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The complex number is usually denoted by $z = a+ib$ which is called standard form of the complex number.

Kinds of Complex numbers:-

i		Imaginary unit
$a+ib$		Standard Complex number
$a+0i$	$b=0$	Real Complex number
$0+ib$	$a=0$	Imaginary Complex No.
$0+0i$	$a,b=0$	Zero Complex number.
$a-ib$		Conjugate of $a+ib$

Conjugate of a Complex Number:-

" If $z = a+ib$ is a Complex number then $a-ib$ is called its Conjugate complex number and it denoted as $\bar{z} = a-ib$

* A complex number $a+ib$ can also be represented as an ordered pair of real number.

i.e. $a+ib = (a,b)$

* Infact a Complex number is a combination of two real numbers with i imaginary unit

$z = a+ib \qquad a,b \in \mathbb{R}$

Properties and Operations:

(i) Equality Property:

$(a+ib)=(c+id) \Longleftrightarrow a=c \wedge b=d$

(ii) Addition:

$(a+ib)+(c+id) = (a+c)+i(b+d)$

(iii) Subtraction:

$(a+ib)-(c+id) = (a-c)+i(b-d)$

(iv) Constant Multiple Property:

$K(a+ib) = Ka + iKb$

(v) Multiplication:

$(a+ib)(c+id) = (ac-bd)+(ad+bc)i$

Exercise 1.2

Q.1 Derive addition Properties:

(i) Commutative Property:-

$(a+ib)+(c+id) = (c+id)+(a+ib)$

Proof:- LHS $= (a+ib)+(c+id)$

$= a+ib+c+id$

$= a+c+ib+id$

$= (a+c)+i(b+d)$ —— (i)

RHS $= (c+id)+(a+ib)$

$= c+id+a+ib$

$= (c+a)+i(d+b)$

$(\because a+b=b+a,\ a,b \in \mathbb{R})$

$= (a+c)+i(b+d)$ —— (ii)

From (i) and (ii)

$(a+ib)+(c+id) = (c+id)+(a+ib)$

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(ii) Associative Property.

$(a,b)+\{(c,d)+(e,f)\} = \{(a,b)+(c,d)\}+(e,f)$

L.H.S $= (a+ib)+\{(c+id)+(e+if)\}$

$= (a+ib)+\{c+id+e+if\}$

$= (a+ib)+\{(c+e)+i(d+f)\}$

$= a+ib+(c+e)+i(d+f)$

$= (a+c+e)+i(b+d+f)$ ——— (1)

RHS $= \{(a+ib)+(c+id)\}+(e+if)$

$= \{a+ib+c+id\}+(e+if)$

$= \{(a+c)+i(b+d)\}+(e+if)$

$= (a+c)+i(b+d)+e+if$

$= (a+c+e)+i(b+d+f)$ ——— (2)

L.H.S. = R.H.S.

Associative Property holds for Complex Nos.

Q.2 Derive the Multiplicative Properties of Complex numbers.

(i) Commutative Property.

$(a,b)(c,d) = (c,d)(a,b)$

LHS $= (a+ib).(c+id)$

$= a.c+a.id+ib.c+ib.id$

$= ac+i(ad+bc)+i^2bd$

$= ac-bd+i(ad+bc)$ $(\because i^2=-1)$

$= (ac-bd)+i(ad+bc)$ ——— (1)

RHS $= (c+id).(a+ib)$

$= ca+id.a+c.ib+id.ib$

$= ca+i(da+cb)+i^2db$

$= ac-bd+i(ad+cb)$ $(\because i^2=-1)$

$= (ac-bd)+i(ad+bc)$ ——— (2)

R.H.S. = L.H.S.

Commutative Property holds for Complex Nos.

(ii) Associative Property.

$(a,b).\{(c,d).(e,f)\} = \{(a,b).(c,d)\}.(e,f)$

LHS $= (a+ib).\{(c+id).(e+if)\}$

$= (a+ib).\{ce+c.if+id.e+id.if\}$

$= (a+ib).\{ce+i(cf+ed)+i^2df\}$ $(\because i^2=-1)$

$= (a+ib).\{(ce-df)+i(cf+de)\}$

$= a.(ce-df)+i(cf+de).a+ib.(ce-df)$
$+i^2b.(cf+de)$

$= (ace-adf)+i(acf+ade)+i(bce-bdf)$
$-(bcf+bde)$

$= \{(ace-adf)-(bcf+bde)\}+i\{(acf+ade)+(bce-bdf)\}$ ——— (1)

RHS $= \{(a+ib).(c+id)\}.(e+if)$

$= \{ac+iad+ibc+i^2bd\}.(e+if)$

$= \{(ac-bd)+i(ad+bc)\}.(e+if)$ $(\because i^2=-1)$

$= (ac-bd).e+i(ad+bc).e+(ac-bd).if$
$+i^2(ad+bc)f$

$= (ace-bde)+i(ade+bce)+i(acf-bdf)$
$-(adf+bcf)$

$= \{(ace-bde)-(adf+bcf)\}+i\{(ade+bce)+(acf-bdf)\}$

$= \{ace-bde-adf-bcf\}+i\{ade+bce+acf-bdf\}$

$= \{(ace-adf)-(bcf+bde)\}+i\{(acf+ade)+(bce-bdf)\}$

RHS = L.HS A.P. holds for Complex. Nos.

