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EXERCISE: 9.4

Prove the Identities:

Q.1 $\tan\theta + \cot\theta = \operatorname{Cosec}\theta \operatorname{Sec}\theta$

$LHS = \tan\theta + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}$ $\quad (\because \sin^2\theta + \cos^2\theta = 1)$

$= \operatorname{Cosec}\theta \cdot \operatorname{Sec}\theta = RHS.$

So $\tan\theta + \cot\theta = \operatorname{Cosec}\theta \cdot \operatorname{Sec}\theta$

(Proved)

Q.2 $\operatorname{Sec}\theta \cdot \operatorname{Cosec}\theta \cdot \sin\theta\cos\theta = 1$

$LHS = \operatorname{Sec}\theta \cdot \operatorname{Cosec}\theta \cdot \sin\theta \cdot \cos\theta$

$= \dfrac{1}{\sin\theta} \cdot \dfrac{1}{\cos\theta} \cdot \sin\theta \cdot \cos\theta$

$= 1 = RHS.$

So $\operatorname{Sec}\theta \cdot \operatorname{Cosec}\theta \cdot \sin\theta \cdot \cos\theta = 1.$

(Proved)

Q.3 $\cos\theta + \tan\theta\sin\theta = \operatorname{Sec}\theta$

$LHS = \cos\theta + \tan\theta \sin\theta$

$= \cos\theta + \dfrac{\sin\theta}{\cos\theta} \cdot \sin\theta$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta}$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= \dfrac{1}{\cos\theta} = \operatorname{Sec}\theta = RHS$

Thus

$\cos\theta + \tan\theta \sin\theta = \operatorname{Sec}\theta$

(Proved)

Q.4 $\operatorname{Cosec}\theta + \tan\theta \operatorname{Sec}\theta = \operatorname{Cosec}\theta \operatorname{Sec}^2\theta$

$LHS = \operatorname{Cosec}\theta + \tan\theta \cdot \operatorname{Sec}\theta$

$= \dfrac{1}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{1}{\cos\theta}$

$= \dfrac{1}{\sin\theta} + \dfrac{\sin\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta \cdot \cos^2\theta}$

$= \dfrac{1}{\sin\theta \cdot \cos^2\theta}$ $\quad (\because \sin^2\theta + \cos^2\theta = 1)$

$= \operatorname{Cosec}\theta \operatorname{Sec}^2\theta$ RHS.

Thus $\operatorname{Cosec}\theta + \tan\theta \operatorname{Sec}\theta = \operatorname{Cosec}\theta \operatorname{Sec}^2\theta.$

(Proved)

Q.5 $\operatorname{Sec}^2\theta - \operatorname{Cosec}^2\theta = \tan^2\theta - \cot^2\theta$

$LHS = \operatorname{Sec}^2\theta - \operatorname{Cosec}^2\theta$

$= (1 + \tan^2\theta) - (1 + \cot^2\theta)$

$= 1 + \tan^2\theta - 1 - \cot^2\theta$

$= \tan^2\theta - \cot^2\theta = RHS.$

Thus $\operatorname{Sec}^2\theta - \operatorname{Cosec}^2\theta = \tan^2\theta - \cot^2\theta$

(Proved)

Q.6 $\cot^2\theta - \cos^2\theta = \cot^2\theta \cdot \cos^2\theta$

$LHS = \cot^2\theta - \cos^2\theta$

$= \dfrac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta$

$= \cos^2\theta \left[\dfrac{1}{\sin^2\theta} - 1\right]$

$= \cos^2\theta \left(\dfrac{1 - \sin^2\theta}{\sin^2\theta}\right)$ $\quad \because 1 - \sin^2\theta = \cos^2\theta$

$= \cos^2\theta \cdot \dfrac{\cos^2\theta}{\sin^2\theta}$

$= \cos^2\theta \cdot \cot^2\theta = RHS.$

Thus

$\cot^2\theta - \cos^2\theta = \cos^2\theta \cdot \cot^2\theta$

(Proved)

Q.7 $(\operatorname{Sec}\theta + \tan\theta)(\operatorname{Sec}\theta - \tan\theta) = 1$

$LHS = (\operatorname{Sec}\theta + \tan\theta)(\operatorname{Sec}\theta - \tan\theta)$

$= \operatorname{Sec}^2\theta - \tan^2\theta$

$= (1 + \tan^2\theta) - \tan^2\theta$

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$= 1 = RHS$

Thus $(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)=1$ (Proved)

