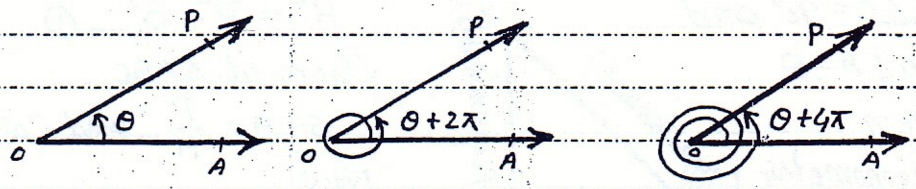
TAHIR MEHMOOD
M.Sc. Math
0345-6510779

TAHIR MEHMOOD
M.Sc. Math
0345-6510779

Coterminal Angles:-

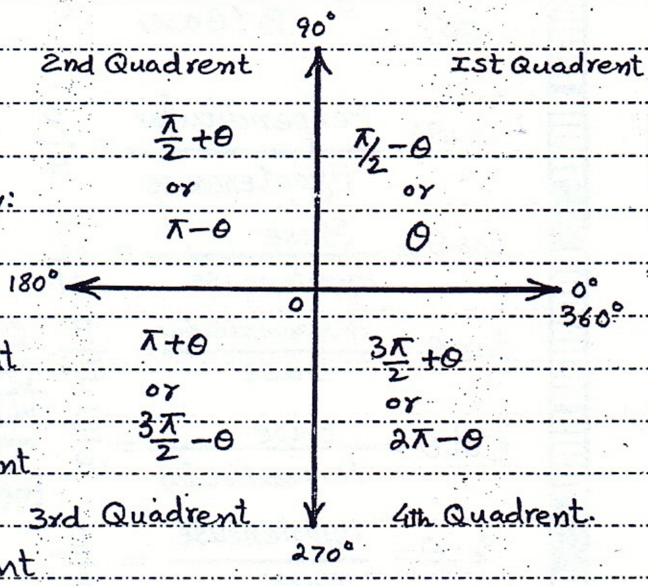
"The angles which have same initial and terminal rays are called Coterminal Angles."

In fact, these angles have same measurement with a difference of period or integral multiples of period ($2k\pi$). for $k \in Z$



The angles θ, $\theta+2\pi$ and $\theta+4\pi$ are Coterminal angles.

Quadrental Angles:-



If $0° < \theta < 90°$ then the angles associated with each quadrent are given by:

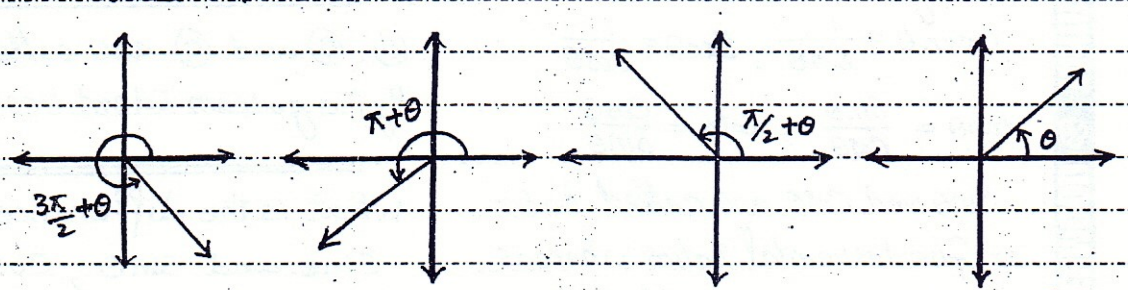
(i) Angle lies in Ist Quadrent if it is θ or $\frac{\pi}{2}-\theta$

(ii) Angle lies in 2nd Quadrent if it is $\frac{\pi}{2}+\theta$ or $\pi-\theta$

(iii) Angle lies in 3rd Quadrent if it is $\pi+\theta$ or $\frac{3\pi}{2}-\theta$

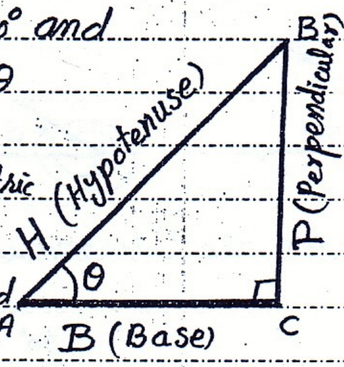
(iv) Angle lies in 4th Quadrent if it is $\frac{3\pi}{2}+\theta$ or $2\pi-\theta$

where 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$ and 2π are called Terminal angles of the Quadrental Axis.