Q.3 Derive Distributive Property of Complex numbers.

$(a,b)\cdot[(c,d)+(e,f)] = (a,b)\cdot(c,d)+(a,b)\cdot(e,f)$

$L.H.S = (a+ib)\cdot\{(c+id)+(e+if)\}$

$= (a+ib)[c+id+e+if]$

$= (a+ib)\cdot[(c+e)+i(d+f)]$

$= a\cdot(c+e)+ia(d+f)+ib\cdot(c+e)+i^2b\cdot(d+f)$ $(\because i^2=-1)$

$= (ac+ae)+i(ad+ \quad +i(bc+be)-(bd+bf)$

$= \{(ac+ae)-(bd+bf)\} + i\{(ad+af)+(bc+be)\}$ —(1)

$RHS = (a+ib)\cdot(c+id)+(a+ib)\cdot(e+if)$

$= \{ac+iad+ibc+i^2bd\}+\{ae+iaf+ibe+i^2bf\}$ $(\because i^2=-1)$

$= \{(ac-bd)+i(ad+bc)\}+\{(ae-bf)+i(af+be)\}$

$= \{(ac-bd)+(ae-bf)\}+i\{(ad+bc)+(af+be)\}$

$= \{ac-bd+ae-bf\}+i\{ad+bc+af+be\}$

$= \{(ac+ae)-(bd+bf)\}+i\{(ad+af)+(bc+be)\}$ —(2)

$RHS = LHS.$

Distributive Property holds for Complex Nos.

Q4 Simplify the followings:

(i) i^9

$= i^8\cdot i$ $\quad (\because i^2=(-1) \wedge i^4=1)$

$= (i^4)^2\cdot i$

$= (1)^2\cdot i \Rightarrow 1\cdot i = i$ Ans.

(ii) i^{14}

$= i^{12}\cdot i^2 \Rightarrow (i^4)^3\cdot i^2$

$= (1)^3\cdot(-1) \Rightarrow 1\cdot(-1) = -1$ Ans.

(iii) $(-i)^{19} \Rightarrow -i^{19}$

$= -i^{16}\cdot i^3$

$= -(i^4)^4\cdot i^2\cdot i$

$= -(1)^4\cdot(-1)\cdot i \Rightarrow -(1)(-1)i$

$= 1\cdot i \Rightarrow i$ Ans.

(iv) $(-1)^{-\frac{21}{2}} \Rightarrow [(-1)^{1/2}]^{-21}$

$= (\sqrt{-1})^{-21} \Rightarrow (i)^{-21}$ $\quad (\because \sqrt{-1}=i)$

$= \dfrac{1}{i^{21}} \Rightarrow \dfrac{1}{i^{20}\cdot i}$

$= \dfrac{1}{(i^4)^5\cdot i} \Rightarrow \dfrac{1}{(1)^5\cdot i}$

$= \dfrac{1}{1\cdot i} \Rightarrow \dfrac{1}{i}$ Ans.

Q.5 Write in terms of i:

(i) $\sqrt{-1}\ b$

$= ib$ Ans. $\quad (\because \sqrt{-1}=i)$

(ii) $\sqrt{-5}$

$= \sqrt{-1\times 5} \Rightarrow \sqrt{-1}\times\sqrt{5}$

$= i\sqrt{5} \Rightarrow \sqrt{5}\,i$ Ans.

(iii) $\sqrt{\dfrac{-16}{25}}$

$= \sqrt{-1\times\dfrac{16}{25}} \Rightarrow \sqrt{-1}\cdot\sqrt{\dfrac{16}{25}}$

$= i\sqrt{\dfrac{4^2}{5^2}}$

$= \dfrac{4}{5}i$ Ans.

(iv) $\sqrt{\dfrac{1}{-4}}$

$= \sqrt{\dfrac{-1}{4}} \Rightarrow \sqrt{-1\times\dfrac{1}{4}}$

$= \sqrt{-1}\times\sqrt{1/4}$

$= i\sqrt{1/4} \Rightarrow i\sqrt{\dfrac{1^2}{2^2}}$

$= i\,\dfrac{1}{2} \Rightarrow \dfrac{1}{2}i$ Ans.

