TAHIR MEHMOOD
M.Sc. Math
0345-6510779

EXERCISE: 9.3

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Q.1 Prove the following identities:-

(i) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$

LHS $= \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$

RHS $= \sin 30^\circ = 1/2$

$\Rightarrow$ LHS = RHS

$\Rightarrow \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$

(ii) $\sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{3} + \tan^2 \frac{\pi}{4} = 2$

LHS $= \sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{3} + \tan^2 \frac{\pi}{4}$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$

$= \frac{1}{4} + \frac{3}{4} + 1$

$= \frac{1+3+4}{4} = \frac{8}{4} = 2$ = RHS

$\Rightarrow$ LHS = RHS

$\Rightarrow \sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{3} + \tan^2 \frac{\pi}{4} = 2$

(iii) $2 \sin 45^\circ + \frac{1}{2} \operatorname{cosec} 45^\circ = \frac{3}{\sqrt{2}}$

LHS $= 2 \sin 45^\circ + \frac{1}{2} \operatorname{cosec} 45^\circ$

$= 2\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}(\sqrt{2})$

$= \frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{2} = \sqrt{2} + \frac{1}{\sqrt{2}}$

$= \frac{(\sqrt{2})^2 + 1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}}$

$= \frac{3}{\sqrt{2}}$ = RHS

$\Rightarrow$ LHS = RHS

So

$2 \sin 45^\circ + \frac{1}{2} \operatorname{cosec} 45^\circ = \frac{3}{\sqrt{2}}$

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(vi) Show that:

$\sin^2 \frac{\pi}{6} : \sin^2 \frac{\pi}{4} : \sin^2 \frac{\pi}{3} : \sin^2 \frac{\pi}{2} = 1:2:3:4$

LHS $= \sin^2 \frac{\pi}{6} : \sin^2 \frac{\pi}{4} : \sin^2 \frac{\pi}{3} : \sin^2 \frac{\pi}{2}$

$= \left(\frac{1}{2}\right)^2 : \left(\frac{1}{\sqrt{2}}\right)^2 : \left(\frac{\sqrt{3}}{2}\right)^2 : (1)^2$

$= \frac{1}{4} : \frac{1}{2} : \frac{3}{4} : 1$

Multiplying by 4, we have:

$= 4 \times \frac{1}{4} : 4 \times \frac{1}{2} : 4 \times \frac{3}{4} : 4 \times 1$

$= 1 : 2 : 3 : 4$ = RHS

$\Rightarrow$ LHS = RHS

$\Rightarrow \sin^2 \frac{\pi}{6} : \sin^2 \frac{\pi}{4} : \sin^2 \frac{\pi}{3} : \sin^2 \frac{\pi}{2} = 1:2:3:4$

Q.2 Evaluate the following:

(i) $\frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{6}}$

$= \frac{(\sqrt{3}) - \left(\frac{1}{\sqrt{3}}\right)}{1 + (\sqrt{3})\left(\frac{1}{\sqrt{3}}\right)} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1+1}$

$= \frac{(\sqrt{3})^2 - 1}{2\sqrt{3}} = \frac{3-1}{2\sqrt{3}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{6}} = \frac{1}{\sqrt{3}}$ Ans.

(ii) $\frac{1 - \tan^2 \frac{\pi}{3}}{1 + \tan^2 \frac{\pi}{3}}$

$= \frac{1 - (\sqrt{3})^2}{1 + (\sqrt{3})^2} = \frac{1-3}{1+3} = \frac{-2}{4}$

$= \frac{-1}{2}$

$\Rightarrow \frac{1 - \tan^2 \frac{\pi}{3}}{1 + \tan^2 \frac{\pi}{3}} = \frac{-1}{2}$

Q.3 Verify the followings at $\theta = 30^\circ$ and $\theta = 45^\circ$

(i) $\sin 2\theta = 2\sin\theta\cos\theta$

For $\theta = 30^\circ$

$LHS = \sin 2(30^\circ)$

$= \sin 60^\circ = \frac{\sqrt{3}}{2}$

$RHS = 2\sin 30^\circ \cos 30^\circ$

$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$

$\Rightarrow LHS = RHS$

For $\theta = 45^\circ$

$LHS = \sin 2(45^\circ)$

$= \sin 90^\circ = 1$

$RHS = 2\sin 45^\circ \cos 45^\circ$

$= 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{2}{2} = 1$

$\Rightarrow LHS = RHS$

Thus $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

For $\theta = 30^\circ$

$LHS = \cos 2(30^\circ)$

$= \cos 60^\circ = 1/2$

$RHS = \cos^2(30^\circ) - \sin^2(30^\circ)$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4} = 1/2$