Trigonometric Ratios:-

The ratios relating to right triangle are called Trigonometric ratios.
Consider the right triangle ABC such that $m\angle C = 90^\circ$ and $m\angle A = \theta$ then Trigonometric ratios are defined as:



$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypoteneouse}} = \frac{P}{H}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypoteneuse}} = \frac{B}{H}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{P}{B}$$

$$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{B}{P}$$

$$\sec\theta = \frac{\text{Hypoteneuse}}{\text{Base}} = \frac{H}{B}$$

$$\operatorname{cosec}\theta = \frac{\text{Hypoteneuse}}{\text{Perpendicular}} = \frac{H}{P}$$

Clearly

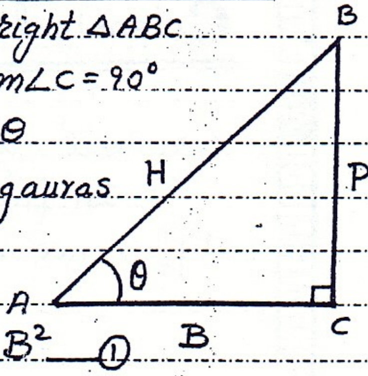
$$\operatorname{cosec}\theta = \frac{1}{\sin\theta},\quad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta},\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$\sin\theta$ and $\cos\theta$ are called basic or Fundamental ratios whereas remainings are called derived Trigonometric ratios.

Fundamental Identities:-

Consider right ΔABC such that $m\angle C = 90^\circ$
$m\angle A = \theta$
Using Pythagauras theorem



$$H^2 = P^2 + B^2 \quad \text{---} (1)$$

From rt. ΔABC

$$\sin\theta = \frac{P}{H} \text{ and } \cos\theta = \frac{B}{H}$$

Now

$$\sin^2\theta + \cos^2\theta = \frac{P^2}{H^2} + \frac{B^2}{H^2}$$

$$= \frac{P^2 + B^2}{H^2}$$

By (1) $P^2 + B^2 = H^2$

$$\Rightarrow \sin^2\theta + \cos^2\theta = \frac{H^2}{H^2} = 1$$

So

$$\boxed{\sin^2\theta + \cos^2\theta = 1} \quad \text{---} (A)$$

Dividing (A) by $\cos^2\theta$, we have

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\Rightarrow \boxed{\tan^2\theta + 1 = \sec^2\theta} \quad \text{---} (B)$$

Dividing (A) by $\sin^2\theta$, we have

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\Rightarrow \boxed{1 + \cot^2\theta = \operatorname{cosec}^2\theta} \rightarrow (C)$$

(A), (B) and (C) are called Pythagorean Identities.

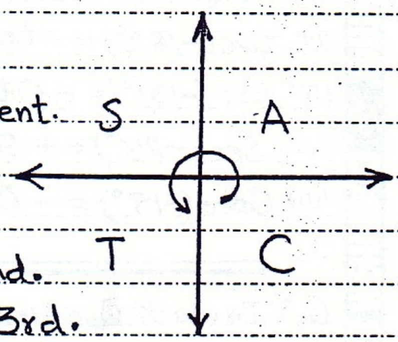
What is the difference between Sine and $\sin\theta$, Cos and $\cos\theta$, tan and $\tan\theta$?

TAHIR MEHMOOD
M.Sc. Math
0345-6510779

TAHIR MEHMOOD
M.Sc. Math
0345-6510779

Signs of Trigonometric ratios in each Quadrent:-

The ratios follow "CAST" rule
from 4th Quadrent to 3rd Quadrent.
C for Cos and its reciprocal +ve in 4th.
A for all +ve in Ist Quadrent.
S for Sin and its reciprocal +ve in 2nd.
T for tan and its reciprocal +ve in 3rd.



