EXERCISE # 14

Chapter No.14

SOLUTIONS OF TRIGONOMETRIC EQUATIONS

Trigonometric Equations:

The equations containing at least one trigonometric functions are called trigonometric equations. e.g., $\sin x = 5$, $\sec x = \tan x$

Trigonometric equations have an infinite number of solutions due to the periodicity of the trigonometric functions.

Example 1: Solve the equation: $\sin x = \frac{1}{2}$

Solution: Since $\sin x$ is +ve in I & II quadrant with the reference angle is $\frac{\pi}{6}$:

For I Quadrant

$$x = \theta = \frac{\pi}{6} = \frac{\pi}{6} + 2n\pi$$

For II Quadrant

$$x = \pi - \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} + 2n\pi$$

$$\text{S.S.} = \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\}, \quad n \in Z$$

Example 2: Solve the equation:

$$1 + \cos x = 0$$

Solution: $1 + \cos x = 0$

$\cos x = -1$

$x = \cos^{-1}(-1)$

$x = \pi \Rightarrow x = \pi + 2n\pi$

$$\text{S.S.} = \{\pi + 2n\pi\}, \quad n \in Z$$

Example 3: Solve the equation: $4\cos^2 x - 3 = 0$

Solution: $4\cos^2 x - 3 = 0 \Rightarrow 4\cos^2 x = 3 \Rightarrow \cos^2 x = \frac{3}{4} \Rightarrow \cos x = \pm\frac{\sqrt{3}}{2}$

Since $\cos x$ is +ve in I & IV quadrant with the reference angle is $\frac{\pi}{6}$:

For I Quadrant

$\theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{6} + 2n\pi$

For IV Quadrant

$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\theta = \frac{11\pi}{6} + 2n\pi$

Since $\cos x$ is −ve in II & III quadrant with the reference angle is $\frac{\pi}{6}$:

For II Quadrant

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\theta = \frac{5\pi}{6} + 2n\pi$

For III Quadrant

$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\theta = \frac{7\pi}{6} + 2n\pi$

$$\text{S.S.} = \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{11\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\} \cup \left\{\frac{7\pi}{6} + 2n\pi\right\}, \quad n \in Z$$

Example 4: Solve the equation: $\sin x + \cos x = 0$

Solution: $\sin x + \cos x = 0$

$\sin x = -\cos x$

$\frac{\sin x}{\cos x} = -1 \Rightarrow \tan x = -1$

Example 5: Solve the equation: $\sin x \cos x = \frac{\sqrt{3}}{4}$

Solution: $\sin x \cos x = \frac{\sqrt{3}}{4}$

$2\sin x \cos x = \frac{\sqrt{3}}{2} \Rightarrow \sin 2x = \frac{\sqrt{3}}{2}$

EXERCISE # 14

Since $\tan x$ is $-$ve in II & IV quadrant with the reference angle is $\pi/4$:

For II Quadrant

$x=\pi-\dfrac{\pi}{4}$

$x=\dfrac{3\pi}{4}+n\pi$

For IV Quadrant

$x=2\pi-\dfrac{\pi}{4}$

$x=\dfrac{7\pi}{4}+n\pi$

$$S.S.=\left\{\frac{3\pi}{4}+n\pi\right\}\cup\left\{\frac{7\pi}{4}+n\pi\right\},\quad n\in Z$$

Since $\sin\theta$ is $+$ve in I & II quadrant with the reference angle is $\pi/3$:

For I Quadrant

$2x=\dfrac{\pi}{3}+2n\pi$

$x=\dfrac{\pi}{6}+n\pi$

For II Quadrant

$2x=\pi-\dfrac{\pi}{3}=\dfrac{2\pi}{3}+2n\pi$

$x=\dfrac{\pi}{3}+n\pi$

$$S.S.=\left\{\frac{\pi}{6}+n\pi\right\}\cup\left\{\frac{\pi}{3}+n\pi\right\},\quad n\in Z$$

Example 6: Solve the equation: $\sin 2x=\cos x$

Solution: $\sin 2x=\cos x \Rightarrow 2\sin x\cos x=\cos x \Rightarrow 2\sin x\cos x-\cos x=0$

$\cos x(2\sin x-1)=0 \Rightarrow$ **Either** $\cos x=0$ **OR** $2\sin x-1=0$

$\cos x=0$

Since $\cos x$ is $+$ve in I & IV quadrant with the reference angle is $\pi/2$:

For I Quadrant

$x=\dfrac{\pi}{2}$

$x=\dfrac{\pi}{2}+2n\pi$

For II Quadrant

$x=2\pi-\dfrac{\pi}{2}$

$x=\dfrac{3\pi}{2}+2n\pi$

$2\sin x=1 \Rightarrow \sin x=1/2$

Since $\sin x$ is $+$ve in I & II quadrant with the reference angle is $\pi/6$:

For III Quadrant

$x=\dfrac{\pi}{6}$

$x=\dfrac{\pi}{6}+2n\pi$

For IV Quadrant

$x=\pi-\dfrac{\pi}{6}$

$x=\dfrac{5\pi}{6}+2n\pi$

$$S.S.=\left\{\frac{\pi}{2}+2n\pi\right\}\cup\left\{\frac{3\pi}{2}+2n\pi\right\}\cup\left\{\frac{\pi}{6}+2n\pi\right\}\cup\left\{\frac{5\pi}{6}+2n\pi\right\},\quad n\in Z$$

Example 7: Solve the equation: $\sin^2 x+\cos x=1$

Solution: $\sin^2 x+\cos x=1$

$1-\cos^2 x+\cos x=1$

$-\cos^2 x+\cos x=1-1$

$-\cos x(\cos x-1)=0$

Either $\cos=0$ **OR** $\cos x-1=0$

Example 8: Solve the equation: $\operatorname{cosec} x=\sqrt{3}+\cot x$

Solution: $\dfrac{1}{\sin x}=\sqrt{3}+\dfrac{\cos x}{\sin x}$

$1=\sqrt{3}\sin x+\cos x \Rightarrow 1-\cos x=\sqrt{3}\sin x$

$1-2\cos x+\cos^2 x=3\sin^2 x$

$1-2\cos x+\cos^2 x=3(1-\cos^2 x)$

$1-2\cos x+\cos^2 x=3-3\cos^2 x$

$4\cos^2 x-2\cos x-2=0$

$4\cos^2 x-4\cos x+2\cos x-2=0$

$4\cos x(\cos x-1)+2(\cos x-1)=0$

Either $\cos x-1=0$ or $4\cos x+2=0$

EXERCISE # 14

Since $\cos x$ is +ve in I & IV quad. with the reference angle is $\frac{\pi}{2}$:

For I Quad.

