

Exercise: 3.3

Q.1 $\int \frac{-2x}{\sqrt{4-x^2}} dx$

$$= \int (4-x^2)^{-\frac{1}{2}} (-2x) dx$$

$$= \frac{(4-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= 2(4-x^2)^{\frac{1}{2}} + c \quad \underline{\text{Ans.}}$$

Q.2 $\int \frac{dx}{x^2+4x+13}$

Put $x+2=z$
 $dx=dz$

$$= \int \frac{dx}{(x+2)^2 + (3)^2}$$

$$\int \frac{dz}{z^2+3^2}$$

By back substitution

$$= \frac{1}{3} \tan^{-1}\left(\frac{z}{3}\right) + c$$

$$= \frac{1}{3} \tan^{-1}\left(\frac{x+2}{3}\right) + c \quad \underline{\text{Ans.}}$$

Q.3 $\int x\sqrt{x+a} dx$

Put $\sqrt{x+a}=z$
 $x+a=z^2$
 $x=z^2-a$
 $dx=2zdz$

$$= \int (z^2-a)z \cdot 2zdz$$

$$= 2 \int (z^2-a)z^2 dz$$

$$= 2 \int z^4 dz - 2 \int z^2 a dz$$

$$= \frac{2z^5}{5} - \frac{2a z^3}{3} + c$$

By back substitutions.

$$= \frac{2}{5} (\sqrt{x+a})^5 - \frac{2a}{3} (\sqrt{x+a})^3 + c \quad \underline{\text{Ans.}}$$

Q.4 $\int \frac{x^2}{4+x^2} dx$

$$= \int \frac{x^2+4-4}{x^2+4} dx$$

$$= \int \frac{x^2+4}{x^2+4} dx - \int \frac{4}{x^2+4} dx$$

$$= \int dx - 4 \int \frac{dx}{x^2+(2)^2}$$

$$= x - 4 \cdot \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + c$$

$$= x - 2 \tan^{-1}\left(\frac{x}{2}\right) + c \quad \underline{\text{Ans.}}$$

Q.5 $\int \frac{1}{x \ln x} dx$

Let $\ln x = z$
 $\frac{1}{x} dx = dz$

$$\int \frac{1}{\ln x} \cdot \left(\frac{1}{x} dx\right)$$

$$\int \frac{1}{z} dz$$

Putting $z = \ln x$ back

$$= |\ln z| + c$$

$$= |\ln |\ln x|| + c \quad \underline{\text{Ans.}}$$

Q.6 $\int \frac{e^x}{e^x+3} dx$

Let $e^x+3=z$
 $e^x dx = dz$

$$= \int \frac{dz}{z}$$

$$= \ln z + c$$

Putting $e^x+3=z$ back

$$= \ln |e^x+3| + c \quad \underline{\text{Ans.}}$$

- * $\sqrt{x^2-a^2}$ - putting $x = a \sec \theta$ or $x = a \csc \theta$
- * $\sqrt{a^2-x^2}$ put $x = a \cos \theta$ or $x = a \sin \theta$
- * $\sqrt{a^2+x^2}$ put $x = a \tan \theta$ or $x = a \cot \theta$

سی ایس ایس اکیڈمی
شاہ پور کھیاں
TAHIR MAHMOOD

سی ایس ایس اکیڈمی
شاہ پور کھیاں
TAHIR MAHMOOD

Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

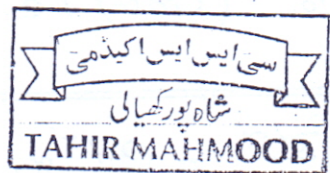
Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

Q.7 $\int \frac{(x+b) dx}{(x^2+2bx+c)^{1/2}}$

$= \frac{1}{2} \int (x^2+2bx+c)^{-1/2} \cdot (2x+2b) \cdot dx$
 $= \frac{1}{2} \int \frac{(x^2+2bx+c)^{-1/2+1}}{-1/2+1} + c$
 $= \frac{1}{2} [2(x^2+2bx+c)] + c$
 $= \sqrt{x^2+2bx+c} + c$ Ans.