Q.8 $2\cos^2\theta-1 = 1-2\sin^2\theta$

$LHS = 2\cos^2\theta - 1$

$= 2(1-\sin^2\theta)-1$

$= 2-2\sin^2\theta-1$

$= 1-2\sin^2\theta = RHS$

Thus $2\cos^2\theta-1 = 1-2\sin^2\theta$ (Proved)

Q.9 $\cos^2\theta-\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$RHS = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}}$

$= \dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta} \times \dfrac{\cos^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta-\sin^2\theta}{1} \times 1$ $\quad (\because \sin^2\theta+\cos^2\theta=1)$

$= \cos^2\theta-\sin^2\theta = LHS$

So $\cos^2\theta-\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$ (Proved)

Q.10 $\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \dfrac{\cot\theta-1}{\cot\theta+1}$

$RHS = \dfrac{\cot\theta-1}{\cot\theta+1}$

$= \dfrac{\dfrac{\cos\theta}{\sin\theta}-1}{\dfrac{\cos\theta}{\sin\theta}+1}$

$= \dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} \times \dfrac{\sin\theta}{\sin\theta}$

$= \dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = LHS$

So $\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \dfrac{\cot\theta-1}{\cot\theta+1}$ (Proved)

Q.11 $\dfrac{\sin\theta}{1+\cos\theta} + \cot\theta = \operatorname{cosec}\theta$

$LHS = \dfrac{\sin\theta}{1+\cos\theta} + \cot\theta$

$= \dfrac{\sin\theta}{1+\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta+\cos\theta(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

$= \dfrac{\sin^2\theta+\cos\theta+\cos^2\theta}{(1+\cos\theta)\sin\theta}$

$= \dfrac{(\sin^2\theta+\cos^2\theta)+\cos\theta}{(1+\cos\theta)\sin\theta}$

$\because \sin^2\theta+\cos^2\theta=1$

$= \dfrac{(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

$= \dfrac{1}{\sin\theta} = \operatorname{cosec}\theta = RHS$

Thus $\dfrac{\sin\theta}{1+\cos\theta} + \cot\theta = \operatorname{cosec}\theta$ (Proved)

Q.12 $\dfrac{\cot^2\theta-1}{1+\cot^2\theta} = 2\cos^2\theta-1$

$LHS = \dfrac{\cot^2\theta-1}{1+\cot^2\theta}$

$= \dfrac{\dfrac{\cos^2\theta}{\sin^2\theta}-1}{1+\dfrac{\cos^2\theta}{\sin^2\theta}} = \dfrac{\cos^2\theta-\sin^2\theta}{\sin^2\theta+\cos^2\theta} \times \dfrac{\sin^2\theta}{\sin^2\theta}$

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$$= \frac{\cos^2\theta - (1-\cos^2\theta)}{1} \times 1$$

$\because \sin^2\theta + \cos^2\theta = 1$ and

$\sin^2\theta = 1 - \cos^2\theta$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$$= 2\cos^2\theta - 1 = RHS$$

So $\frac{\cot^2\theta - 1}{1+\cot^2\theta} = 2\cos^2\theta - 1$ (Proved)

Q.13 $\frac{1+\cos\theta}{1-\cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$

$RHS = (\operatorname{cosec}\theta + \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1+\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1+\cos\theta)^2}{\sin^2\theta} = \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1+\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1+\cos\theta}{1-\cos\theta} = LHS.$$

So

$$\frac{1+\cos\theta}{1-\cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$$

(Proved)

Q.14 $(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$

$LHS = (\sec\theta - \tan\theta)^2$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{1-\sin\theta}{1+\sin\theta} = RHS$$

Thus

$$(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta} \text{ (Proved)}$$

Q.15 $\frac{2\tan\theta}{1+\tan^2\theta} = 2\sin\theta\cos\theta$

$LHS = \frac{2\tan\theta}{1+\tan^2\theta}$

$$= \frac{2\,\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{2\sin\theta/\cos\theta}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{2\sin\theta \cdot \cos^2\theta}{(\cos^2\theta+\sin^2\theta)\cos\theta}$$

$\because \sin^2\theta + \cos^2\theta = 1$

$$= \frac{2\sin\theta\cos\theta}{1}$$

$$= 2\sin\theta\cos\theta = RHS.$$

Thus $\frac{2\tan\theta}{1+\tan^2\theta} = 2\sin\theta\cos\theta$

(Proved)