$x=\frac{\pi}{2}$

$x=\frac{\pi}{2}+2n\pi$

For IV Quad.

$x=2\pi-\frac{\pi}{2}$

$x=\frac{3\pi}{2}+2n\pi$

$\cos x=1$

$x=\cos^{-1}(1)$

$x=2\pi$

$x=2\pi+2n\pi$

$\because \{2\pi+2n\pi\}\subset\{2n\pi\}$

$\therefore\ x=2n\pi,\quad \forall n\in z$

$$\text{S.S.}=\left\{\frac{\pi}{2}+2n\pi\right\}\cup\left\{\frac{3\pi}{2}+2n\pi\right\}\cup\{2n\pi\},\ n\in z$$

$\cos x=1$

$x=\cos^{-1}(1)$

$x=0\ \&\ x=2\pi$

As, both cosecx & cotx are not defined for $x=0\ \&\ x=2\pi$

$\therefore\ x=0\ \&\ x=2\pi$ are not admissible.

$4\cos x=-2\Rightarrow\cos x=-\frac{2}{4}\Rightarrow\cos x=-\frac{1}{2}$

Since $\cos x$ is −ve in II & III quadrant with the reference angle is $\frac{\pi}{3}$:

For II Quad.

$\theta=\pi-\frac{\pi}{3}$

$\theta=\frac{2\pi}{3}+2n\pi$

For IV Quad.

$\theta=\pi+\frac{\pi}{3}$

$\theta=\frac{4\pi}{3}+2n\pi$

$$\text{S.S.}=\left\{\frac{2\pi}{3}+2n\pi\right\},\quad n\in Z$$

Exercise # 14

Q.1: **Find the solutions of the following equations which lie in [0, 2π]:**

(i) $\sin x=-\sqrt{3}/2$ **(ii)** $\operatorname{cosec}\theta=2$ **(iii)** $\sec x=-2$ **(iv)** $\cot\theta=1/\sqrt{3}$

Q.2: **Solve the following trigonometric equations:**

(i) $\tan^2\theta=\frac{1}{3}$ **(ii)** $\operatorname{cosec}^2\theta=\frac{4}{3}$ **(iii)** $\sec^2\theta=\frac{4}{3}$ **(iv)** $\cot^2\theta=\frac{1}{3}$

Find the value of θ satisfying the following equations:

Q.3: $3\tan^2\theta+2\sqrt{3}\tan\theta+1=0$

Q.4: $\tan^2\theta-\sec\theta-1=0$

Q.5: $2\sin\theta+\cos^2\theta-1=0$

Q.6: $2\sin^2\theta-\sin\theta=0$

Q.7: $3\cos^2\theta-2\sqrt{3}\sin\theta\cos\theta-3\sin^2\theta=0$

Q.8: $4\sin^2\theta-8\cos\theta+1=0$

Find the solution sets of the following equations:

Q.9: $\sqrt{3}\tan x-\sec x-1=0$

Q.10: $\cos 2x=\sin 3x$

Q.11: $\sec 3\theta=\sec\theta$

Q.12: $\tan 2\theta+\cot\theta=0$

Q.13: $\sin 2x+\sin x=0$

Q.14: $\sin 4x-\sin 2x=\cos 3x$

Q.15: $\sin x+\cos 3x=\cos 5x$

Q.16: $\sin 3x+\sin 2x+\sin x=0$

Q.17: $\sin 7x-\sin x=\sin 3x$

Q.18: $\sin x+\sin 3x+\sin 5x=0$

Q.19: $\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta=0$

Q.20: $\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta=0$

SOLUTION OF EXERCISE # 14

Exercise # 14

Q.1: Find the solutions of the following equations which lie in [0, 2π]:

(i) $\sin x = -\sqrt{3}/2$ **(ii)** $\operatorname{cosec}\theta = 2$ **(iii)** $\sec x = -2$ **(iv)** $\cot\theta = \frac{1}{\sqrt{3}}$

Solution:

(i) $\sin x = -\sqrt{3}/2$

Since $\sin x$ is $-$ve in III & IV quadrant with the reference angle is $\pi/3$:

For III Quadrant

$$x = \pi + \frac{\pi}{3} \Rightarrow \boxed{x = \frac{4\pi}{3}}$$

For IV Quadrant

$$x = 2\pi - \frac{\pi}{3} \Rightarrow \boxed{x = \frac{5\pi}{3}}$$

(ii) $\operatorname{cosec}\theta = 2$

Since $\operatorname{cosec}\theta$ is $+$ve in I & II quadrant with the reference angle is $\pi/6$:

For I Quadrant

$$\boxed{x = \frac{\pi}{6}}$$

For II Quadrant

$$x = \pi - \frac{\pi}{6} \Rightarrow \boxed{x = \frac{5\pi}{6}}$$

(iii) $\sec x = -2$

Since $\sec x$ is $-$ve in II & III quadrant with the reference angle is $\pi/3$:

For II Quadrant

$$x = \pi - \frac{\pi}{3} \Rightarrow \boxed{x = \frac{2\pi}{3}}$$

For III Quadrant

$$x = \pi + \frac{\pi}{3} \Rightarrow \boxed{x = \frac{4\pi}{3}}$$

(iv) $\cot\theta = 1/\sqrt{3}$

Since $\cot\theta$ is $+$ve in I & III quadrant with the reference angle is $\pi/3$:

For I Quadrant

$$\boxed{x = \frac{\pi}{3}}$$

For III Quadrant

$$x = \pi + \frac{\pi}{3} \Rightarrow \boxed{x = \frac{4\pi}{3}}$$

Q.2: Solve the following trigonometric equations:

(i) $\tan^2\theta = 1/3$ **(ii)** $\operatorname{cosec}^2\theta = 4/3$ **(iii)** $\sec^2\theta = 4/3$ **(iv)** $\cot^2\theta = 1/3$

Solution:

(i) $\tan^2\theta = \frac{1}{3} \Rightarrow \sqrt{\tan^2\theta} = \pm\sqrt{\frac{1}{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$ and $\tan\theta = \frac{-1}{\sqrt{3}}$

Since $\tan\theta$ is $+$ve in I & III quadrant with the reference angle is $\pi/6$:

For I Quadrant

$$\theta = \frac{\pi}{6}$$
$$\theta = \frac{\pi}{6} + n\pi$$

For III Quadrant

$$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$
$$\theta = \frac{7\pi}{6} + n\pi$$

Since $\tan\theta$ is $-$ve in II & IV quadrant with the reference angle is $\pi/6$:

