FUNDAMENTALS OF TRIGONOMETRY.

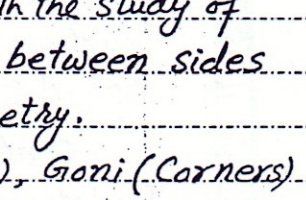
TRIGONOMETRY.

The branch of Mathematics which deals with the study of sides and angles of a triangle and relation between sides and angles of the triangle is called Trigonometry.

Trigonometry is derived from Trie (Three), Goni (Corners) and Metron (To measure).

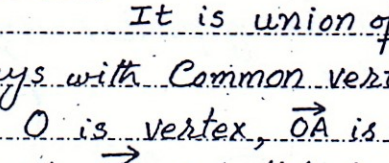
ANGLE

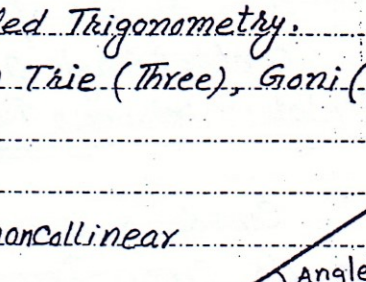
It is union of two two noncollinear rays with Common vertex.



O is vertex, $\overrightarrow{OA}$ is called initial ray and $\overrightarrow{OP}$ is called terminating ray.

* Angle is Considered +ve if its rotation is anticlockwise
* Angle is Considered -ve if its rotation is clockwise.





* Angle is usually denoted by $\alpha, \beta, \gamma, \theta, \phi, \psi$ etc Greek Letters.

Measurement of Angle:-

Usually an angle is measured into two Systems:

(i) English (Sexagesimal) System.
(ii) Circular (Radian) System.

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(i) English (Sexagesimal) System:-

In this System, angle is measured in degree, minute, seconds. One degree is the $\frac{1}{360}$ th part of the circle angle. $1^\circ = 60' (\text{min}) = 3600'' (\text{seconds})$

Convertion:- $x' = \left(\frac{x}{60}\right)^\circ$

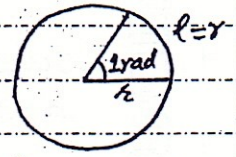
$y'' = \left(\frac{y}{3600}\right)^\circ$

(ii) Circular (Radian) System:-

In this system, angle is measured in radians.

Radian:-

"The central angle is said to be unit radian due to which length of arc becomes equal to radius of Circle."



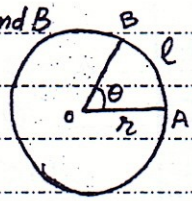
Central Angle $\theta = 1$ radian for arc length(l) = radius(r)

Relation between Radius, Arc Length and Central Angle:-

Consider a Circle of Centre O having points A and B on its Circumference such that

r is the radius of Circle

$\angle AOB = \theta$ is Central angle and $\overset{\frown}{AB} = l$



Consider the ratio:

$$\frac{\text{length of arc}}{\text{length of Circle}} = \frac{\text{Central Angle}}{\text{Angle of Circle}}$$

$$\Rightarrow \frac{l}{2\pi r} = \frac{\theta}{2\pi} \quad \text{where angles are in Circular measure}$$

$$\Rightarrow \frac{l}{r} = \theta \quad \text{or} \quad l = r\theta$$

which is required relation between l, r and θ.

* l and r should have same units.
* θ should be in Circular measure (radians).

Relation between Degree and Radian:-

We know that $l = r\theta$

$l = 2\pi r$ for One complete rotation so $\theta = \frac{l}{r} = \frac{2\pi r}{r}$

$\Rightarrow \theta = 2\pi$ radians for complete rotation.

Also $\theta = 360^\circ$ for Complete rotation.

$\Rightarrow 2\pi$ radians $= 360^\circ \Rightarrow \pi$ rad $= 180^\circ$

$\Rightarrow 1 \text{ rad} = \frac{180^\circ}{\pi} = \frac{180^\circ}{3.14} = 57.3^\circ$

and

$$1^\circ = \frac{\pi}{180} \text{ rad} = \frac{3.14}{180} = 0.01745 \text{ rad.}$$

Thus

$$\boxed{1^\circ = \frac{\pi}{180} \text{ rad} = 0.01745 \text{ rad}} \quad \text{and} \quad \boxed{1 \text{ rad} = \frac{180^\circ}{\pi} = 57.3^\circ}$$

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EXERCISE: 9.1

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M.Sc. Math
0345-6510779

Q: Express the following sexagesimal Measurements in Radians:

(i) 30°
$= 30 \times \frac{\pi}{180} = \frac{\pi}{6}$ rad.