From these rules

$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$

$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$
$\sec(-\theta) = \sec\theta$
$\cot(-\theta) = -\cot\theta$

EXERCISE: 9.2

Q:1 Find the Sign of the followings:-

(i) $\sin 160^\circ$
Clearly $90^\circ < 160^\circ < 180^\circ$
which lies in 2nd Quadrent
Thus $\sin 160^\circ$ is +ve in 2nd.

(ii) $\cos 190^\circ$
Clearly $180^\circ < 190^\circ < 270^\circ$
which lies in 3rd Quadrent
Thus $\cos 190^\circ$ is −ve in 3rd.

(iii) $\tan 115^\circ$
Clearly $90^\circ < 115^\circ < 180^\circ$
which lies in 2nd Quadrent
Thus $\tan 115^\circ$ is −ve in 2nd.

(iv) $\sec 245^\circ$
Clearly $180^\circ < 245^\circ < 270^\circ$
which lies in 3rd Quadrent
Thus $\sec 245^\circ$ is −ve in 3rd

(v) $\cot 80^\circ$
Clearly $0 < 80^\circ < 90^\circ$
which lies in Ist Quadrent
Thus $\cot 80^\circ$ is +ve in Ist.

(vi) $\operatorname{cosec} 297^\circ$
Clearly $270^\circ < 297^\circ < 360^\circ$
which lies in 4th Quadrent
Thus $\operatorname{cosec} 297^\circ$ is −ve in 4th.

Q.2 Fill in the blanks:

(i) $\sin(-310°) = -\sin 310°$

(ii) $\cos(-75°) = +\cos 75°$

(iii) $\tan(-182°) = -\tan 182°$

(iv) $\cot(-137°) = -\cot(137°)$

(v) $\sec(-216°) = +\sec 216°$

(vi) $\operatorname{cosec}(-15°) = -\operatorname{cosec} 15°$

Q.3 In which Quadrent θ lies:

(i) $\sin\theta < 0$ and $\cos\theta > 0$

∵ Sinθ is −ve in III and IV

and Cosθ is +ve in I and IV

So θ lies in IV Quadrent.

(ii) $\cot\theta > 0$ and $\operatorname{cosec}\theta > 0$

∵ Cotθ is +ve in I and III

and Cosecθ is +ve in I and II

So θ lies in I Quadrent.

(iii) $\tan\theta < 0$ and $\cos\theta > 0$

∵ tanθ is −ve in II and IV

and Cosθ is +ve in I and IV

So θ lies in IV Quadrent.

(iv) $\sec\theta < 0$ and $\sin\theta < 0$

∵ Secθ is −ve in II and III

and Sinθ is −ve in III and IV

So θ lies in III Quadrent.

(v) $\cot\theta > 0$ and $\sin\theta < 0$

∵ Cotθ is +ve in I and III

and Sinθ is −ve in III and IV

So θ lies in III Quadrent.

(iv) $\cos\theta < 0$ and $\tan\theta < 0$

∵ Cosθ is −ve in II and III

and tanθ is −ve in II and IV

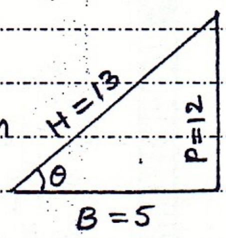
So θ lies in II Quadrent.

Q.4 Find the remaining trigonometric ratios if:

(i) $\sin\theta = \frac{12}{13}$ (θ lies in Ist Quadrent)

∵ $\sin\theta = \frac{P}{H}$

So $P = 12$ and $H = 13$



By Pythagrean theorem

$$H^2 = P^2 + B^2$$

$$\Rightarrow (13)^2 = (12)^2 + B^2$$

$$\Rightarrow B^2 = 169 - 144 \Rightarrow B = \sqrt{25} = 5$$

Trigonometric Ratios in Ist Quadrent

$\sin\theta = \frac{P}{H} = \frac{12}{13}$ $\quad$ $\operatorname{cosec}\theta = \frac{H}{P} = \frac{13}{12}$