$\Rightarrow LHS = RHS.$

For $\theta = 45^\circ$

$LHS = \cos 2(45^\circ)$

$= \cos 90^\circ = 0$

$RHS = \cos^2 45^\circ - \sin^2 45^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{2} - \frac{1}{2} = 0$

$\Rightarrow LHS = RHS$

Thus $\cos 2\theta = \cos^2\theta - \sin^2\theta$

(iii) $\cos 2\theta = 2\cos^2\theta - 1$

For $\theta = 30^\circ$

$LHS = \cos 2(30^\circ)$

$= \cos 60^\circ = 1/2$

$RHS = 2\cos^2(30^\circ) - 1$

$= 2(\sqrt{3}/2)^2 - 1 = 2(3/4) - 1$

$= \frac{3}{2} - 1 = \frac{3-2}{2} = 1/2$

$\Rightarrow LHS = RHS$

For $\theta = 45^\circ$

$LHS = \cos 2(45^\circ)$

$= \cos 90^\circ = 0$

$RHS = 2\cos^2(45^\circ) - 1$

$= 2(1/\sqrt{2})^2 - 1 = 2\left(\frac{1}{2}\right) - 1$

$= 1 - 1 = 0$

$\Rightarrow LHS = RHS$

So $\cos 2\theta = 2\sin\theta\cos\theta$

(iv) $\cos 2\theta = 1 - 2\sin^2\theta$

For $\theta = 30^\circ$

$LHS = \cos 2(30^\circ)$

$= \cos 60^\circ = 1/2$

$RHS = 1 - 2\sin^2(30^\circ)$

$= 1 - 2(1/2)^2 = 1 - 2(1/4) = 1 - 1/2$

$= \frac{2-1}{2} = 1/2$

$\Rightarrow LHS = RHS$

For $\theta = 45^\circ$

$LHS = \cos 2(45^\circ)$

$= \cos 90^\circ = 0$

$RHS = 1 - 2\sin^2 45^\circ$

$= 1 - 2(1/\sqrt{2})^2 = 1 - 2(1/2)$

$= 1 - 1 = 0$

$\Rightarrow LHS = RHS$

$\Rightarrow \cos 2\theta = 1 - 2\sin^2\theta.$

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(v) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

For $\theta = 30°$

$LHS = \tan 2(30°)$

$= \tan 60° = \sqrt{3}$

$RHS = \dfrac{2\tan(30°)}{1-\tan^2(30°)}$

$= \dfrac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

$= \dfrac{2/\sqrt{3}}{1-1/3} = \dfrac{2/\sqrt{3}}{\frac{3-1}{3}}$

$= \dfrac{2\times 3}{2\sqrt{3}} = \dfrac{3}{\sqrt{3}} = \sqrt{3}$

$\Rightarrow LHS = RHS$

For $\theta = 45°$

$LHS = \tan 2(45°)$

$= \tan 90° = \infty$

$RHS = \dfrac{2\tan 45°}{1-\tan^2 45°}$

$= \dfrac{2(1)}{1-(1)^2}$

$= \dfrac{2}{1-1} = \dfrac{2}{0}$

$= \infty$

$\Rightarrow LHS = RHS$

Thus $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Q.4 Find x if:

$\tan^2 45° - \cos^2 60° = x \sin 45° \cos 45° \tan 60°$

Sol:- $x = \dfrac{\tan^2 45° - \cos^2 60°}{\sin 45° \cos 45° \tan 60°}$

$x = \dfrac{(1)^2 - \left(\frac{1}{2}\right)^2}{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(\sqrt{3})}$

$x = \dfrac{1-\frac{1}{4}}{\sqrt{3}/2} = \dfrac{4-1}{4} \times \dfrac{2}{\sqrt{3}}$