(ii) 45°
$= 45 \times \frac{\pi}{180} = \frac{\pi}{4}$ rad

(iii) 60°
$= 60 \times \frac{\pi}{180} = \frac{\pi}{3}$ rad.

(iv) 75°
$= 75 \times \frac{\pi}{180} = \frac{5\pi}{12}$ rad.

(v) 90°
$= 90 \times \frac{\pi}{180} = \frac{\pi}{2}$ rad.

(vi) 105°
$= 105 \times \frac{\pi}{180} = \frac{7\pi}{12}$ rad.

(vii) 120°
$= 120 \times \frac{\pi}{180} = \frac{2\pi}{3}$ rad.

(viii) 135°
$= 135 \times \frac{\pi}{180} = \frac{3\pi}{4}$ rad.

(ix) 150°
$= 150 \times \frac{\pi}{180} = \frac{5\pi}{6}$ rad.

(x) $10^\circ 15'$
$= \left(10 + \frac{15}{60}\right)^\circ \times \frac{\pi}{180^\circ}$
$= \left(10 + \frac{1}{4}\right) \times \frac{\pi}{180}$
$= \left(\frac{40+1}{4}\right) \times \frac{\pi}{180} = \frac{41}{4} \times \frac{\pi}{180}$
$= \frac{41}{720}\pi$ rad.

(xi) $75^\circ 6' 30''$
$= \left(75 + \frac{6}{60} + \frac{30}{3600}\right)^\circ \times \frac{\pi}{180^\circ}$
$= \left(75 + \frac{1}{10} + \frac{1}{120}\right)^\circ \times \frac{\pi}{180}$
$= \left(\frac{9000 + 12 + 1}{120}\right) \times \frac{\pi}{180} = \frac{9013}{120} \times \frac{\pi}{180}$
$= \frac{9013\pi}{21600}$ rad.

(xii) $120' 40''$
$= \left(\frac{120}{60} + \frac{40}{3600}\right)^\circ \times \frac{\pi}{180^\circ}$
$= \left(2 + \frac{1}{90}\right) \times \frac{\pi}{180} = \left(\frac{180+1}{90}\right) \times \frac{\pi}{180}$
$= \frac{181}{16200}\pi$ rad.

(xiii) $154^\circ 20''$
$= \left(154 + \frac{20}{3600}\right)^\circ \times \frac{\pi}{180}$
$= \left(154 + \frac{1}{180}\right) \times \frac{\pi}{180} = \left(\frac{27000+1}{180}\right) \times \frac{\pi}{180}$
$= \frac{27001}{32400}\pi$ rad.

(xiv) 0°
$= 0 \times \frac{\pi}{180} = 0$ rad.

(xv) $35^\circ 20'$
$= \left(35 + \frac{20}{60}\right)^\circ \times \frac{\pi}{180^\circ} = \left(35 + \frac{1}{3}\right) \times \frac{\pi}{180}$
$= \frac{105+1}{3} \times \frac{\pi}{180} = \frac{106}{540}\pi$
$= \frac{53}{270}\pi$ rad.

(xvi) $3''$
$= \frac{3}{3600} \times \frac{\pi}{180} = \frac{\pi}{216000}$ rad.

Q.2 Convert the radian measures into Sexagesimal System.