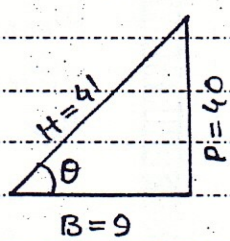
$\cos\theta = \frac{B}{H} = \frac{5}{13}$ $\quad$ $\sec\theta = \frac{H}{B} = \frac{13}{5}$

$\tan\theta = \frac{P}{B} = \frac{12}{5}$ $\quad$ $\cot\theta = \frac{B}{P} = \frac{5}{12}$

(ii) $\cos\theta = \frac{9}{41}$ (θ in 4th Quadrent)

∵ $\cos\theta = \frac{B}{H}$

So $B = 9$ and $H = 41$



By Pythagorean theorem

$$H^2 = P^2 + B^2$$

$$(41)^2 = P^2 + (9)^2$$

$$P^2 = 1681 - 81 = 1600 \Rightarrow P = 40$$

Trigonometric Ratios in 4th Quadrent.

$\sin\theta = -\frac{P}{H} = -\frac{40}{41}$ $\quad$ $\operatorname{cosec}\theta = -\frac{H}{P} = -\frac{41}{40}$

$\cos\theta = \frac{B}{P} = \frac{9}{41}$ $\quad$ $\sec\theta = \frac{P}{B} = \frac{41}{9}$

$\tan\theta = -\frac{P}{B} = \frac{-40}{9}$ $\quad$ $\cot\theta = -\frac{B}{P} = \frac{-9}{40}$

TAHIR
0345-6510779
طاہر محمود
ایم - ایس - سی (میتھ)

TAHIR MEHMOOD
M.Sc. Math
0345-6510779

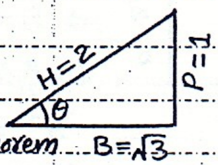
TAHIR MEHMOOD
M.Sc. Math
0345-6510779

(iii)

$\cos\theta = -\frac{\sqrt{3}}{2}$ (θ in III Quadrent)

$\because \cos\theta = \frac{B}{H}$

$\Rightarrow B = \sqrt{3}$ and $H = 2$



Using Pythagorean theorem

$H^2 = P^2 + B^2$

$(2)^2 = P^2 + (\sqrt{3})^2$

$P^2 = 4 - 3 = 1 \Rightarrow P = 1$

Trigonometric ratios in III Quadrent

$\sin\theta = \frac{-P}{H} = \frac{-1}{2}$ $\quad \mathrm{Cosec}\,\theta = \frac{-H}{P} = -2$

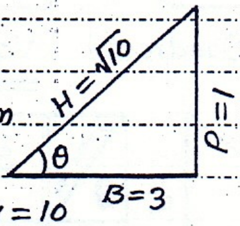
$\cos\theta = \frac{-B}{H} = \frac{-\sqrt{3}}{2}$ $\quad \sec\theta = \frac{-H}{B} = \frac{-2}{\sqrt{3}}$

$\tan\theta = \frac{+P}{B} = \frac{1}{\sqrt{3}}$ $\quad \cot\theta = \frac{B}{P} = \sqrt{3}$

(iv) $\tan\theta = \frac{-1}{3}$ (θ in II Quadrent)

$\because \tan\theta = \frac{P}{B}$

$\Rightarrow P = 1$ and $B = 3$



By Pythagorean Theorem

$H^2 = P^2 + B^2$

$H^2 = (1)^2 + (3)^2 = 1 + 9 = 10$

$\Rightarrow H = \sqrt{10}$

Trigonometric ratios in II Quadrent:

$\sin\theta = \frac{P}{H} = \frac{1}{\sqrt{10}}$ $\quad \mathrm{Cosec}\,\theta = \frac{H}{P} = \sqrt{10}$

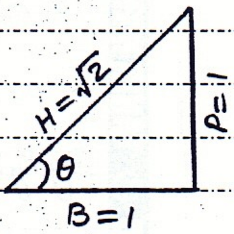
$\cos\theta = \frac{-B}{H} = \frac{-3}{\sqrt{10}}$ $\quad \sec\theta = \frac{-H}{B} = \frac{-\sqrt{10}}{3}$