For II Quadrant

$$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$
$$\theta = \frac{5\pi}{6} + n\pi$$

For IV Quadrant

$$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$
$$\theta = \frac{11\pi}{6} + n\pi$$

$$\text{S.S.} = \left\{\frac{\pi}{6} + n\pi\right\} \cup \left\{\frac{7\pi}{6} + n\pi\right\} \cup \left\{\frac{5\pi}{6} + n\pi\right\} \cup \left\{\frac{11\pi}{6} + n\pi\right\}, \quad n \in Z$$

SOLUTION OF EXERCISE # 14

(ii) $\operatorname{cosec}^2\theta = \frac{4}{3} \Rightarrow \sqrt{\operatorname{cosec}^2\theta} = \pm\sqrt{\frac{4}{3}} \Rightarrow \operatorname{cosec}\theta = \frac{2}{\sqrt{3}}$ and $\operatorname{cosec}\theta = -\frac{2}{\sqrt{3}}$

Since $\operatorname{cosec}\theta$ is $+$ve in I & II quadrant with the reference angle is $\pi/3$:

For I Quadrant

$\theta = \frac{\pi}{3}$

$\theta = \frac{\pi}{3} + 2n\pi$

For II Quadrant

$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\theta = \frac{2\pi}{3} + 2n\pi$

Since $\operatorname{cosec}\theta$ is $-$ve in III & IV quadrant with the reference angle is $\pi/3$:

For III Quadrant

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$\theta = \frac{4\pi}{3} + 2n\pi$

For IV Quadrant

$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\theta = \frac{5\pi}{3} + 2n\pi$

$$\text{S.S.} = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\}, \quad n \in Z$$

(iii) $\sec^2\theta = \frac{4}{3} \Rightarrow \sqrt{\sec^2\theta} = \pm\sqrt{\frac{4}{3}} \Rightarrow \sec\theta = \frac{2}{\sqrt{3}}$ and $\sec\theta = -\frac{2}{\sqrt{3}}$

Since $\sec x$ is $+$ve in I & IV quadrant with the reference angle is $\pi/6$:

For I Quadrant

$\theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{6} + 2n\pi$

For IV Quadrant

$\theta = 2\pi - \frac{\pi}{6}$

$\theta = \frac{11\pi}{6} + 2n\pi$

Since $\sec x$ is $-$ve in II & III quadrant with the reference angle is $\pi/6$:

For II Quadrant

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\theta = \frac{5\pi}{6} + 2n\pi$

For III Quadrant

$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\theta = \frac{7\pi}{6} + 2n\pi$

$$\text{S.S.} = \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{11\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\} \cup \left\{\frac{7\pi}{6} + 2n\pi\right\}, \quad n \in Z$$

(iv) $\cot^2\theta = \frac{1}{3} \Rightarrow \sqrt{\cot^2\theta} = \pm\sqrt{\frac{1}{3}} \Rightarrow \cot\theta = \frac{1}{\sqrt{3}}$ and $\cot\theta = -\frac{1}{\sqrt{3}}$

Since $\cot\theta$ is $+$ve in I & III quadrant with the reference angle is $\pi/3$:

For I Quadrant

$\theta = \frac{\pi}{3}$

$\theta = \frac{\pi}{3} + n\pi$

For III Quadrant

$\theta = \pi + \frac{\pi}{3}$

$\theta = \frac{4\pi}{3} + n\pi$

Since $\cot\theta$ is $-$ve in II & IV quadrant with the reference angle is $\pi/3$:

For II Quadrant

$\theta = \pi - \frac{\pi}{3}$

$\theta = \frac{2\pi}{3} + n\pi$

For IV Quadrant

$\theta = 2\pi - \frac{\pi}{3}$

$\theta = \frac{5\pi}{3} + n\pi$

$$\text{S.S.} = \left\{\frac{\pi}{3} + n\pi\right\} \cup \left\{\frac{4\pi}{3} + n\pi\right\} \cup \left\{\frac{2\pi}{3} + n\pi\right\} \cup \left\{\frac{5\pi}{3} + n\pi\right\}, \quad n \in Z$$

SOLUTION OF EXERCISE # 14

Find the values of θ satisfying the following equations:

Q.3: $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$

Solution: Here $a = 3, \quad b = 2\sqrt{3}, \quad c = 1$

By using quadratic formula : $\tan\theta = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\tan\theta = \frac{-2\sqrt{3} \pm \sqrt{\left(2\sqrt{3}\right)^2 - 4(3)(1)}}{2(3)}$$

$$\tan\theta = \frac{-2\sqrt{3} \pm \sqrt{12-12}}{6} = \frac{-2\sqrt{3}}{6} = \frac{-1}{\sqrt{3}}$$

Since $\tan\theta$ is $-$ve in II & IV quadrant

with the reference angle is $\pi/6$:

For II Quadrant	For IV Quadrant
$\theta = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$	$\theta = 2\pi - \dfrac{\pi}{6} = \dfrac{11\pi}{6}$
$\theta = \dfrac{5\pi}{6} + n\pi$	$\theta = \dfrac{11\pi}{6} + n\pi$

$$S.S = \left\{\frac{5\pi}{6} + n\pi\right\} \cup \left\{\frac{4\pi}{6} + n\pi\right\}, \quad n \in Z$$

Q.4: $\tan^2\theta - \sec\theta - 1 = 0$

Solution: $\tan^2\theta - \sec\theta - 1 = 0$

$\sec^2\theta - 1 - \sec\theta - 1 = 0 \Rightarrow \sec^2\theta - \sec\theta - 2 = 0$

$\sec^2\theta - 2\sec\theta + \sec\theta - 2 = 0$ by factorization

$\sec^2\theta(\sec\theta - 2) + 1(\sec\theta - 2) = 0$

$(\sec\theta - 2)(\sec\theta + 1) = 0$

Either

$\sec\theta = 2 \quad \Rightarrow \quad \cos\theta = \dfrac{1}{2}$

Since $\cos\theta$ is $+$ve in I & IV quadrant

with the reference angle is $\pi/3$:

For I Quadrant	For IV Quadrant
$\theta = \dfrac{\pi}{3}$	$\theta = 2\pi - \dfrac{\pi}{3} = \dfrac{5\pi}{3}$
$\theta = \dfrac{\pi}{3} + 2n\pi$	$\theta = \dfrac{5\pi}{3} + 2n\pi$

OR

$\sec\theta = -1$

$\cos\theta = -1$

$\theta = \cos^{-1}(-1)$

$\theta = \pi + 2n\pi$

$\theta = 2n\pi + \pi$

$\theta = (2n+1)\pi$

$$S.S. = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\} \cup \{(2n+1)\pi\}, \quad n \in Z$$