Simplify the followings:

Q.6 $(7,9)+(3,-5)$

$=(7+i9)+(3-i5)$

$=7+3+i9-i5$

$=(7+3)+i(9-5)$

$=10+i4$ Ans or $(10,4)$

Q.7 $(8,-5)-(-7,4)$

$=(8-5i)-(-7+i4)$

$=(8-5i)+(7-i4)$

$=8-5i+7-i4$

$=(8+7)-i(5+4)$

$=15-i9$ Ans. or $(15,-9)$

Q.8 $(2,6).(3,7)$

$=(2+i6)(3+i7)$

$=2.3+i6.3+i2.7+i^2 6.7$

$=6+i18+i14-42$

$=(6-42)+i(18+14)$

$=-36+i32$ Ans. or $(-36,32)$

Q.9 $(5,-4).(-3,-2)$

$=(5-4i).(-3-2i)$

$=5.-3-i5.2+i4.3+i^2.4.2$

$=-15-i10+i12-8$

$=(-15-8)+i(12-10)$

$=-23+2i$ or $(-23,2)$ Ans.

Q.10 $(0,3)(0,5)$

$=(0+3i).(0+5i)$

$=0+i5.0+i0.3+i^2 5.3$

$=0+0i+i0-15$

$=(0-15)+i(0+0)$

$=-15+0i$ or $(-15,0)$ Ans.

Q.11 $(2,6)\div(3,7)$

$=(2+i6)\div(3+i7)$

$=\frac{(2+i6)}{(3+i7)}$

Multiply and dividing by $3-i7$

$=\frac{2+i6}{3+i7}\times\frac{3-i7}{3-i7}$

$=\frac{(2+i6)(3-i7)}{(3)^2-(i7)^2}$

$=\frac{2.3+i6.3-i2.7-i^2 6.7}{9-i^2 49}$

$=\frac{6+i18-i14+42}{9+49}$ $(\because i^2=-1)$

$=\frac{(6+42)+i(18-14)}{58}$

$=\frac{48+i4}{58} \Rightarrow \frac{48}{58}+i\frac{4}{58}$

$=\frac{24}{29}+i\frac{2}{29}$ or $\left(\frac{24}{29},\frac{2}{29}\right)$ Ans.

Q.12 $(5,-4)\div(-3,-8)$

$=\frac{5-4i}{-3-8i}\times\frac{-3+8i}{-3+8i}$ Multiply and dividing by $(-3+8i)$

$=\frac{(5-4i)\times(-3+8i)}{(-3)^2-(8i)^2}$

$=\frac{-5.3+i4.3-4.8i^2+5.8i}{9-i^2 64}$ $(\because i^2=-1)$

$=\frac{-15+i12+32+i40}{9+64}$

$=\frac{(-15+32)+i(12+40)}{73} \Rightarrow \frac{17+i52}{73}$

$=\frac{17}{73}+i\frac{52}{73}$ or $\left(\frac{17}{73},\frac{52}{73}\right)$ Ans.

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Q.13 Prove that the Sum as well as the product of any two Conjugate Complex Nos. is a Real Number.

Proof: Let $a+ib$ and $a-ib$ are the Complex Conjugate of each other.

(i) $(a+ib)+(a-ib)$

$= a+\cancel{ib}+a-\cancel{ib} \Rightarrow 2a$

which is a real number because no i is involve in the Answer.

(ii) $(a+ib)(a-ib)$

$= a.a + ib.a - ia\,b - i^2 b.b$

$= a^2 + i\cancel{ab} - i\cancel{ab} + b^2 \quad (\because i^2=-1)$

$= a^2+b^2$

which is also real number.

Q.14 Find the multiplicative inverse:

(i) $(-4,7)$

By defination $\frac{1}{a}$ is multiplicative inverse of a

$\therefore \frac{1}{(-4,7)}$ is m.i. of $(-4,7)$

$= \frac{1}{-4+7i} \times \frac{-4-7i}{-4-7i}$

$= \frac{-4-7i}{(-4)^2-(7i)^2}$

$= \frac{-4-7i}{16-49i^2}$

$= \frac{-4-7i}{16+49} \quad (\because i^2=-1)$

$= \frac{-4-7i}{65} \Rightarrow \frac{-4}{65} - \frac{7}{65}i$

$= \left(\frac{-4}{65}, \frac{-7}{65}\right)$ is multip. inverse of $(-4,7)$

(ii) $(\sqrt{2}, -\sqrt{5})$

Multip. Inverse $= \frac{1}{\sqrt{2}-i\sqrt{5}}$

$= \frac{1}{\sqrt{2}-i\sqrt{5}} \times \frac{\sqrt{2}+i\sqrt{5}}{\sqrt{2}+i\sqrt{5}}$

