+D}{x^2+x+2}dx$

$$\frac{3x-8}{(x^2-x+2)(x^2+x+2)} = \frac{Ax+B}{x^2-x+2} + \frac{Cx+D}{x^2+x+2}$$

$3x-8 = A(x^3+x^2+2x) + B(x^2+x+2) + C(x^3-x^2+2x) + D(x^2-x+2)$

Comparing the Coefficients of

$x^3 \Rightarrow 0 = A + C$ —— ① $\quad x^2 \Rightarrow 0 = A+B-C+D$ —— ②

$x \Rightarrow 3 = 2A+B+2C-D$ —— ③ $\quad$ Constant $\Rightarrow -8 = 2B+2D$

$B+D = -4$ —— ④

③ $\Rightarrow 3 = 2(A+C) + B - D$

$3 = 2(0) + B - D \quad (\because A+C=0)$

$B - D = 3$ —— ⑤

② $\Rightarrow A - C + (B+D) = 0$

$A - C + (-4) = 0 \quad (\because B+D=-4)$

$A - C = 4$ —— ⑥

Adding ① and ⑥

$$A + C = 0$$
$$A - C = 4$$
$$\overline{2A = 4} \Rightarrow \boxed{A = 2}$$

Adding ④ and ⑤

$$B + D = -4$$
$$B - D = 3$$
$$\overline{2B = -1} \Rightarrow \boxed{B = -\tfrac{1}{2}}$$

$\because A + C = 0 \Rightarrow \boxed{C = -2}$

$B - D = 3 \Rightarrow D = B - 3 = \frac{-1}{2} - 3 =$ $\boxed{D = \frac{-7}{2}}$

Thus
$$\int \frac{3x-8}{(x^2-x+2)(x^2+x+2)}\,dx = \int \frac{2x - \frac{1}{2}\,dx}{x^2-x+2} + \int \frac{-2x-\frac{7}{2}}{x^2+x+2}\,dx$$

$$= \int \frac{2x-1+(1-\frac{1}{2})\,dx}{x^2-x+2} - \int \frac{2x+1-1+\frac{7}{2}}{x^2+x+2}\,dx$$

$$= \int \frac{2x-1}{x^2-x+2}\,dx + \int \frac{\frac{1}{2}\,dx}{x^2-x+2} - \int \frac{2x+1\,dx}{x^2+x+2} - \int \frac{\frac{5}{2}\,dx}{x^2+x+2}$$

$$= \ln(x^2-x+2) + \frac{1}{2}\int \frac{dx}{x^2-x+\frac{1}{4}+\frac{7}{4}} - \ln(x^2+x+2) - \frac{5}{2}\int \frac{dx}{x^2+x+\frac{1}{4}+\frac{7}{4}}$$

$$= \ln\left(\frac{x^2-x+2}{x^2+x+2}\right) + \frac{1}{2}\int \frac{dx}{(x-\frac{1}{2})^2+(\frac{\sqrt{7}}{2})^2} - \frac{5}{2}\int \frac{dx}{(x+\frac{1}{2})^2+(\frac{\sqrt{7}}{2})^2}$$

$$= \ln\left(\frac{x^2-x+2}{x^2+x+2}\right) + \frac{1}{2}\cdot\frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\sqrt{7}/2}\right) - \frac{5.2}{2\sqrt{7}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\sqrt{7}/2}\right) + c$$

$$= \ln\left(\frac{x^2-x+2}{x^2+x+2}\right) + \frac{1}{\sqrt{7}}\tan^{-1}\left(\frac{2x-1}{\sqrt{7}}\right) - \frac{5}{\sqrt{7}}\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right) + c \quad \text{Ans.}$$

Definate Integration

If $f'(x)$ is a derivative of $F(x)$ then

$$\int_a^b f'(x)\,dx = F(b) - F(a)$$

is called its definate integration without arbitrary constant." where a, b are called lower and upper limits respectively.

Indefinate integration

If $f'(x)$ is a derivative of $F(x)$ then

$$\int f'(x)\,dx = F(x) + c$$

is called indefinate integration of $f'(x)$ having an arbitrary constant c also called constant of integration