Q.16 $\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$

$LHS = \frac{1-\sin\theta}{\cos\theta}$

Multiplying and dividing by $(1+\sin\theta)$

$$= \frac{1-\sin\theta}{\cos\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$$

$$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{\cos\theta}{1+\sin\theta} = RHS$$

Thus

$$\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta} \text{ (Proved)}$$

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Q.17 $(\tan\theta+\cot\theta)^2=\operatorname{Cosec}^2\theta\cdot\sec^2\theta$

$LHS=(\tan\theta+\cot\theta)^2$

$=\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)^2$

$=\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right)^2$

$\because \sin^2\theta+\cos^2\theta=1$

$=\left(\frac{1}{\sin\theta\cos\theta}\right)^2$

$=(\operatorname{Cosec}\theta\cdot\sec\theta)^2$

$=\operatorname{Cosec}^2\theta\cdot\sec^2\theta=RHS.$

Thus

$(\tan\theta+\cot\theta)^2=\operatorname{Cosec}^2\theta\cdot\sec^2\theta$ (Proved)

Q.18 $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\tan\theta+\sec\theta$

$LHS=\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$

$\because 1+\tan^2\theta=\sec^2\theta$

$1=\sec^2\theta-\tan^2\theta$

$1=(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)$

$=\frac{(\tan\theta+\sec\theta)-(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{(\tan\theta-\sec\theta+1)}$

$=\frac{(\tan\theta+\sec\theta)[1-(\sec\theta-\tan\theta)]}{(1+\tan\theta-\sec\theta)}$

$=\frac{(\tan\theta+\sec\theta)(1+\tan\theta-\sec\theta)}{(1+\tan\theta-\sec\theta)}$

$=\tan\theta+\sec\theta=RHS.$

Thus

$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\tan\theta+\sec\theta$ (Proved)

Q.19 $\frac{1}{\operatorname{Cosec}\theta-\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{Cosec}\theta+\cot\theta}$

$\frac{1}{\operatorname{Cosec}\theta-\cot\theta}+\frac{1}{\operatorname{Cosec}\theta+\cot\theta}=\frac{1}{\sin\theta}+\frac{1}{\sin\theta}$

$\frac{1}{\operatorname{Cosec}\theta-\cot\theta}+\frac{1}{\operatorname{Cosec}\theta+\cot\theta}=\frac{2}{\sin\theta}$

$LHS=\frac{1}{\operatorname{Cosec}\theta-\cot\theta}+\frac{1}{\operatorname{Cosec}\theta+\cot\theta}$

$=\frac{\operatorname{Cosec}\theta+\cot\theta+\operatorname{Cosec}\theta-\cot\theta}{(\operatorname{Cosec}\theta-\cot\theta)(\operatorname{Cosec}\theta+\cot\theta)}$

$=\frac{2\operatorname{Cosec}\theta}{\operatorname{Cosec}^2\theta-\cot^2\theta}$

$=\frac{2\operatorname{Cosec}\theta}{(1+\cot^2\theta)-\cot^2\theta}$

$=\frac{2\operatorname{Cosec}\theta}{1}=\frac{2}{\sin\theta}=RHS$

Thus,

$\frac{1}{\operatorname{Cosec}\theta-\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{Cosec}\theta+\cot\theta}$ (Proved)

Q.20 $\sin^3\theta-\cos^3\theta=(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)$

$LHS=\sin^3\theta-\cos^3\theta$

$\because a^3-b^3=(a-b)(a^2+ab+b^2)$

$=(\sin\theta-\cos\theta)[\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta]$

$=(\sin\theta-\cos\theta)[(\sin^2\theta+\cos^2\theta)+\sin\theta\cos\theta]$

$\because \sin^2\theta+\cos^2\theta=1$

$=(\sin\theta-\cos\theta)[1+\sin\theta\cos\theta]$

$=RHS$

Thus

$\sin^3\theta-\cos^3\theta=(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)$ (Proved)

Q.21 $\sin^6\theta-\cos^6\theta=(\sin^2\theta-\cos^2\theta)(1-\sin^2\theta\cos^2\theta)$

$LHS=\sin^6\theta-\cos^6\theta$

$=(\sin^2\theta)^3-(\cos^2\theta)^3$

$\because a^3-b^3=(a-b)(a^2+ab+b^2)$

$=(\sin^2\theta-\cos^2\theta)[\sin^4\theta+\sin^2\theta\cos^2\theta+\cos^4\theta]$