Q.5: $2\sin\theta + \cos^2\theta - 1 = 0$

Solution: $2\sin\theta + \cos^2\theta - 1 = 0$

$2\sin\theta + (1 - \sin^2\theta) - 1 = 0$

$2\sin\theta + \cancel{1} - \sin^2\theta - \cancel{1} = 0$

$2\sin\theta - \sin^2\theta = 0$

$\sin\theta(2 - \sin\theta) = 0$

Either $\sin\theta = 0$ **OR** $2 - \sin\theta = 0$

$\sin\theta = 0$	$2 - \sin\theta = 0$
$\theta = \sin^{-1}(0)$	$\sin\theta = 2$
$\theta = 0$	Which is not possible
$\theta = n\pi$	$\because \ -1 \le \sin\theta \le 1$

$$S.S. = \{n\pi, \ \forall\ n \in z\}, \quad n \in Z$$

Q.6: $2\sin^2\theta - \sin\theta = 0$

Solution: $2\sin^2\theta - \sin\theta = 0 \Rightarrow \sin\theta(2\sin\theta - 1) = 0$

Either $\sin\theta = 0$

$\sin\theta = 0$

$\theta = \sin^{-1}(0)$

$\theta = 0$

$\theta = 0 + n\pi$

$\theta = n\pi$

OR $2\sin\theta - 1 = 0$

$\Rightarrow \quad \sin\theta = 1/2$

Since $\sin x$ is $+$ve in I & II quadrant

with the reference angle is $\pi/6$:

For I Quadrant	For II Quadrant
$\theta = \dfrac{\pi}{6}$	$\theta = \pi - \dfrac{\pi}{6}$
$\theta = \dfrac{\pi}{6} + 2n\pi$	$\theta = \dfrac{5\pi}{6} + 2n\pi$

$$S.S. = \{n\pi\} \cup \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\}, \quad n \in Z$$

SOLUTION OF EXERCISE # 14

Q.7: $3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

Solution: $3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

$3\cos^2\theta - 3\sqrt{3}\sin\theta\cos\theta + \sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

$3\cos\theta\left(\cos\theta - \sqrt{3}\sin\theta\right) + \sqrt{3}\sin\theta\left(\cos\theta - \sqrt{3}\sin\theta\right) = 0$

$\left(\cos\theta - \sqrt{3}\sin\theta\right)\left(3\cos\theta + \sqrt{3}\sin\theta\right) = 0$

Either $\cos\theta - \sqrt{3}\sin\theta = 0$ OR $3\cos\theta + \sqrt{3}\sin\theta = 0$

$\cos\theta = \sqrt{3}\sin\theta$

$\tan\theta = 1/\sqrt{3}$

Since $\tan x$ is $+$ve in I & III quad. with the reference angle is $\pi/6$:

For I Quad.

$\theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{6} + n\pi$

For III Quad.

$\theta = \pi + \frac{\pi}{6}$

$\theta = \frac{7\pi}{6} + n\pi$

$\sqrt{3}\sin\theta = -3\cos\theta$

$\tan\theta = -\sqrt{3}$

Since $\tan x$ is $-$ve in II & IV quad. with the reference angle is $\pi/3$:

For II Quad.

$\theta = \pi - \frac{\pi}{3}$

$\theta = \frac{2\pi}{3} + n\pi$

For IV Quad.

$\theta = 2\pi - \frac{\pi}{3}$

$\theta = \frac{5\pi}{3} + n\pi$

$$S.S. = \left\{\frac{\pi}{6} + n\pi\right\} \cup \left\{\frac{7\pi}{6} + n\pi\right\} \cup \left\{\frac{2\pi}{3} + n\pi\right\} \cup \left\{\frac{5\pi}{3} + n\pi\right\},\quad n \in Z$$

Q.8: $4\sin^2\theta - 8\cos\theta + 1 = 0$

Solution: $4\sin^2\theta - 8\cos\theta + 1 = 0$

$4\left(1 - \cos^2\theta\right) - 8\cos\theta + 1 = 0$

$4\cos^2\theta + 8\cos\theta - 5 = 0$

$4\cos^2\theta + 10\cos\theta - 2\cos\theta - 5 = 0$

$2\cos\theta(2\cos\theta + 5) - 1(2\cos\theta + 5) = 0$

$(2\cos\theta + 5)(2\cos\theta - 1) = 0$

Either $2\cos\theta + 5 = 0$ OR $2\cos\theta - 1 = 0$

$2\cos\theta + 5 = 0$

$2\cos\theta = -5$

$\cos\theta = -\frac{5}{2}$

$\cos\theta = -2.5$

Which is not possible

$\because -1 \le \cos\theta \le 1$

$\cos\theta = \frac{1}{2}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ = \frac{\pi}{3}$

Since $\cos x$ is $+$ve in I & IV quad. with the reference angle is $\pi/3$:

For I Quadrant

$\theta = \frac{\pi}{3}$

$\theta = \frac{\pi}{3} + 2n\pi$

For IV Quadrant

$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\theta = \frac{5\pi}{3} + 2n\pi$

$$S.S. = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\},\quad n \in Z$$

Find the solution sets of the following equations:

Q.9: $\sqrt{3}\tan x - \sec x - 1 = 0$

Solution: $\sqrt{3}\tan x - \sec x - 1 = 0 \Rightarrow \sqrt{3}\tan x = 1 + \sec x \Rightarrow \left(\sqrt{3}\tan x\right)^2 = \left(1 + \sec x\right)^2$

$3\tan^2 x = 1 + \sec^2 x + 2\sec x \Rightarrow 3\left(\sec^2 x - 1\right) = 1 + \sec^2 x + 2\sec x$

$3\sec^2 x - 3 - 1 - \sec^2 x - 2\sec x = 0 \Rightarrow 2\sec^2 x - 2\sec x - 4 = 0$

$\sec^2 x - \sec x - 2 = 0 \Rightarrow \sec^2 x - 2\sec x + \sec x - 2 = 0$

$\Rightarrow \sec x(\sec x - 2) + 1(\sec x - 2) = 0 \Rightarrow (\sec x - 2)(\sec x + 1) = 0$

Either $\sec x - 2 = 0$ OR $\sec x + 1 = 0$

$\sec x = 2 \Rightarrow \cos x = 1/2$

Since $\cos x$ is + ve in I & IV quadrant with the reference angle is $\pi/3$:

For I Quadrant

For I Quadrant

$\sec x = -1$

$\cos x = -1$

$x = \cos^{-1}(-1)$

$x = \pi$

$x = \pi + 2n\pi, \quad \forall n \in z$

SOLUTION OF EXERCISE # 14

$x=\frac{\pi}{3} \Rightarrow x=\frac{\pi}{3}+2n\pi$

$x=2\pi-\frac{\pi}{3} \Rightarrow x=\frac{5\pi}{3}+2n\pi$

It is not admissible

$\because$ it does not satisfied the given eq.

$$S.S.=\left\{\frac{\pi}{3}+2n\pi\right\}\cup\{\pi+2n\pi\},\quad n\in Z$$

Q.10: $\cos 2x=\sin 3x$

Solution:

$\cos 2x=\sin 3x$

$\cos^2 x-\sin^2 x=3\sin x-4\sin^3 x$

$1-\sin^2 x-\sin^2 x-3\sin x+4\sin^3 x=0$

$4\sin^3 x-2\sin^2 x-3\sin x+1=0$

By synthetic division, we have:

	4	-2	-3	1	
1		4	2	-1	
	4	2	-1	0	

The above equation is satisfied by: $\sin x=1 \Rightarrow x=\frac{\pi}{2}+2n\pi,\quad \forall n\in z$

& $4\sin^2 x+2\sin x-1=0$

By using quadratic formula: $\sin x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Here $a=4, b=2, c=-1$

$$\sin x=\frac{-2\pm\sqrt{(2)^2-4(4)(-1)}}{2(4)}=\frac{-2\pm\sqrt{4+16}}{8}=\frac{-2\pm\sqrt{4\times5}}{8}=\frac{-2\pm2\sqrt{5}}{8}=\frac{2\left(-1\pm\sqrt{5}\right)}{8}=\frac{-1\pm\sqrt{5}}{4}$$

Either $\sin x=\frac{-1+\sqrt{5}}{4}$

$x=\sin^{-1}\left(\frac{-1+\sqrt{5}}{4}\right) \Rightarrow x=18^\circ=\frac{\pi}{10}$

Since $\sin x$ is $+$ve in I & II quadrant

with the reference angle is $\frac{\pi}{10}$:

OR $\sin x=\frac{-1-\sqrt{5}}{4}$

$x=\sin^{-1}\left(\frac{-1-\sqrt{5}}{4}\right) \Rightarrow x=54^\circ=\frac{3\pi}{10}$

Since $\sin x$ is $-$ve in II & IV quadrant

with the reference angle is $\frac{3\pi}{10}$:

For I Quadrant

$x=\frac{\pi}{10}$

$x=\frac{\pi}{10}+2n\pi$

For II Quadrant

$x=\pi-\frac{\pi}{10}$

$x=\frac{9\pi}{10}+2n\pi$

For III Quadrant

$x=\pi+\frac{3\pi}{10}$

$x=\frac{13\pi}{10}+2n\pi$

For IV Quadrant

$x=2\pi-\frac{3\pi}{10}$

$x=\frac{17\pi}{10}+2n\pi$

SOLUTION OF EXERCISE # 14

$$S.S.=\left\{\frac{\pi}{2}+2n\pi\right\}\cup\left\{\frac{\pi}{10}+2n\pi\right\}\cup\left\{\frac{9\pi}{10}+2n\pi\right\}\cup\left\{\frac{13\pi}{10}+2n\pi\right\}\cup\left\{\frac{17\pi}{10}+2n\pi\right\},\quad n\in Z$$

Q.11: $\sec 3\theta=\sec\theta$

Solution: $\sec 3\theta=\sec\theta \Rightarrow \dfrac{1}{\cos 3\theta}=\dfrac{1}{\cos\theta}$

$\cos 3\theta=\cos\theta \Rightarrow \cos 3\theta-\cos\theta=0$

$-2\sin\left(\dfrac{3\theta+\theta}{2}\right)\sin\left(\dfrac{3\theta-\theta}{2}\right)=0$

$-2\sin 2\theta\sin\theta=0$

Either $\sin 2\theta=0$ **OR** $\sin\theta=0$

$2\theta=\sin^{-1}(0)$ | $\theta=\sin^{-1}(0)$

$2\theta=n\pi\Rightarrow\theta=\dfrac{n\pi}{2}$ | $\theta=n\pi$

$$S.S.=\left\{\frac{n\pi}{2}\right\}\cup\{n\pi\},\quad n\in Z$$

Q.12: $\tan 2\theta+\cot\theta=0$

Solution: $\tan 2\theta+\cot\theta=0$

$\tan 2\theta=-\cot\theta \Rightarrow \dfrac{\sin 2\theta}{\cos 2\theta}=-\dfrac{\cos\theta}{\sin\theta}$

$\sin 2\theta\sin\theta=-\cos 2\theta\cos\theta$

$\sin 2\theta\sin\theta+\cos 2\theta\cos\theta=0$

$\cos 2\theta\cos\theta+\sin 2\theta\sin\theta=0$

$\cos(2\theta-\theta)=0 \quad \because\left\{\begin{array}{l}\cos(\alpha+\beta)=\\ \cos\alpha\cos\beta+\sin\alpha\sin\beta\end{array}\right\}$

$\cos\theta=0$

$\theta=\cos^{-1}(0) \Rightarrow \theta=(2n+1)\dfrac{\pi}{2}$

$$S.S.=\left\{(2n+1)\frac{\pi}{2}\right\},\quad n\in Z$$

Q.13: $\sin 2x+\sin x=0$

Solution: $\sin 2x+\sin x=0$

$2\sin x\cos x+\sin x=0 \quad \because\{\sin 2x=2\sin x\cos x\}$

$\sin x(2\cos x+1)=0$

Either $\sin x=0$ **OR** $2\cos x+1=0$

$\sin x=0$

$x=\sin^{-1}(0)$

$x=n\pi$

$2\cos x=-1 \Rightarrow \cos x=-\frac{1}{2}$

Since $\cos x$ is $-$ve in II & III quad.

with the reference angle is $\dfrac{\pi}{3}$:

For II Quadrant

$x=\pi-\dfrac{\pi}{3}$

$x=\dfrac{2\pi}{3}+2n\pi$

For III Quadrant

$x=\pi+\dfrac{\pi}{3}$

$x=\dfrac{4\pi}{3}+2n\pi$

$$S.S.=\{n\pi\}\cup\left\{\frac{2\pi}{3}+2n\pi\right\}\cup\left\{\frac{4\pi}{3}+2n\pi\right\},\quad n\in Z$$