$\tan\theta = \frac{-P}{B} = \frac{-1}{3}$ $\quad \cot\theta = \frac{-B}{P} = -3$

(v) $\sin\theta = \frac{-1}{\sqrt{2}}$ (θ in III Quadrent)

$\because \sin\theta = \frac{P}{H}$

$\Rightarrow P = 1$ and $H = \sqrt{2}$



By Pythagorean theorem

$H^2 = P^2 + B^2$

$(\sqrt{2})^2 = (1)^2 + B^2$

$\Rightarrow B^2 = 2 - 1 = 1 \Rightarrow B = 1$

Trigonometric ratios in III Quadrent.

$\sin\theta = \frac{-P}{H} = \frac{-1}{\sqrt{2}}$ $\quad \mathrm{Cosec}\,\theta = \frac{-H}{P} = -\sqrt{2}$

$\cos\theta = \frac{-B}{H} = \frac{-1}{\sqrt{2}}$ $\quad \sec\theta = \frac{-H}{B} = -\sqrt{2}$

$\tan\theta = \frac{P}{B} = \frac{1}{1} = 1$ $\quad \cot\theta = \frac{B}{P} = 1$

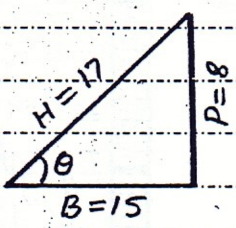
Q.5 If $\cot\theta = \frac{15}{8}$ and θ does not lie in Ist Quadrent. Find $\mathrm{Cosec}\,\theta$ and $\cos\theta$.

Sol:- $\cot\theta = \frac{15}{8}$ (θ lies in III Quadrent)

$\because \cot\theta = \frac{B}{P}$

$\Rightarrow$

$B = 15$ and $P = 8$



By Pythagorean Theorem

$H^2 = P^2 + B^2$

$H^2 = (15)^2 + (8)^2 = 225 + 64 = 289$

$\Rightarrow H = \sqrt{289} = 17$

$\mathrm{Cosec}\,\theta$ and $\cos\theta$ in III Quadrent

$\cos\theta = \frac{-B}{H} = \frac{-15}{17}$

$\mathrm{Cosec}\,\theta = \frac{-H}{P} = \frac{-17}{8}$

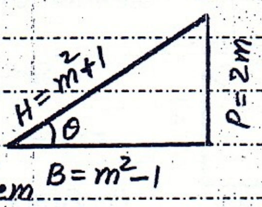
TAHIR

V.Imp

Q.7 $\operatorname{Cosec}\theta = \dfrac{m^2+1}{2m}$ $\quad 0<\theta<\pi/2$

$\because \operatorname{Cosec}\theta = \dfrac{H}{P}$

$\Rightarrow P = 2m \quad H = m^2+1$



By Pythagorean Theorem

$$H^2 = P^2 + B^2$$

$$(m^2+1)^2 = (2m)^2 + B^2$$

$$B^2 = m^4 + 2m^2 + 1 - 4m^2$$

$$B^2 = m^4 - 2m^2 + 1 = (m^2-1)^2$$

$$\Rightarrow \quad B = m^2 - 1$$

Trigonometric ratios in Ist Quadrent.

$\operatorname{Sin}\theta = \dfrac{P}{H} = \dfrac{2m}{m^2+1} \qquad \operatorname{Cosec}\theta = \dfrac{H}{P} = \dfrac{m^2+1}{2m}$

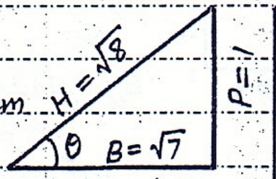
$\operatorname{Cos}\theta = \dfrac{B}{H} = \dfrac{m^2-1}{m^2+1} \qquad \operatorname{Sec}\theta = \dfrac{H}{B} = \dfrac{m^2+1}{m^2-1}$