(i) $\frac{\pi}{8}$ radian $= \frac{180^\circ}{8} = 22.5^\circ = 22^\circ 30'$

(ii) $\frac{\pi}{6}$ radian $= \frac{180^\circ}{6} = 30^\circ$

(iii) $\frac{\pi}{4}$ radian $= \frac{180^\circ}{4} = 45^\circ$

(iv) $\frac{\pi}{3}$ radian $= \frac{180^\circ}{3} = 60^\circ$

(v) $\frac{\pi}{2}$ radian $= \frac{180^\circ}{2} = 90^\circ$

(vi) $\frac{2\pi}{3}$ radian $= \frac{2(180^\circ)}{3} = 2(60^\circ) = 120^\circ$

(vii) $\frac{3\pi}{4}$ radian $= \frac{3(180^\circ)}{4} = 3(45^\circ) = 135^\circ$

(viii) $\frac{5\pi}{6}$ radian $= \frac{5(180^\circ)}{6} = 5(30^\circ) = 150^\circ$

(ix) $\frac{7\pi}{12}$ radian $= \frac{7(180^\circ)}{12} = 7(15^\circ) = 105^\circ$

(x) $\frac{9\pi}{5}$ radian $= \frac{9(180^\circ)}{5} = 9(36) = 324^\circ$

(xi) $\frac{11\pi}{27} = \frac{11(180^\circ)}{27} = 73.33^\circ = 73^\circ 20'$

(xii) $\frac{13}{16}\pi = \frac{13(180^\circ)}{16} = 146.25^\circ = 146^\circ 15'$

(xiii) $\frac{17\pi}{24} = \frac{17(180^\circ)}{24} = 127.5^\circ = 127^\circ 30'$

(xiv) $\frac{25\pi}{36} = \frac{25(180^\circ)}{36} = 25(5^\circ) = 125^\circ$

(xv) $\frac{19\pi}{32} = \frac{19(180^\circ)}{32} = 106.875^\circ = 106^\circ 52' 30''.$

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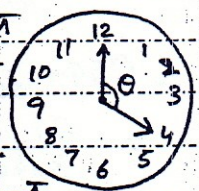
Q.3

12 hours Cover angle $= 2\pi$

1 hour Covers angle $= \frac{2\pi}{12} = \frac{\pi}{6}$ rad.



4 hours Cover angle $= 4 \times \frac{\pi}{6}$

$= \frac{2\pi}{3}$ rad.

Q.4 (i) Find θ if $l = 1.5$ cm $r = 2.5$ cm

$\because l = r\theta \Rightarrow \theta = \frac{l}{r}$

$\theta = \frac{1.5}{2.5} = \frac{15}{25} \times \frac{10}{10} = \frac{3}{5}$

$\theta = 0.6$ radian.

(ii) $l = 3.2$ m $r = 2$ m

$\because l = r\theta \Rightarrow \theta = \frac{l}{r}$

$\theta = \frac{3.2}{2} = 1.6$ radian.

Q.5 Find l if (i) $\theta = \pi$ rad $r = 6$ cm

$\because l = r\theta$

$l = 6(\pi) = 6\pi$ cm $= 6(3.1416)$

$l = 18.85$ cm

(ii) $\theta = 65^\circ 20' = (65 + \frac{20}{60}) \times 0.01745 = 1.1401$ rad

$r = 18$ mm

$\because l = r\theta = 18(1.1401)$

$l = 20.52$ cm

Q.6 Find r if

(i) $\ell = 5\,cm$ $\quad \theta = \frac{1}{2}$ radian

$\because \ell = r\theta \Rightarrow r = \frac{\ell}{\theta}$

$\Rightarrow r = \frac{5}{(1/2)} = 5 \times 2 = 10\,cm$

(ii) $\ell = 56\,cm$ $\quad \theta = 45^\circ \times 0.01745$

$= 0.78525$ rad.

$\because \ell = r\theta \Rightarrow r = \frac{\ell}{\theta}$

$r = \frac{56}{0.78525} = 71.31\,cm$

Q.7

$\ell = ?$

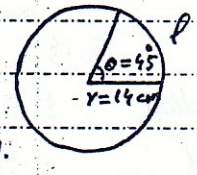
$r = 14\,cm$

$\theta = 45^\circ \times 0.01745$

$= 0.78525$ rad.

$\because \ell = r\theta$

$\ell = 14(0.78525) = 10.9935\,cm$



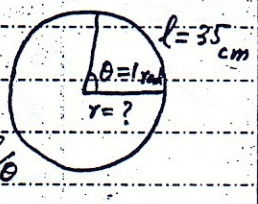
Q.8 $r = ?$

$\ell = 35\,cm$

$\theta = 1$ rad.

