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Integration:-

"The process of finding anti-derivative of a function is known as Integration or Anti-differentiation."

By the defination of derivative $\frac{dy}{dx} = f'(x)$

$\Rightarrow dy = f'(x)\,dx$

Also $\delta y = f(x+\delta x) - f(x)$

where dy and dx are called differentials of y and x.

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Exercise: 3.1

Q.1 Find δy and dy in the following cases.

(i) $y = x^2 - 1$ — ①

$y + \delta y = (x+\delta x)^2 - 1$ — ②

Eq ② − Eq ①

$x + \delta x = 3.02$

$x = 3$

$\delta x = 3.02 - 3$

$\Delta x = \delta x = 0.02$

$\delta y = (x+\delta x)^2 - 1 - x^2 + 1$

$\delta y = x^2 + \delta x^2 + 2x\delta x - 1 - x^2 + 1$

$\delta y = \delta x^2 + 2x\delta x$

$\delta y = (0.02)^2 + 2(3)(0.02)$

$\delta y = 0.0004 + 0.12$

$\boxed{\delta y = 0.1204}$

$y = x^2 - 1$

$\frac{dy}{dx} = 2x - 0$ $\quad \because dx \simeq \delta x$

$dy = 2x\,dx$

$dy = 2(3)(0.02)$

$\boxed{dy = 0.12}$

(ii) $y = x^2 + 2x$ — ①

$y + \delta y = (x+\delta x)^2 + 2(x+\delta x)$ — ②

Eq ② − Eq ①

$x + \delta x = 1.8$

$x = 2$

$\delta x = 1.8 - 2 = -0.2$

$\delta y = (x+\delta x)^2 + 2(x+\delta x) - x^2 - 2x$

$\delta y = x^2 + \delta x^2 + 2x\delta x + 2x + 2\delta x - x^2 - 2x$

$\delta y = \delta x^2 + 2x\delta x + 2\delta x$

$\delta y = (-0.2)^2 + 2(2)(-0.2) + 2(-0.2)$

$\delta y = 0.04 - 0.8 - 0.4$

$\boxed{\delta y = -1.16}$

$y = x^2 + 2x$

$\frac{dy}{dx} = 2x + 2 \cdot 1$

$dy = (2x+2)\,dx$

$dy = [2(2)+2]\cdot(-0.2)$ $\quad \because dx \simeq \delta x$

$\boxed{dy = -1.2}$

(iii) $y = \sqrt{x}$ — ① x changes from 4 to 4.41

$y+\delta y = \sqrt{x+\delta x}$ — ②

Eq ② − Eq ①

$\delta y = \sqrt{x+\delta x} - \sqrt{x}$

$\delta y = \sqrt{4.41} - \sqrt{4} \Rightarrow \delta y = 2.1 - 2$

$\boxed{\delta y = 0.1}$

$x+\delta x = 4.41$ & $x = 4$

$\Delta x = \delta x = 4.41 - 4 = 0.41$

Now $y = \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow dy = \frac{dx}{2\sqrt{x}}$

$dy = \frac{(0.41)}{2\sqrt{4}} \Rightarrow dy = \frac{0.41}{2.2} \Rightarrow dy = \frac{0.41}{4}$

$\boxed{dy = 0.1025}$

Use the differentials to find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the following cases:

(i) $xy + x = 4$ Taking differentials.

$d[xy] + d[x] = d[4]$

$x\,dy + dx \cdot y + dx = 0$

$x\,dy + (y+1)\,dx = 0$

$\Rightarrow x\,dy = -(y+1)\,dx$

$\boxed{\frac{dx}{dy} = \frac{-x}{y+1}}$

Also $x\,dy = -(y+1)\,dx$

$\boxed{\frac{dy}{dx} = \frac{-(y+1)}{x}}$

$\Rightarrow x\,dx + y\,dy = 0$

$x\,dx = -y\,dy$

$\boxed{\frac{dx}{dy} = \frac{-y}{x}}$

Also $\boxed{\frac{dy}{dx} = \frac{-x}{y}}$

(ii) $x^2 + y^2 = 16$ Taking differentials.

$d[x^2] + 2d[y^2] = d[16]$

$2x\,dx + 2 \cdot 2y\,dy = 0$

$2(x\,dx + 2y\,dy) = 0 \quad 2 \neq 0$

$x\,dx + 2y\,dy = 0$

(iii) $x^4 + y^2 = xy^2$ Taking differentials.

$d(x^4) + d(y^2) = d(xy^2)$

$4x^3\,dx + 2y\,dy = x\,d(y^2) + dx \cdot y^2$

$4x^3\,dx + 2y\,dy = 2xy\,dy + y^2\,dx$

$4x^3\,dx - y^2\,dx = 2xy\,dy - 2y\,dy$

$(4x^3 - y^2)\,dx = (2xy - 2y)\,dy$

$\boxed{\frac{dx}{dy} = \frac{2y(x-1)}{4x^3 - y^2}}$

$\boxed{\frac{dy}{dx} = \frac{4x^3 - y^2}{2y(x-1)}}$

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(iv) $xy - \ln x = e$

$d(xy) - d(\ln x) = d(c)$ (Taking differentials.