Q.8 $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$ Let $\tan x = z$
 $\sec^2 x dx = dz$
 $= \int \frac{dz}{\sqrt{z}}$
 $= \int z^{-1/2} dz$
 $= \frac{z^{-1/2+1}}{-1/2+1} + c$
 $= 2z^{1/2} + c \Rightarrow 2\sqrt{z} + c$
 Put $z = \tan x$ back
 $= 2\sqrt{\tan x} + c$ Ans.

Q.9 $\int \frac{3x(2x^2+1) dx}{x^4+x^2+1}$
 $= \int \frac{6x^3+3x}{x^4+x^2+1} dx$
 $= \frac{3}{2} \int \frac{4x^3+2x}{x^4+x^2+1} dx$
 $= \frac{3}{2} \cdot \ln|x^4+x^2+1| + c$
 $= \frac{3}{2} \ln|x^4+x^2+1| + c$ Ans.



Q.10 @ Show that $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln|x+\sqrt{x^2-a^2}| + c$

LHS = $\int \frac{dx}{\sqrt{x^2-a^2}}$
 Let $x = a \sec \theta \Rightarrow dx = a \sec \theta \tan \theta d\theta$
 $= \int \frac{a \sec \theta \tan \theta d\theta}{\sqrt{a^2 \sec^2 \theta - a^2}} \Rightarrow \int \frac{a \sec \theta \tan \theta d\theta}{a^2 (\sec^2 \theta - 1)}$
 $= \int \frac{a \sec \theta \tan \theta d\theta}{a \sqrt{\tan^2 \theta}} \Rightarrow \int \frac{a \sec \theta \tan \theta d\theta}{a \tan \theta}$
 $= \int \sec \theta d\theta$
 $= \ln|\sec \theta + \tan \theta| + c_1$
 $\sec \theta = \frac{x}{a} \Rightarrow \tan \theta = \sqrt{\sec^2 \theta - 1}$
 $\tan \theta = \frac{\sqrt{x^2-a^2}}{a}$
 $= \ln\left|\frac{x}{a} + \frac{\sqrt{x^2-a^2}}{a}\right| + c_1$
 $= \ln\left|\frac{x+\sqrt{x^2-a^2}}{a}\right| + c_1$
 $= \ln(x+\sqrt{x^2-a^2}) + (c_1 - \ln a)$
 where $c_1 - \ln a = c$ (any constant)
 $= \ln(x+\sqrt{x^2-a^2}) + c$

Thus $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln(x+\sqrt{x^2-a^2}) + c$ (Proved)

(b) $\int \sqrt{a^2-x^2} dx$
 Let $x = a \sin \theta \Rightarrow dx = a \cos \theta d\theta$
 $= \int \sqrt{a^2-a^2 \sin^2 \theta} \cdot a \cos \theta d\theta$
 $= \int \sqrt{a^2(1-\sin^2 \theta)} \cdot a \cos \theta d\theta$
 $= \int a^2 \sqrt{\cos^2 \theta} \cos \theta d\theta$
 $= a^2 \int \cos^3 \theta d\theta$
 $= a^2 \int \frac{1+\cos 2\theta}{2} d\theta$

$\therefore \cos^2 \theta = \frac{1+\cos 2\theta}{2}$
 $\cos^3 \theta = \frac{1+\cos 2\theta}{2} \cdot \cos \theta$

Tahir Mahmood
 M.Sc. (Maths)
 Mob No. 93007414159

$$Q.12 \int \frac{1}{(1+x^2)\tan^{-1}x} dx \quad \text{let}$$

$$\tan^{-1}x = z$$

$$= \int \frac{1}{\tan^{-1}x} \cdot \left(\frac{1}{1+x^2} dx\right) \quad \frac{1}{1+x^2} dx = dz$$

$$\int \frac{dz}{z}$$

$$= (n|z| + c) \quad \text{Putting } z = \tan^{-1}x \text{ back.}$$

$$= (n|\tan^{-1}x| + c)$$

$$Q.13 \int \sqrt{\frac{1+x}{1-x}} dx$$

$$= \int \sqrt{\frac{1+x}{1-x}} \times \frac{\sqrt{1+x}}{\sqrt{1+x}} dx$$

$$= \int \frac{1+x}{\sqrt{1-x^2}} dx$$

$$= \int \frac{1}{\sqrt{1-x^2}} dx + \int \frac{x}{\sqrt{1-x^2}} dx$$

$$= \int \frac{1}{\sqrt{1-x^2}} dx - \frac{1}{2} \int (1-x^2)^{-\frac{1}{2}} (-2x) dx$$