Q.14: $\sin 4x-\sin 2x=\cos 3x$

Solution: $\sin 4x-\sin 2x=\cos 3x$

$2\cos\left(\dfrac{4x+2x}{2}\right)\sin\left(\dfrac{4x-2x}{2}\right)=\cos 3x$

$2\cos 3x\sin x-\cos 3x=0 \Rightarrow \cos 3x(2\sin x-1)=0$

Either $\cos 3x=0$ **OR** $2\sin x-1=0$

$3x=\cos^{-1}(0)$

$3x=(2n+1)\dfrac{\pi}{2}$

$x=(2n+1)\dfrac{\pi}{6}$

$2\sin x=1 \Rightarrow \sin x=\dfrac{1}{2}$

Since $\sin x$ is $+$ve in I & II quad.

with the reference angle is $\dfrac{\pi}{6}$:

For I Quadrant

$x=\dfrac{\pi}{6}$

$x=\dfrac{\pi}{6}+2n\pi$

For II Quadrant

$x=\pi-\dfrac{\pi}{6}$

$x=\dfrac{5\pi}{6}+2n\pi$

$$S.S.=\left\{(2n+1)\frac{\pi}{6}\right\}\cup\left\{\frac{\pi}{6}+2n\pi\right\}\cup\left\{\frac{5\pi}{6}+2n\pi\right\},\quad n\in Z$$

SOLUTION OF EXERCISE # 14

Q.15: $\sin x+\cos 3x=\cos 5x$

Solution: $\sin x+\cos 3x=\cos 5x$

$\sin x=\cos 5x-\cos 3x$

$\sin x=-2\sin\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)$

$\sin x=-2\sin 4x\sin x \Rightarrow \sin x+2\sin 4x\sin x=0$

$\sin x(1+2\sin 4x)=0$

Either $\sin x=0$ **OR** $1+2\sin 4x=0$

$\sin x=0$

$x=\sin^{-1}(0)$

$x=n\pi$

$2\sin 4x=-1 \Rightarrow \sin 4x=-\frac{1}{2}$

Since $\sin x$ is $-$ve in II & IV quad.

with the reference angle is $\frac{\pi}{6}$:

For III Quadrant

$4x=\pi+\frac{\pi}{6}$

$4x=\frac{7\pi}{6}+2n\pi$

$x=\frac{7x}{24}+\frac{n\pi}{2}$

For IV Quadrant

$4x=2\pi-\frac{\pi}{6}$

$4x=\frac{11\pi}{6}+2n\pi$

$x=\frac{11\pi}{24}+\frac{n\pi}{2}$

$$S.S.=\{n\pi\}\cup\left\{\frac{7\pi}{24}+\frac{n\pi}{2}\right\}\cup\left\{\frac{11\pi}{24}+\frac{n\pi}{2}\right\},\quad n\in Z$$

Q.16: $\sin 3x+\sin 2x+\sin x=0$

Solution: $\sin 3x+\sin 2x+\sin x=0$

$\sin 3x+\sin x+\sin 2x=0$

$2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)+\sin 2x=0$

$2\sin 2x\cos x+\sin 2x=0$

$\sin 2x(2\cos x+1)=0$

Either $\sin 2x=0$ **OR** $2\cos x+1=0$

$\sin 2x=0$

$2x=\sin^{-1}(0)$

$2x=n\pi$

$x=\frac{n\pi}{2}$

$2\cos x=-1 \Rightarrow x=\cos^{-1}\left(-\frac{1}{2}\right)$

Since $\cos x$ is $-$ve in II & III quad.

with the reference angle is $\frac{\pi}{3}$:

For II Quad.

$x=\pi-\frac{\pi}{3}$

$x=\frac{2\pi}{3}+2n\pi$

For III Quad.

$x=\pi+\frac{\pi}{3}$

$x=\frac{4\pi}{3}+2n\pi$

$$S.S.=\left\{\frac{n\pi}{2}\right\}\cup\left\{\frac{2\pi}{3}+2n\pi\right\}\cup\left\{\frac{4\pi}{3}+2n\pi\right\},\quad n\in Z$$

Q.17: $\sin 7x-\sin x=\sin 3x$

Solution: $\sin 7x-\sin x=\sin 3x$

$2\cos\left(\frac{7x+x}{2}\right)\sin\left(\frac{7x-x}{2}\right)=\sin 3x$

$2\cos 4x\sin 3x-\sin 3x=0$

$\sin 3x(2\cos 4x-1)=0$

Either $\sin 3x=0$ **OR** $2\cos 4x-1=0$

$\sin 3x=0$

$3x=\sin^{-1}(0)$

$3x=n\pi$

$x=\frac{n\pi}{3}$

$2\cos 4x=1 \Rightarrow \cos 4x=\frac{1}{2}$

Since $\cos 4x$ is $+$ve in I & IV quad.

with the reference angle is $\frac{\pi}{3}$:

Q.18: $\sin x+\sin 3x+\sin 5x=0$

Solution: $\sin 5x+\sin x+\sin 3x=0$

$2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right)+\sin 3x=0$

$2\sin 3x\cos 2x+\sin 3x=0$

$\sin 3x(2\cos 2x+1)=0$

Either, $\sin 3x=0$ **OR** $2\cos 2x+1=0$

$\sin 3x=0$

$3x=\sin^{-1}(0)$

$3x=n\pi$

$x=\frac{n\pi}{3}$

$2\cos 2x=-1 \Rightarrow \cos 2x=-\frac{1}{2}$

Since $\cos 2x$ is $-$ve in II & III quad.

with the reference angle is $\frac{\pi}{3}$:

SOLUTION OF EXERCISE # 14

For I Quadrant	For IV Quadrant
$4x=\frac{\pi}{3}$	$4x=2\pi-\frac{\pi}{3}$
$4x=\frac{\pi}{3}+2n\pi$	$4x=\frac{5\pi}{12}+2n\pi$
$x=\frac{\pi}{12}+\frac{n\pi}{2}$	$x=\frac{5\pi}{12}+\frac{n\pi}{2}$

$$S.S.=\left\{\frac{n\pi}{3}\right\}\cup\left\{\frac{\pi}{12}+\frac{n\pi}{2}\right\}\cup\left\{\frac{5\pi}{12}+\frac{n\pi}{2}\right\},\quad n\in Z$$

For II Quadrant	For III Quadrant
$2x=\pi-\frac{\pi}{3}$	$2x=\pi+\frac{\pi}{3}$
$2x=\frac{2\pi}{3}+2n\pi$	$2x=\frac{4\pi}{3}+2n\pi$
$x=\frac{\pi}{3}+n\pi$	$x=\frac{2\pi}{3}+n\pi$

$$S.S.=\left\{\frac{n\pi}{3}\right\}\cup\left\{\frac{\pi}{3}+n\pi\right\}\cup\left\{\frac{2\pi}{3}+n\pi\right\},\quad n\in Z$$