$x\,dy + dx\,y - \frac{1}{x}\,dx = 0 \Rightarrow x\,dy + (y - 1/x)\,dx = 0$

$\Rightarrow x\,dy = -(y - 1/x)\,dx \Rightarrow x\,dy = (1/x - y)\,dx$

$\Rightarrow x\,dy = \left(\frac{1-xy}{x}\right)dx \Rightarrow \boxed{\frac{dy}{dx} = \frac{1-xy}{x^2}}$

$\Rightarrow \boxed{\frac{dx}{dy} = \frac{x^2}{1-xy}}$

Q3/ Use the differential to approximate the values of:

(i) $\sqrt[4]{17}$

Let $f(x) = \sqrt[4]{x} = x^{1/4}$

$f'(x) = \frac{1}{4}x^{1/4-1} \cdot 1$

$f'(x) = \frac{1}{4}x^{-3/4}$

Let $x = 16$, $x + \delta x = 17$, $\delta x = 17 - 16 = 1$

$\because f'(x) = dy/dx$

$dy = f'(x)\,dx$

$dy = \frac{1}{4}(16)^{-3/4} \cdot 1$

$dy = 0.03125$ — (1)

Also $dy = f(x+\delta x) - f(x)$

$0.03125 = \sqrt[4]{17} - \sqrt[4]{16}$

$\sqrt[4]{17} = 0.03125 + \sqrt[4]{16}$

$\sqrt[4]{17} = 0.03125 + 2$

$\boxed{\sqrt[4]{17} = 2.03125}$ Ans.

(ii) $(8.2)^{1/3}$

Let $f(x) = (x)^{1/3}$

$f'(x) = \frac{1}{3}x^{1/3-1} \cdot 1$

$f'(x) = \frac{1}{3}x^{-2/3}$

$\because dy/dx = f'(x)$

$dy = f'(x)\,dx$

$dy = \frac{1}{3}(8)^{-2/3}(0.2)$

$dy = 0.016667$

Also $dy = f(x+\delta x) - f(x)$

$0.01667 = (8.2)^{1/3} - (8)^{1/3}$

$(8.2)^{1/3} = 0.01667 + (8)^{1/3}$

$(8.2)^{1/3} = 0.01667 + 2$

$\boxed{(8.2)^{1/3} = 2.01667}$ Ans.

Let $x = 8$

$x + \delta x = 8.2$

$\delta x = 8.2 - 8$

$\delta x = 0.2$

(iii) $(31)^{1/5}$

Let $f(x) = (x)^{1/5}$ | Let $x = 32$

$f'(x) = \frac{1}{5}x^{-4/5}$ | $x + \delta x = 31$

$\because f'(x) = dy/dx$ | $\delta x = 31 - 32 = -1$

$dy = f'(x)\,dx$

$dy = \frac{1}{5}(32)^{-4/5}\cdot(-1)$

$dy = \frac{1}{5}\left(\frac{1}{16}\right)(-1) = -0.0125$

Also $dy = f(x+\delta x) - f(x)$

$-0.0125 = (31)^{1/5} - (32)^{1/5}$

$(31)^{1/5} = -0.0125 + (32)^{1/5}$

$(31)^{1/5} = -0.0125 + 2$

$\boxed{(31)^{1/5} = 1.9875}$ Ans.

(iv) $\cos 29^\circ$

Let $f(x) = \cos x$ | Let $x = 30^\circ$

$f'(x) = -\sin x$ | $x + \delta x = 29^\circ$

$\because dy/dx = f'(x)$ | $\delta x = 29^\circ - 30^\circ = -1^\circ$

$dy = f'(x)\,dx$ | $\delta x = -1 \times 0.0175$

$\delta x = -0.0175$

$dy = -\sin 30^\circ \cdot (-0.0175)$

$dy = -\frac{1}{2}(-0.0175) = 0.00875$

Also $dy = f(x+\delta x) - f(x)$

$0.00875 = \cos 29^\circ - \cos 30^\circ$

$\cos 29^\circ = 0.00875 + \cos 30^\circ$

$\cos 29^\circ = 0.00875 + 0.866$

$\boxed{\cos 29^\circ = 0.87478}$ Ans.

(v) $\sin 61^\circ$

Let $f(x) = \sin x$ | Let $x = 60^\circ$

$f'(x) = \cos x$ | $x + \delta x = 61^\circ$

$dy/dx = f'(x)$ | $\delta x = 61^\circ - 60^\circ = 1^\circ$

$dy = f'(x)\cdot dx$ | $\delta x = 1 \times 0.0175$

$dy = \cos(60^\circ)\cdot(0.0175)$ | $\delta x = 0.0175$

$dy = 0.00875$

also $dy = f(x+\delta x) - f(x)$

$0.00875 = \sin 61^\circ - \sin 60^\circ$

$\sin 61^\circ = 0.00875 + \sin 60^\circ$

$\sin 61^\circ = 0.00875 + 0.866$

$\boxed{\sin 61^\circ = 0.87478}$

Q.4 Let x be the sides of Cube so

$V = x^3$ | also $x + \delta x = 5.02$

$dV = 3x^2\,dx$ | $x = 5$

$dV = 3(5)^2(0.02)$ | $\delta x = 5.02 - 5$

$dV = 3 \times 25 \times 0.02$ | $\delta x = 0.02$

$dV = 1.5$ (cubic unit)

Q.5 Let x be the diameter of disc

$A = \pi(\text{radius})^2$ | $\because$ radius $= \frac{\text{diameter}}{2}$

$A = \pi\left(\frac{x}{2}\right)^2$ | $x + \delta x = 44.4$ cm

$dA = \pi \cdot 2\left(\frac{x}{2}\right)\cdot\frac{dx}{2}$ | $x = 44$ cm

$dA = \pi\left[\frac{44}{2}\right]\cdot(0.4)$ | $\delta x = 44.4 - 44$

$dA = 22\pi(0.4)$ | $\delta x = 0.4$

$dA = 8.8\,\pi \Rightarrow dA = 8.8(3.14)$

$\boxed{dA = 27.65\ cm^2}$