$x = \dfrac{3}{4} \times \dfrac{2}{\sqrt{3}} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow x = \dfrac{\sqrt{3}}{2}$

Q.5 Find the values of trigonometric ratios:

(i) $\theta = -\pi$

$\sin\theta = \sin(-\pi) = -\sin\pi = 0$

$\cos\theta = \cos(-\pi) = \cos\pi = -1$

$\tan\theta = \tan(-\pi) = -\tan\pi = 0$

$\cot\theta = \cot(-\pi) = -\cot\pi = -\infty$

$\sec\theta = \sec(-\pi) = \sec\pi = -1$

$\operatorname{cosec}\theta = \operatorname{cosec}(-\pi) = -\operatorname{cosec}\pi = -\infty$

(ii) $\theta = -3\pi = -2\pi - \pi$

$\sin\theta = \sin(-2\pi-\pi) = \sin(-\pi) = -\sin\pi = 0$

$\cos\theta = \cos(-2\pi-\pi) = \cos(-\pi) = \cos\pi = -1$

$\tan\theta = \tan(-2\pi-\pi) = \tan(-\pi) = -\tan\pi = 0$

$\cot\theta = \cot(-2\pi-\pi) = \cot(-\pi) = -\cot\pi = -\infty$

$\sec\theta = \sec(-2\pi-\pi) = \sec(-\pi) = \sec\pi = -1$

$\operatorname{cosec}\theta = \operatorname{cosec}(-2\pi-\pi) = \operatorname{cosec}(-\pi) = -\operatorname{cosec}\pi = -\infty$

$\theta = \frac{5}{2}\pi = (2\pi + \pi/2)$

$\sin\theta = \sin(2\pi + \pi/2) = \sin\pi/2 = 1$

$\cos\theta = \cos(2\pi + \pi/2) = \cos\pi/2 = 0$

$\tan\theta = \tan(2\pi + \pi/2) = \tan\pi/2 = \infty$

$\cot\theta = \cot(2\pi + \pi/2) = \cot\pi/2 = 0$

$\sec\theta = \sec(2\pi + \pi/2) = \sec\pi/2 = \infty$

$\operatorname{cosec}\theta = \operatorname{cosec}(2\pi + \pi/2) = \operatorname{cosec}\pi/2 = 1$

(iv) $\theta = \frac{-9}{2}\pi = -4\pi - \frac{\pi}{2}$

$\sin\theta = \sin(-4\pi - \pi/2) = \sin(-\frac{\pi}{2}) = -1$

$\cos\theta = \cos(-4\pi - \pi/2) = \cos(-\pi/2) = 0$

$\tan\theta = \tan(-4\pi - \pi/2) = \tan(-\pi/2) = -\infty$

$\cot\theta = \cot(-4\pi - \pi/2) = \cot(-\pi/2) = 0$

$\sec\theta = \sec(-4\pi - \pi/2) = \sec(-\pi/2) = \infty$

$\operatorname{cosec}\theta = \operatorname{cosec}(-4\pi - \pi/2) = \operatorname{cosec}(-\pi/2) = -\infty$

(v) $\theta = -15\pi = -14\pi - \pi$

$\sin\theta = \sin(-14\pi - \pi) = \sin(-\pi) = -\sin\pi = 0$

$\cos\theta = \cos(-14\pi - \pi) = \cos(-\pi) = \cos\pi = -1$

$\tan\theta = \tan(-14\pi - \pi) = \tan(-\pi) = -\tan\pi = 0$

$\cot\theta = \cot(-14\pi - \pi) = \cot(-\pi) = -\cot\pi = -\infty$

$\sec\theta = \sec(-14\pi - \pi) = \sec(-\pi) = \sec\pi = -1$

$\operatorname{cosec}\theta = \operatorname{cosec}(-14\pi - \pi) = \operatorname{cosec}(-\pi) = -\operatorname{cosec}\pi = -\infty$