Q.19: $\sin\theta+\sin3\theta+\sin5\theta+\sin7\theta=0$

Solution: $\sin\theta+\sin3\theta+\sin5\theta+\sin7\theta=0$

$$[\sin7\theta+\sin\theta]+[\sin5\theta+\sin3\theta]=0$$

$$\left[2\sin\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)\right]+\left[2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)\right]=0$$

$$2\sin4\theta\cos3\theta+2\sin4\theta\cos\theta=0$$

$$2\sin4\theta[\cos3\theta+\cos\theta]=0$$

$$2\sin4\theta\left[2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right]=0$$

$$4\sin4\theta\cos2\theta\cos\theta=0$$

Either	OR	OR
$\sin4\theta=0$	$\cos2\theta=0$	$\cos\theta=0$
$4\theta=n\pi$	$2\theta=(2n+1)\frac{\pi}{2}$	$\theta=(2n+1)\frac{\pi}{2}$
$\theta=\frac{n\pi}{4}$	$\theta=(2n+1)\frac{\pi}{4}$	

$$S.S.=\left\{\frac{n\pi}{4}\right\}\cup\left\{(2n+1)\frac{\pi}{4}\right\}\cup\left\{(2n+1)\frac{\pi}{2}\right\},\quad n\in Z$$

Q.20: $\cos\theta+\cos3\theta+\cos5\theta+\cos7\theta=0$

Solution: $\cos\theta+\cos3\theta+\cos5\theta+\cos7\theta=0$

$$[\cos7\theta+\cos\theta]+[\cos5\theta+\cos3\theta]=0$$

$$2\cos\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)+2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)=0$$

$$2\cos4\theta\cos3\theta+2\cos4\theta\cos\theta=0$$

$$2\cos4\theta[\cos3\theta+\cos\theta]=0$$

$$2\cos4\theta\left[2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right]=0$$

$$4\cos4\theta\cos2\theta\cos\theta=0$$

Either	OR	OR
$\cos4\theta=0$	$\cos2\theta=0$	$\cos\theta=0$
$4\theta=(2n+1)\frac{\pi}{2}$	$2\theta=(2n+1)\frac{\pi}{2}$	$\theta=(2n+1)\frac{\pi}{2}$
$\theta=(2n+1)\frac{\pi}{8}$	$\theta=(2n+1)\frac{\pi}{4}$	

$$S.S.=\left\{(2n+1)\frac{\pi}{8}\right\}\cup\left\{(2n+1)\frac{\pi}{4}\right\}\cup\left\{(2n+1)\frac{\pi}{2}\right\},\quad n\in Z$$

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ANSWER:

1. An equation containing at least one trigonometric function is called __________ equation:
[a] Algebraic [b] Exponential [c] Trigonometric [d] None of these

2. If the Reference angle is θ, then solution in third quadrant is:
[a] θ [b] $\pi-\theta$ [c] $\pi+\theta$ [d] $2\pi-\theta$

3. If the Reference angle is θ, then solution in second quadrant is:
[a] θ [b] $\pi-\theta$ [c] $\pi+\theta$ [d] $2\pi-\theta$

4. If $\sin x=\frac{\sqrt{3}}{2}$, then reference angle is: [a] $\frac{\pi}{2}$ [b] $\frac{\pi}{3}$ [c] $\frac{\pi}{4}$ [d] $\frac{\pi}{6}$

5. If $\cos x=\frac{1}{\sqrt{2}}$, then reference angle is: [a] $\frac{\pi}{2}$ [b] $\frac{\pi}{3}$ [c] $\frac{\pi}{4}$ [d] $\frac{\pi}{6}$

6. If $\tan x=-1$, then reference angle is: [a] $\frac{\pi}{2}$ [b] $\frac{\pi}{3}$ [c] $\frac{\pi}{4}$ [d] $\frac{\pi}{6}$

7. If $\sec x=-2$, then reference angle is: [a] $\frac{\pi}{6}$ [b] $\frac{\pi}{3}$ [c] $-\frac{\pi}{3}$ [d] $\frac{\pi}{4}$

8. If $\csc x=2$, then reference angle is: [a] $\frac{\pi}{6}$ [b] $\frac{\pi}{3}$ [c] $\frac{\pi}{4}$ [d] $\frac{\pi}{3}$

9. If $\cot^2 x=\frac{1}{3}$, then reference angle is: [a] $\frac{\pi}{2}$ [b] $\frac{\pi}{3}$ [c] $\frac{\pi}{4}$ [d] $\frac{\pi}{6}$

10. Solution set of 1 + cos x = 0 is: [a] $\left\{\frac{1}{2}+2n\pi\right\}$ [b] $\{\pi+2n\pi\}$ [c] $\left\{\frac{3\pi}{2}+2n\pi\right\}$ [d] $\left\{-\frac{\pi}{2}+2n\pi\right\}$

11. If $\sin x=\frac{1}{2}$, then x = ? [a] $\frac{\pi}{6},\frac{7\pi}{6}$ [b] $\frac{\pi}{6},\frac{5\pi}{6}$ [c] $\frac{\pi}{3},\frac{\pi}{6}$ [d] $\frac{\pi}{3},\frac{2\pi}{3}$

12. If $\sin x=-\frac{\sqrt{3}}{2}$, then solution is: [a] $\frac{4\pi}{6},\frac{5\pi}{6}$ [b] $\frac{4\pi}{3},\frac{5\pi}{3}$ [c] $\frac{5\pi}{6},\frac{\pi}{6}$ [d] $\frac{\pi}{3},\frac{7\pi}{3}$

13. If $\sec^2\theta=\frac{4}{3}$, then $\tan^2\theta=?$ [a] $\frac{1}{3}$ [b] $-\frac{1}{3}$ [c] $\frac{2}{3}$ [d] $-\frac{2}{3}$

14. $\tan^{-1}\left(-\sqrt{3}\right)=?$ [a] $\frac{2\pi}{3}$ [b] $-\frac{2\pi}{3}$ [c] $-\frac{\pi}{6}$ [d] $-\frac{\pi}{3}$