$=(\sin^2\theta-\cos^2\theta)[\sin^4\theta+\cos^4\theta+(\sin^2\theta\cos^2\theta-\sin^2\theta\cos^2\theta)+\sin^2\theta\cos^2\theta]$

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$= (Sin^2\theta - Cos^2\theta)\,[\,(Sin^4\theta + Cos^4\theta + 2Sin^2\theta Cos^2\theta) - Sin^2\theta Cos^2\theta\,]$

$= (Sin^2\theta - Cos^2\theta)\,[\,(Sin^2\theta + Cos^2\theta)^2 - Sin^2\theta Cos^2\theta\,]$

$= (Sin^2\theta - Cos^2\theta)\,[\,(1)^2 - Sin^2\theta Cos^2\theta\,]$ $\quad \because Sin^2\theta + Cos^2\theta = 1$

$= (Sin^2\theta - Cos^2\theta)(1 - Sin^2\theta Cos^2\theta) = RHS.$

Thus $Sin^6\theta - Cos^6\theta = (Sin^2\theta - Cos^2\theta)(1 - Sin^2\theta Cos^2\theta)$ (Proved)

Q.22 $Sin^6\theta + Cos^6\theta = 1 - 3Sin^2\theta Cos^2\theta$

LHS $= Sin^6\theta + Cos^6\theta$

$= (Sin^2\theta)^3 + (Cos^2\theta)^3$ $\quad \because a^3+b^3 = (a+b)(a^2-ab+b^2)$

$= (Sin^2\theta + Cos^2\theta)\,[\,Sin^4\theta - Sin^2\theta Cos^2\theta + Sin^4\theta\,]$

$= (1)\,[\,Sin^4\theta + Cos^4\theta + 2Sin^2Cos^2\theta - 2Sin^2\theta Cos^2\theta - Sin^2\theta Cos^2\theta\,]$

$= [\,(Sin^2\theta + Cos^2\theta)^2 - 3Sin^2\theta Cos^2\theta\,]$ $\quad \because Sin^2\theta + Cos^2\theta = 1$

$= [\,(1)^2 - 3Sin^2\theta Cos^2\theta\,]$

$= 1 - 3Sin^2\theta Cos^2\theta = RHS$

Thus $Sin^6\theta + Cos^6\theta = 1 - 3Sin^2\theta Cos^2\theta$ (Proved)

Q.23 $\dfrac{1}{1+Sin\theta} + \dfrac{1}{1-Sin\theta} = 2Sec^2\theta$

LHS $= \dfrac{1}{1+Sin\theta} + \dfrac{1}{1-Sin\theta} = \dfrac{1-Sin\theta+1+Sin\theta}{(1+Sin\theta)(1-Sin\theta)}$

$= \dfrac{2}{1-Sin^2\theta} = \dfrac{2}{Cos^2\theta} = 2Sec^2\theta = RHS$

Thus $\dfrac{1}{1+Sin\theta} + \dfrac{1}{1-Sin\theta} = 2Sec^2\theta$ (Proved)

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Q.24 $\dfrac{Cos\theta+Sin\theta}{Cos\theta-Sin\theta} + \dfrac{Cos\theta-Sin\theta}{Cos\theta+Sin\theta} = \dfrac{2}{1-2Sin^2\theta}$

LHS $= \dfrac{Cos\theta+Sin\theta}{Cos\theta-Sin\theta} + \dfrac{Cos\theta-Sin\theta}{Cos\theta+Sin\theta}$

$= \dfrac{(Cos\theta+Sin\theta)^2 + (Cos\theta-Sin\theta)^2}{(Cos\theta-Sin\theta)(Cos\theta+Sin\theta)}$

$= \dfrac{Cos^2\theta+Sin^2\theta+2Sin\theta Cos\theta + Cos^2\theta+Sin^2\theta-2Sin\theta Cos\theta}{Cos^2\theta - Sin^2\theta}$ $\quad (\because Sin^2\theta+Cos^2\theta=1)$

$= \dfrac{1+1}{(1-Sin^2\theta)-Sin^2\theta} = \dfrac{2}{1-2Sin^2\theta} = RHS.$

Thus $\dfrac{Cos\theta+Sin\theta}{Cos\theta-Sin\theta} + \dfrac{Cos\theta-Sin\theta}{Cos\theta+Sin\theta} = \dfrac{2}{1-2Sin^2\theta}$ (Proved)

THE END.