(vi) $\theta = 1530^\circ = 1440^\circ + 90^\circ$

$\sin\theta = \sin(1440^\circ + 90^\circ) = \sin 90^\circ = 1$

$\cos\theta = \cos(1440^\circ + 90^\circ) = \cos 90^\circ = 0$

$\tan\theta = \tan(1440^\circ + 90^\circ) = \tan 90^\circ = \infty$

$\cot\theta = \cot(1440^\circ + 90^\circ) = \cot 90^\circ = 0$

$\sec\theta = \sec(1440^\circ + 90^\circ) = \sec 90^\circ = \infty$

$\operatorname{cosec}\theta = \operatorname{cosec}(1440^\circ + 90^\circ) = \operatorname{cosec} 90^\circ = 1$

(vii) $\theta = -2430^\circ = -2520^\circ + 90^\circ$

$\sin\theta = \sin(-2520^\circ + 90^\circ) = \sin 90^\circ = 1$

$\cos\theta = \cos(-2520^\circ + 90^\circ) = \cos 90^\circ = 0$

$\tan\theta = \tan(-2520^\circ + 90^\circ) = \tan 90^\circ = \infty$

$\cot\theta = \cot(-2520^\circ + 90^\circ) = \cot 90^\circ = 0$

$\sec\theta = \sec(-2520^\circ + 90^\circ) = \sec 90^\circ = \infty$

$\operatorname{cosec}\theta = \operatorname{cosec}(-2520^\circ + 90^\circ) = \operatorname{cosec} 90^\circ = 1$

(viii) $\theta = \frac{235}{2}\pi = 116\pi + \frac{3\pi}{2}$

$\sin\theta = \sin(116\pi + \frac{3\pi}{2}) = \sin\frac{3\pi}{2} = -1$

$\cos\theta = \cos(116\pi + \frac{3\pi}{2}) = \cos\frac{3\pi}{2} = 0$

$\tan\theta = \tan(116\pi + \frac{3\pi}{2}) = \tan\frac{3\pi}{2} = -\infty$

$\cot\theta = \cot(116\pi + \frac{3\pi}{2}) = \cot\frac{3\pi}{2} = 0$

$\sec\theta = \sec(116\pi + \frac{3\pi}{2}) = \sec\frac{3\pi}{2} = -\infty$

$\operatorname{cosec}\theta = \operatorname{cosec}(116\pi + \frac{3\pi}{2}) = \operatorname{cosec}\frac{3\pi}{2} = -1$

(ix) $\theta = \frac{407\pi}{2} = 202\pi + \frac{3\pi}{2}$

$\sin\theta = \sin(202\pi + \frac{3\pi}{2}) = \sin\frac{3\pi}{2} = -1$

$\cos\theta = \cos(202\pi + \frac{3\pi}{2}) = \cos(\frac{3\pi}{2}) = 0$

$\tan\theta = \tan(202\pi + \frac{3\pi}{2}) = \tan\frac{3\pi}{2} = -\infty$

$\cot\theta = \cot(202\pi + \frac{3\pi}{2}) = \cot\frac{3\pi}{2} = 0$

$\sec\theta = \sec(202\pi + \frac{3\pi}{2}) = \sec\frac{3\pi}{2} = -\infty$

$\operatorname{cosec}\theta = \operatorname{cosec}(202\pi + \frac{3\pi}{2}) = \operatorname{cosec}\frac{3\pi}{2} = -1$

Q.6 Find the values of trigonometric ratios:

(i) $\theta = 390^\circ = 360^\circ + 30^\circ = 30^\circ$

$\sin\theta = \sin 30^\circ = 1/2$

$\cos\theta = \cos 30^\circ = \sqrt{3}/2$

$\tan\theta = \tan 30^\circ = 1/\sqrt{3}$

$\cot\theta = \cot 30^\circ = \sqrt{3}$

$\sec\theta = \sec 30^\circ = 2/\sqrt{3}$

$\operatorname{cosec}\theta = \operatorname{cosec} 30^\circ = 2$.