$$= \sin^{-1}x - \frac{1}{2} \cdot \frac{(1-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= \sin^{-1}x - \frac{2}{2} (1-x^2)^{\frac{1}{2}} + c$$

$$= \sin^{-1}x - \sqrt{1-x^2} + c \quad \text{Ans}$$

$$Q.14 \int \frac{\sin \theta}{1+\cos^2 \theta} d\theta \quad \text{let}$$

$$\cos \theta = z$$

$$-\sin \theta d\theta = dz$$

$$\sin \theta d\theta = -dz$$

$$\int \frac{-dz}{1+z^2}$$

$$= \cot^{-1}(z) + c \quad \text{putting } z = \cos \theta \text{ back.}$$

$$= \cot^{-1}(\cos \theta) + c \quad \text{Ans}$$

$$\left(\therefore \int \frac{-dx}{1+x^2} = \cot^{-1}x + c \right)$$

$$= \frac{a^2}{2} \int d\theta + \frac{a^2}{2} \int \cos 2\theta d\theta$$

$$= \frac{a^2}{2} \theta + \frac{a^2}{2} \cdot \frac{\sin 2\theta}{2} + c$$

$$= \frac{a^2}{2} \theta + \frac{a^2}{4} \cdot 2 \sin \theta \cos \theta + c$$

$$= \frac{a^2}{2} \theta + \frac{a^2}{2} \sin \theta \cos \theta + c$$

$$\text{Now } x = a \sin \theta \Rightarrow \sin \theta = \frac{x}{a}$$

$$\theta = \sin^{-1}\left(\frac{x}{a}\right)$$

$$\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \frac{x^2}{a^2}}$$

$$\cos \theta = \frac{\sqrt{a^2 - x^2}}{a}$$

$$= \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + \frac{a^2}{2} \cdot \frac{x}{a} \cdot \frac{\sqrt{a^2 - x^2}}{a} + c$$

$$= \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2} \sqrt{a^2 - x^2} + c$$

(Proved)

$$Q.11 \int \frac{dx}{(1+x^2)^{3/2}}$$

$$\text{let } x = \tan \theta$$

$$dx = \sec^2 \theta d\theta$$

$$= \int \frac{\sec^2 \theta d\theta}{(1+\tan^2 \theta)^{3/2}}$$

$$= \int \frac{\sec^2 \theta d\theta}{(\sec^2 \theta)^{3/2}} \quad \because 1+\tan^2 \theta = \sec^2 \theta$$

$$= \int \frac{\sec^2 \theta}{\sec^3 \theta} d\theta \Rightarrow \int \frac{1}{\sec \theta} d\theta$$

$$= \int \cos \theta d\theta$$

$$= \sin \theta + c$$

$$= \frac{x}{\sqrt{x^2+1}} + c$$

Ans.

Now

$$\cot \theta = \frac{1}{x}$$

$$\sqrt{1+\cot^2 \theta} = \csc \theta$$

$$\frac{\sqrt{x^2+1}}{x} = \csc \theta$$

$$\sin \theta = \frac{x}{\sqrt{x^2+1}}$$

Q.15 $\int \frac{ax}{\sqrt{a^2-x^2}} dx$

Let $x^2 = z$

$2x dx = dz$

$\int \frac{adz}{2\sqrt{a^2-z}}$

$x dx = \frac{dz}{2}$

$= \frac{a}{2} \int \frac{dz}{\sqrt{a^2-z}}$

Tahir Mahmood

M.Sc. (Math)

Mob No: 0345-6510779

$= \frac{a}{2} \sin^{-1}\left(\frac{z}{a}\right) + c$

Putting $z = x^2$ back.

$= \frac{a}{2} \sin^{-1}\left(\frac{x^2}{a}\right) + c$ Ans.