15. The solution set of tan x = 0 is given by: [a] $\{2n\pi\}$ [b] $\{\pi+2n\pi\}$ [c] $\{n\pi\}$ [d] $\{3n\pi\}$

16. Trigonometric equation has solutions: [a] 0 [b] 1 [c] 2 [d] Infinite

17. The equation $\cos^2 x=\frac{3}{4}$ has: [a] One [b] Two [c] Four [d] Infinite

18. If $\sin x=-\frac{1}{2}$, then x = ? [a] $\frac{\pi}{3},\frac{2\pi}{3}$ [b] $\frac{7\pi}{6},\frac{11\pi}{6}$ [c] $\frac{\pi}{6},\frac{5\pi}{6}$ [d] $\frac{4\pi}{3},\frac{5\pi}{3}$

19. The solution of equation 1 + cos θ = 0 are in quadrants:
[a] I and IV [b] II and III [c] II and IV [d] None of these

OBJECTIVE TYPE QUESTIONS

20. The solution of equation $\frac{1}{2}+\sin\theta=0$ are in quadrants:

[a] I & IV **[b]** I & III **[c]** III & IV **[d]** II & IV

21. The solution of the equation $\tan x=\frac{1}{\sqrt{3}}$ are in quadrants:

[a] I and II **[b]** I and III **[c]** II and IV **[d]** I and IV

22. The solution set of the equation cos x – 1 = 0 in the interval $[0,\ 2\pi]$ is:

[a] $\frac{\pi}{2}$ **[b]** $0,\ 2\pi$ **[c]** $-\pi$ **[d]** $\frac{\pi}{2}$

23. The solution set of the equation cosec θ = 2 in the interval $[0,\ 2\pi]$ is:

[a] $\left\{\frac{\pi}{3},\ \frac{\pi}{4}\right\}$ **[b]** $\left\{\frac{\pi}{6},\ \frac{5\pi}{6}\right\}$ **[c]** $\left\{\frac{\pi}{2},\ \frac{2\pi}{3}\right\}$ **[d]** $\left\{\frac{\pi}{4},\ \frac{3\pi}{4}\right\}$

24. The Solution set of the equation tan 2x = 1 in the interval $[0,\ 2\pi]$ is:

[a] $\left\{\frac{\pi}{4},\ \frac{5\pi}{4}\right\}$ **[b]** $\left\{\frac{\pi}{8},\ \frac{5\pi}{8}\right\}$ **[c]** $\left\{\frac{\pi}{4},\ \frac{3\pi}{4}\right\}$ **[d]** $\left\{\frac{\pi}{8},\ \frac{9\pi}{8}\right\}$

25. If $\sin x=\frac{\sqrt{3}}{2}$ and $x\in[0,\ 2\pi]$ then x = ?

[a] $\frac{\pi}{3},\ \frac{2\pi}{3}$ **[b]** $\frac{\pi}{4},\ \frac{3\pi}{4}$ **[c]** $\frac{5\pi}{3},\ \frac{4\pi}{3}$ **[d]** $\frac{7\pi}{6},\ \frac{5\pi}{6}$

26. If cos 2x = 0, then solution in 1st quadrant is:

[a] $30°$ **[b]** $45°$ **[c]** $60°$ **[d]** $15°$

27. If $\sin x=\frac{1}{2}$, then x is a member of:

[a] $[0,\ \pi]$ **[b]** $[-\pi,\ 0]$ **[c]** $[\pi,\ 2\pi]$ **[d]** $\left[\frac{\pi}{3}+n\pi\right]$

28. If $\tan x=-1$, then general value of x is:

[a] $\left\{\frac{3\pi}{4}+n\pi\right\}$ **[b]** $\left\{\frac{3\pi}{6}+n\pi\right\}$ **[c]** $\left\{\frac{\pi}{2}+n\pi\right\}$ **[d]** $\left\{\frac{\pi}{3}+n\pi\right\}$

ANSWER KEYS

1	c	2	c	3	b	4	b	5	c	6	c	7	b
8	a	9	d	10	b	11	b	12	b	13	b	14	d
15	c	16	d	17	d	18	b	19	d	20	c	21	b
22	b	23	b	24	d	25	a	26	b	27	a	28	a

FREQUENTLY ASKED AND IMPORTANT QUESTIONS

MODEL TEST CHAPTER NO. 14

Time Allowed: 45 Minutes **Maximum Marks:25**

Q.1: Choose the correct answer: $1 \times 5 = 5$

1. If $\cos x = -\frac{1}{2}$, then reference angle is: **[a]** $\frac{\pi}{3}$ **[b]** $-\frac{\pi}{3}$ **[c]** $\frac{\pi}{6}$ **[d]** $-\frac{\pi}{6}$

2. If $\sin x + \cos x = 0$, then x = ? **[a]** $\frac{\pi}{4}$ **[b]** $-\frac{\pi}{4}$ **[c]** $\frac{3\pi}{4}$ **[d]** $\frac{3\pi}{4} + n\pi$

3. If the Reference angle is θ, then solution in fourth quadrant is:
[a] θ **[b]** $\pi - \theta$ **[c]** $\pi + \theta$ **[d]** $2\pi - \theta$

4. Solution of equation $\tan x = \frac{1}{\sqrt{3}}$ lie in:
[a] I and II quad. **[b]** I and III quad. **[c]** II and IV quad. **[d]** I and IV quad.

5. The solution set of the equation cosec θ = 2 in the interval $[0, 2\pi]$ is:
[a] $\left\{\frac{\pi}{3}, \frac{\pi}{4}\right\}$ **[b]** $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$ **[c]** $\left\{\frac{\pi}{2}, \frac{2\pi}{3}\right\}$ **[d]** $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

Q.2: Write short answers to any five(05) questions: $2 \times 05 = 10$

(i) Solve the trigonometric equation: $\sin x + \cos x = 0$. **(ii)** Find the value of θ satisfying the equation: $2\sin^2\theta - \sin\theta = 0$. **(iii)** Solve the trigonometric equation: $1 + \cos x = 0$. **(iv)** Solve the trigonometric equation: $\sin x \cos x = \frac{\sqrt{3}}{4}$. **(v)** Solve the trigonometric equation: $\sin x = \frac{1}{2}$. **(vi)** Solve the trigonometric equation: $\sec x = -2$. **(vii)** Find the value of θ satisfying the equation: $2\sin\theta + \cos^2\theta - 1 = 0$.

Attempt any one question: $1 \times 10 = 10$

Q.3 **[a]:** Find the value of θ satisfying the equation: $\tan^2\theta - \sec\theta + 1 = 0$.
[b]: Solve the trigonometric equation: $\tan^2\theta = 1/3$.

Q.4 **[a]:** Solve the trigonometric equation: $\cos 2x = \sin 3x$.
[b]: Solve the trigonometric equation: $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$.

THE END