(ii) $\theta = -330^\circ = -360^\circ + 30^\circ = 30^\circ$

$\sin\theta = \sin 30^\circ = 1/2$

$\cos\theta = \cos 30^\circ = \sqrt{3}/2$

$\tan\theta = \tan 30^\circ = 1/\sqrt{3}$

$\cot\theta = \cot 30^\circ = \sqrt{3}$

$\sec\theta = \sec 30^\circ = 2/\sqrt{3}$

$\operatorname{cosec}\theta = \operatorname{cosec} 30^\circ = 2$

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(iii) $\theta = 765^\circ = 720^\circ + 45^\circ = 45^\circ$

$\sin\theta = \sin 45^\circ = 1/\sqrt{2}$

$\cos\theta = \cos 45^\circ = 1/\sqrt{2}$

$\tan\theta = \tan 45^\circ = 1$

$\cot\theta = \cot 45^\circ = 1$

$\sec\theta = \sec 45^\circ = \sqrt{2}$

$\operatorname{cosec}\theta = \operatorname{cosec} 45^\circ = \sqrt{2}$

(iv) $\theta = -675^\circ = -720^\circ + 45^\circ = 45^\circ$

$\sin\theta = \sin 45^\circ = 1/\sqrt{2}$

$\cos\theta = \cos 45^\circ = 1/\sqrt{2}$

$\tan\theta = \tan 45^\circ = 1$

$\cot\theta = \cot 45^\circ = 1$

$\sec\theta = \sec 45^\circ = \sqrt{2}$

$\operatorname{cosec}\theta = \operatorname{cosec} 45^\circ = \sqrt{2}$

(v) $\theta = \frac{-17}{3}\pi = -18\pi + \frac{\pi}{3} = \pi/3$

$\sin\theta = \sin \pi/3 = \sqrt{3}/2$

$\cos\theta = \cos \pi/3 = 1/2$

$\tan\theta = \tan \pi/3 = \sqrt{3}$

$\cot\theta = \cot \pi/3 = 1/\sqrt{3}$

$\sec\theta = \sec \pi/3 = 2$

$\operatorname{cosec}\theta = \operatorname{cosec} \pi/3 = 2/\sqrt{3}$

(vi) $\theta = \frac{13\pi}{3} = 4\pi + \pi/3 = \pi/3$

$\sin\theta = \sin \pi/3 = \sqrt{3}/2$

$\cos\theta = \cos \pi/3 = 1/2$

$\tan\theta = \tan \pi/3 = \sqrt{3}$

$\cot\theta = \cot \pi/3 = 1/\sqrt{3}$

$\sec\theta = \sec \pi/3 = 2$

$\operatorname{cosec}\theta = \operatorname{cosec} \pi/3 = \frac{2}{\sqrt{3}}$

(vii) $\theta = \frac{25}{6}\pi = 4\pi + \frac{\pi}{6} = \pi/6$

$\sin\theta = \sin \pi/6 = 1/2$

$\cos\theta = \cos \pi/6 = \sqrt{3}/2$

$\tan\theta = \tan \pi/6 = 1/\sqrt{3}$

$\cot\theta = \cot \pi/6 = \sqrt{3}$

$\sec\theta = \sec \pi/6 = 2/\sqrt{3}$

$\operatorname{cosec}\theta = \operatorname{cosec} \pi/6 = 2$

(viii) $\theta = \frac{-71\pi}{6} = -12\pi + \frac{\pi}{6} = \pi/6$

$\sin\theta = \sin \pi/6 = 1/2$

$\cos\theta = \cos \pi/6 = \sqrt{3}/2$

$\tan\theta = \tan \pi/6 = 1/\sqrt{3}$

$\cot\theta = \cot \pi/6 = \sqrt{3}$

$\sec\theta = \sec \pi/6 = 2/\sqrt{3}$

$\operatorname{cosec}\theta = \operatorname{cosec} \pi/6 = 2$

(ix) $\theta = -1035^\circ = -1080^\circ + 45^\circ = 45^\circ$

$\sin\theta = \sin 45^\circ = 1/\sqrt{2}$

$\cos\theta = \cos 45^\circ = 1/\sqrt{2}$

$\tan\theta = \tan 45^\circ = 1$

$\cot\theta = \cot 45^\circ = 1$

$\sec\theta = \sec 45^\circ = \sqrt{2}$

$\operatorname{cosec}\theta = \operatorname{cosec} 45^\circ = \sqrt{2}$

USEFUL FORMULAE:-

(i) $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta \wedge \cos^2\theta = 1 - \sin^2\theta$

(ii) $1 + \tan^2\theta = \sec^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

(iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$