$= \frac{\sqrt{2}+i\sqrt{5}}{(\sqrt{2})^2-(i\sqrt{5})^2}$

$= \frac{\sqrt{2}+i\sqrt{5}}{2-i^2 5} \Rightarrow \frac{\sqrt{2}+i\sqrt{5}}{2+5}$

$= \frac{\sqrt{2}+i\sqrt{5}}{7} \quad (\because i^2=-1)$

$= \frac{\sqrt{2}}{7} + i\frac{\sqrt{5}}{7}$ or $\left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right)$ is m.i. of $(\sqrt{2}, -\sqrt{5})$

(iii) $(1,0)$

Multip. Inverse $= \frac{1}{1+i0}$

$= \frac{1}{1+i0} \times \frac{1-i0}{1-i0}$

$= \frac{1-i0}{(1)^2-(i0)^2}$

$= \frac{1-i0}{1-i^2 0} \Rightarrow \frac{1-i0}{1+0}$

$= \frac{1-i0}{1}$

$= 1-0i$

$= (1,0)$ is multip. Inverse of $(1,0)$

Q.15 Factorize the followings:

(i) a^2+4b^2

$(\because -1=i^2)$

$= a^2-(-4)b^2 \Rightarrow (a)^2-(-1\times 4b^2)$

$= (a)^2-(i^2 4b^2) \Rightarrow (a)^2-(2ib)^2$

$= (a+i2b)(a-i2b)$ Ans.

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(ii) $9a^2+16b^2$

$= 9a^2-(-16b^2) \Rightarrow 9a^2-(-1\times 16b^2)$

$= 9a^2-(i^2 16b^2)$ $\quad (\because -1 = i^2)$

$= (3a)^2-(i4b)^2$

$= (3a+i4b)(3a-i4b)$ Ans.

(iii) $3x^2+3y^2 \Rightarrow 3(x^2+y^2)$

$= 3\{x^2-(-y^2)\} \Rightarrow 3\{x^2-(-1\cdot y^2)\}$

$= 3\{x^2-(i^2 y^2)\}$ $\quad (\because i^2=-1)$

$= 3\{(x)^2-(iy)^2\}$

$= 3\{(x-iy)(x+iy)\}$ Ans.

Q.16 Separate real and imaginary Part.

(i) $\frac{2-7i}{4+5i}$

$= \frac{2-7i}{4+5i}\times\frac{4-5i}{4-5i}$ Multip. and dividing by $4-5i$

$= \frac{(2-7i)(4-5i)}{(4)^2-(5i)^2}$

$(\because i^2=-1)$

$= \frac{8-10i-28i+i^2 35}{16-i^2 25} \Rightarrow \frac{8-i(10+28)-35}{16+25}$

$= \frac{(8-35)-i(38)}{41} \Rightarrow \frac{-27-i38}{41}$

$= \frac{-27}{41}-i\frac{38}{41}$ or $\left(\frac{-27}{41},\frac{-38}{41}\right)$ Ans.

(ii) $\Rightarrow \frac{(-2+3i)^2}{(1+i)}$

$= \frac{(-2)^2+(3i)^2-2(2)(3i)}{1+i}\times\frac{1-i}{1-i}$

$= \frac{(4+9i^2-12i)(1-i)}{(1)^2-(i)^2} \Rightarrow \frac{(4-9-12i)(1-i)}{1-i^2}$

$= \frac{(4-9-i12)(1-i)}{1+1}$ $\quad (\because i^2=-1)$

$= \frac{(-5-i12)(1-i)}{2} \Rightarrow \frac{-5+5i-i12+i^2 12}{2}$

$= \frac{-5+i(5-12)-12}{2} \Rightarrow \frac{(-5-12)-7i}{2}$

$= \frac{-17}{2}-\frac{7}{2}i$ or $\left(\frac{-17}{2},\frac{-7}{2}\right)$ Ans.

(iii) $\frac{i}{1+i}$ Multip. and dividing by $1-i$

$= \frac{i}{1+i}\times\frac{1-i}{1-i}$

$= \frac{i(1-i)}{(1)^2-(i)^2} \Rightarrow \frac{i-i^2}{1-i^2}$

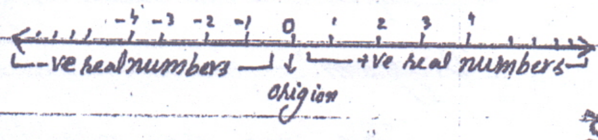
$= \frac{i+1}{1+1}$ $\quad (\because i^2=-1)$

$= \frac{1+i}{2} \Rightarrow \frac{1}{2}+i\frac{1}{2}$ or $\left(\frac{1}{2},\frac{1}{2}\right)$ Ans.

Real Line:-

"A line with a real number associated with each point and vice versa is called real number line or real line."

The point with Co-ordinate 0 is called origion. The numbers lie on right side of origion are called +ve real numbers and the numbers lie on left side of origion are called negative real numbers.



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