$\tan\theta = \dfrac{P}{B} = \dfrac{2m}{m^2-1} \qquad \operatorname{Cot}\theta = \dfrac{B}{P} = \dfrac{m^2-1}{2m}$

Q.8 $\tan\theta = \dfrac{1}{\sqrt{7}}$ (θ lies in Ist Quadrent)

$\because \tan\theta = \dfrac{P}{B}$

$\Rightarrow P = 1$ and $B = \sqrt{7}$



By Pythagorean theorem

$$H^2 = P^2 + B^2$$

$$H^2 = (1)^2 + (\sqrt{7})^2 = 1 + 7 = 8 \Rightarrow H = \sqrt{8}$$

Now $\operatorname{Cosec}\theta = \dfrac{H}{P} = \dfrac{\sqrt{8}}{1} = \sqrt{8}$

$\operatorname{Sec}\theta = \dfrac{H}{B} = \dfrac{\sqrt{8}}{\sqrt{7}} = \sqrt{8/7}$

Consider

$$\frac{\operatorname{Cosec}^2\theta - \operatorname{Sec}^2\theta}{\operatorname{Cosec}^2\theta + \operatorname{Sec}^2\theta} = \frac{(\sqrt{8})^2 - (\sqrt{8/7})^2}{(\sqrt{8})^2 + (\sqrt{8/7})^2}$$

$$= \frac{8 - 8/7}{8 + 8/7} = \frac{56-8}{56+8} \times \frac{7}{7}$$

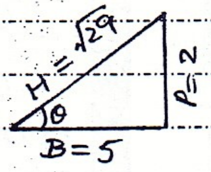
$$= \frac{48}{64} = \frac{3}{4}$$

So $\boxed{\dfrac{\operatorname{Cosec}^2\theta - \operatorname{Sec}^2\theta}{\operatorname{Cosec}^2\theta + \operatorname{Sec}^2\theta} = 3/4}$

Q.9 $\operatorname{Cot}\theta = \dfrac{5}{2}$ (θ in Ist Quadrent)

$\because \operatorname{Cot}\theta = \dfrac{B}{P}$

$\Rightarrow B = 5 \quad P = 2$



By Pythagorean theorem

$$H^2 = P^2 + B^2$$

$$H^2 = (2)^2 + (5)^2 = 4 + 25 = 29$$

$$H = \sqrt{29}$$

So $\operatorname{Sin}\theta = \dfrac{P}{H} = \dfrac{2}{\sqrt{29}}$

$\operatorname{Cos}\theta = \dfrac{B}{H} = \dfrac{5}{\sqrt{29}}$

Now

$$\frac{3\operatorname{Sin}\theta + 4\operatorname{Cos}\theta}{\operatorname{Cos}\theta - \operatorname{Sin}\theta} = \frac{3\left(\frac{2}{\sqrt{29}}\right) + 4\left(\frac{5}{\sqrt{29}}\right)}{\frac{5}{\sqrt{29}} - \frac{2}{\sqrt{29}}}$$

$$= \frac{6+20}{\sqrt{29}} \Big/ \frac{5-2}{\sqrt{29}} = \frac{26}{3}$$

$$\Rightarrow \boxed{\frac{3\operatorname{Sin}\theta + 4\operatorname{Cos}\theta}{\operatorname{Cos}\theta - \operatorname{Sin}\theta} = \frac{26}{3}}$$

Values of Trigonometric ratios

θ \ function	Sinθ	cosθ	tanθ
0°	0	1	0
30°	$\frac{1}{2} = 0.5$	$\frac{\sqrt{3}}{2} = 0.866$	$\frac{1}{\sqrt{3}} = 0.577$
45°	$\frac{1}{\sqrt{2}} = 0.707$	$\frac{1}{\sqrt{2}} = 0.707$	1
60°	$\frac{\sqrt{3}}{2} = 0.866$	$\frac{1}{2} = 0.5$	$\sqrt{3} = 1.732$
90°	1	0	∞
180°	0	−1	0
270°	−1	0	−∞
360°	0	1	0

TAHIR

Tahir Mahmood
M.Sc. (Math)
03456510779