Q.16 $\int \frac{dx}{\sqrt{7-6x-x^2}}$

$= \int \frac{dx}{\sqrt{7+9-9-6x-x^2}}$

$= \int \frac{dx}{\sqrt{16-(x^2+6x+9)}} \Rightarrow \int \frac{dx}{\sqrt{(4)^2-(x+3)^2}}$

$= \sin^{-1}\left(\frac{x+3}{4}\right) + c$ Ans.

Q.17 $\int \frac{\cos x}{\sin x |\ln \sin x|} dx$

Let $(\ln |\sin x|) = z$

$= \int \frac{1}{(\ln |\sin x|)} \cdot \left(\frac{\cos x}{\sin x} dx\right)$

$\frac{1}{\sin x} \cdot \cos x dx = dz$

$= \int \frac{1}{z} dz$

$\frac{\cos x}{\sin x} dx = dz$

$= \ln |z| + c$

Putting $z = (\ln |\sin x|)$ back.

$= (\ln |\ln \sin x|) + c$ Ans.

Q.18 $\int \cos x \left(\frac{\ln \sin x}{\sin x}\right) dx$

$= \int \left(\frac{\cos x}{\sin x}\right) dx \cdot \ln \sin x$

Let $z = \ln \sin x$

$dz = \frac{\cos x}{\sin x} dx$

$= \int z dz \Rightarrow \frac{z^2}{2} + c$

$\frac{1}{2} (\ln |\sin x|)^2 + c$ Ans.

Q.19 $\int \frac{x dx}{4+2x+x^2}$

$= \frac{1}{2} \int \frac{2x}{x^2+2x+4} dx$

$= \frac{1}{2} \int \frac{2x+2-2}{x^2+2x+4} dx$

$= \frac{1}{2} \int \frac{2x+2}{x^2+2x+4} - \frac{2}{2} \int \frac{dx}{x^2+2x+4}$

$= \frac{1}{2} \int \frac{2x+2}{x^2+2x+4} - \int \frac{dx}{x^2+2x+1+3}$

$= \frac{1}{2} (\ln |x^2+2x+4|) - \int \frac{dx}{(x+1)^2 + (\sqrt{3})^2}$

$= \frac{1}{2} (\ln |x^2+2x+4|) - \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right) + c$ Ans.

Q.20 $\int \frac{x}{x^4+2x^2+5} dx$

$= \int \frac{dz/2}{z^2+2z+5}$ Let $x^2 = z$

$2x dx = dz$

$= \frac{1}{2} \int \frac{dz}{z^2+2z+1+4}$ $x dx = \frac{dz}{2}$

$= \frac{1}{2} \int \frac{dz}{(z+1)^2 + (2)^2}$

$= \frac{1}{2} \cdot \frac{1}{2} \tan^{-1}\left(\frac{z+1}{2}\right) + c$

$= \frac{1}{4} \tan^{-1}\left(\frac{z+1}{2}\right) + c$

Putting $z = x^2$ back, we have

$= \frac{1}{4} \tan^{-1}\left(\frac{x^2+1}{2}\right) + c$ Ans.

Q.21 $\int \sin^5 x dx$

$= \int \sin^4 x \cdot \sin x dx$

$= \int (\sin^2 x)^2 \cdot \sin x dx$

$\int (1-\cos^2 x)^2 \cdot \sin x dx$

$\int (1+\cos^2 x - 2\cos^2 x) \sin x dx$

سی ایس ایم

شاہ پورکیاں

TAHIR MAHMOOD

TAHIR

$$= \int \sin x dx + \int \cos^4 x \sin x dx - 2 \int \cos^2 x \sin x dx$$

$$= \int \sin x dx - \int \cos^4 x (-\sin x) dx + 2 \int \cos^2 x (-\sin x) dx$$

$$= -\cos x - \frac{\cos^5 x}{5} + \frac{2}{3} \cos^3 x + C$$

$$= -\cos x - \frac{1}{5} \cos^5 x + \frac{2}{3} \cos^3 x + C \text{ Ans.}$$

Q22 $\int \cos^7 x dx$

$$= \int \cos^6 x \cos x dx \Rightarrow \int (\cos^2 x)^3 \cos x dx$$

$$= \int (1 - \sin^2 x)^3 \cos x dx$$

$$= \int (1 - \sin^6 x - 3 \sin^4 x + 3 \sin^2 x) \cos x dx$$

$$= \int \cos x dx - \int \sin^6 x \cos x dx - 3 \int \sin^4 x \cos x dx + 3 \int \sin^2 x \cos x dx$$

$$= \sin x - \frac{1}{7} \sin^7 x - \frac{3}{3} \sin^3 x + \frac{3}{5} \sin^5 x + C$$

$$= \sin x - \frac{1}{7} \sin^7 x - \sin^3 x + \frac{3}{5} \sin^5 x + C \text{ Ans.}$$

Q23 $\int \cos(\sqrt{x} - \frac{x}{2}) \cdot (\frac{1}{\sqrt{x}} - 1) dx$

let $\sqrt{x} - \frac{x}{2} = z$

$$\frac{1}{2\sqrt{x}} - \frac{1}{2} dx = dz$$

$$\frac{dx}{\sqrt{x}} (\frac{1}{\sqrt{x}} - 1) dx = 2 dz$$

$$(\frac{1}{\sqrt{x}} - 1) dx = 2 dz$$

$$\int 2 \cos z dz \Rightarrow 2 \sin z + C$$

$$= 2 \sin(\sqrt{x} - \frac{x}{2}) + C \text{ Ans.}$$

Q24 $\int \frac{x+2}{\sqrt{x+3}} dx$ let $\sqrt{x+3} = z$

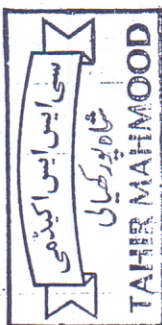
$$x+3 = z^2$$

$$x = z^2 - 3$$

$$dx = 2z dz$$

$$= \int \frac{(z^2 - 3 + 2)}{z} 2z dz$$

$$= \int (z^2 - 1) 2 dz$$



Tahir Mahmood
M.Sc. (Math)
Mob No: 0345-6510779

$$= 2 \int z^2 dz - 2 \int dz$$

$$= 2 \frac{z^3}{3} - 2z + C$$

Putting $z = \sqrt{x+3}$ back

$$= \frac{2}{3} (\sqrt{x+3})^3 - 2\sqrt{x+3} + C \text{ Ans.}$$

Q25 $\int \frac{\sqrt{2}}{\sin x + \cos x} dx$

Multipl. and dividing by $\sqrt{2}$ Den.

$$= \int \frac{\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x}{(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x) \sqrt{2}} dx$$

$$= \int \frac{dx}{\cos x \frac{1}{\sqrt{2}} + \sin x \frac{1}{\sqrt{2}}}$$

$\therefore \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$= \int \frac{dx}{\cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4}}$$

$\therefore \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

$$= \int \frac{dx}{\cos(x - \frac{\pi}{4})} \Rightarrow \int \sec(x - \frac{\pi}{4}) dx$$

$$= \ln | \sec(x - \frac{\pi}{4}) + \tan(x - \frac{\pi}{4}) | + C$$

Ans.

Q26 $\int \frac{dx}{\cos x \frac{\sqrt{3}}{2} + \sin x \cdot \frac{1}{2}}$

$\therefore \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ & $\sin \frac{\pi}{6} = \frac{1}{2}$

$$= \int \frac{dx}{\cos x \cos \frac{\pi}{6} + \sin x \sin \frac{\pi}{6}}$$

$$= \int \frac{dx}{\cos(x - \frac{\pi}{6})} \Rightarrow \int \sec(x - \frac{\pi}{6}) dx$$

$$= \ln | \sec(x - \frac{\pi}{6}) + \tan(x - \frac{\pi}{6}) | + C$$

Ans.

Q.27 $\int x^3 \sqrt{1+x^2} dx$

let $x = \tan \theta$

$dx = \sec^2 \theta d\theta$

$= \int \tan^3 \theta \cdot \sqrt{1+\tan^2 \theta} \sec^2 \theta d\theta$
 $= \int \tan^3 \theta \cdot \sec \theta \sec^2 \theta d\theta$
 $= \int \tan^3 \theta \cdot \sec \theta \sec^2 \theta d\theta$
 $= \int \tan^3 \theta \sec^3 \theta d\theta$
 $= \int \tan^2 \theta \sec^2 \theta \cdot (\tan \theta \sec \theta) d\theta$
 $= \int (\sec^2 \theta - 1) \sec^2 \theta (\tan \theta \sec \theta) d\theta$
 $= \int (\sec^4 \theta - \sec^2 \theta) \tan \theta \sec \theta d\theta$
 $= \int \sec^4 \theta (\tan \theta \sec \theta) d\theta - \int \sec^2 \theta (\tan \theta \sec \theta) d\theta$
 $= \frac{1}{5} \sec^5 \theta - \frac{1}{3} \sec^3 \theta + c$

Now $x = \tan \theta \Rightarrow \sec \theta = \sqrt{1+\tan^2 \theta}$

$\sec \theta = \sqrt{1+x^2}$

$= \frac{1}{5} (\sqrt{1+x^2})^5 - \frac{1}{3} (\sqrt{1+x^2})^3 + c$
 $= \frac{1}{5} (1+x^2)^{5/2} - \frac{1}{3} (1+x^2)^{3/2} + c$ Ans.

Q.28 $\int x \sqrt{1+x} dx$ let $\sqrt{1+x} = z$

$\int (z^2-1) z \cdot (2z dz)$ $1+x = z^2$

$= \int (z^2-1) 2z^2 dz$ $x = z^2 - 1$

$= \int (2z^4 - 2z^2) dz$ $dx = 2z dz$

$= 2 \int z^4 dz - 2 \int z^2 dz$

$= \frac{2}{5} z^5 - \frac{2}{3} z^3 + c$ Putting $z = \sqrt{1+x}$ back.

$= \frac{2}{5} (\sqrt{1+x})^5 - \frac{2}{3} (\sqrt{1+x})^3 + c$

$= \frac{2}{5} (1+x)^{5/2} - \frac{2}{3} (1+x)^{3/2} + c$ Ans.

Q.29 $\int dx$

let $x = \cos \theta$

$dx = -\sin \theta d\theta$

$= \int \frac{-\sin \theta d\theta}{(1+\cos \theta)^{3/2} (1-\cos \theta)^{1/2}}$

$= \int \frac{-\sin \theta d\theta}{(2\cos^2 \frac{\theta}{2})^{3/2} (2\sin^2 \frac{\theta}{2})^{1/2}}$

$= \int \frac{-\sin \theta d\theta}{2^{3/2} \cos^3 \frac{\theta}{2} \cdot 2^{1/2} \sin \frac{\theta}{2}}$

$= \frac{1}{2^{4/2}} \int \frac{-2 \sin \frac{\theta}{2} \cos^3 \frac{\theta}{2} d\theta}{\cos^3 \frac{\theta}{2} \cdot \sin \frac{\theta}{2}}$

$= -\frac{2}{4} \int \frac{d\theta}{\cos^2 \frac{\theta}{2}}$

$= -\frac{2}{4} \int \sec^2 \frac{\theta}{2} d\theta$

$= -\frac{1}{2} \frac{\tan \frac{\theta}{2}}{1/2} + c$

$= -\frac{2}{2} \tan \frac{\theta}{2} + c$

$= -\tan \frac{\theta}{2} + c$

$\cos^2 \frac{\theta}{2} = \frac{x+1}{2} \Rightarrow \sec^2 \frac{\theta}{2} = \frac{2}{x+1}$

$\sqrt{\sec^2 \frac{\theta}{2} - 1} = \tan \frac{\theta}{2}$

$\sqrt{\left(\frac{2}{x+1}\right) - 1} = \tan \frac{\theta}{2}$

$\tan \frac{\theta}{2} = \sqrt{\frac{2-x-1}{x+1}} = \sqrt{\frac{1-x}{1+x}}$

$\Rightarrow -\sqrt{\frac{1-x}{1+x}} + c$ Ans.

$\cos \theta = 1 - 2\cos^2 \frac{\theta}{2}$
 $= 2\cos^2 \frac{\theta}{2} - 1$

Tahir Mahmood

M.Sc. (Math)

Phone No. 0300-5510779

$x = \cos \theta$
 $x+1 = 2\cos^2 \frac{\theta}{2}$
 \Rightarrow