$\because \ell = r\theta \Rightarrow r = \ell/\theta$

$r = \frac{35}{1} = 35\,cm.$



Q.9 $\frac{\ell}{t} = 30\,km/h$

$= \frac{30 \times 1000}{3600}$

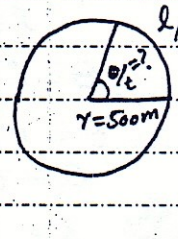
$\frac{\ell}{t} = \frac{25}{3}\,m/sec$

$r = 500\,m$

$\because \ell = r\theta$

$\frac{\ell}{t} = r\frac{\theta}{t} \Rightarrow \frac{\theta}{t} = \frac{\ell}{rt}$

$\frac{\theta}{t} = \frac{25}{3} \times \frac{1}{500} = \frac{1}{60}$ rad/sec



Thus θ in one second $= \frac{1}{60}$ rad.

θ in 10 seconds $= \frac{1}{60} \times 10 = \frac{1}{6}$ rad.

Q.10 $\ell = ?$

$r = 9\,m$

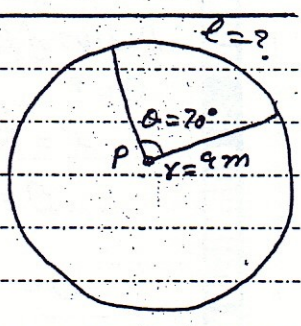
$\theta = 70^\circ \times 0.01745$

$= 1.2215$ rad.

$\because \ell = r\theta$

$\ell = 9(1.2215)$

$\ell = 10.9935\,m$



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Q.11 $\ell = ?$

$r = 20\,cm$

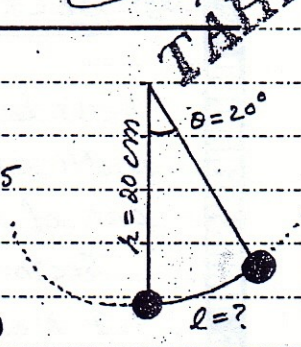
$\theta = 20^\circ \times 0.01745$

$= 0.349$ rad.

$\because \ell = r\theta$

$\ell = 20(0.349)$

$\ell = 6.98\,cm.$



Q.12 ℓ = diameter of Sun = ?

$r = 148 \times 10^6$ Km

$\theta = 9.3 \times 10^{-3}$ rad.

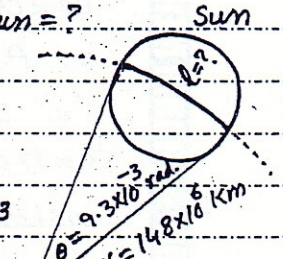
$\because \ell = r\theta$

$\ell = 148 \times 10^6 \times 9.3 \times 10^{-3}$

$\ell = 148 \times 9.3 \times 10^3$

$\ell = 1376.4 \times 10^3\,Km$

$\ell = 1376400\,Km.$



Knowledge is a great Strength which can Destroy the Evil.

Q.13/ Radius of wire = 6cm

After cutting the wire strightend

length of wire = $2\pi r = 2\pi(6) = 12\pi$ cm

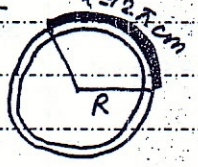
Radius of hoop (R) = 24cm

$\theta = ?$

$\because \ell = R\theta \Rightarrow \theta = \frac{\ell}{R}$

$\theta = \frac{12\pi \text{ cm}}{24 \text{ cm}}$

$\theta = \frac{\pi}{2}$ rad.

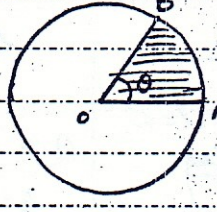


Q.14 SECTOR:-

The region bounded between two radii and their corresponding arc of a Circle is called Sector of a Circle.

let A and B be two points on a Circle of radius "r" Centre at O.

let A be the area of Sector OAB having Central angle θ.



Consider the ratio

$\frac{\text{Area of Sector}}{\text{Area of Circle}} = \frac{\text{Angle of Sector}}{\text{Angle of Circle}}$

$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$

$\Rightarrow A = \frac{\theta}{2\pi} \times \pi r^2$

$\Rightarrow A = \frac{1}{2} r^2 \theta$

where θ is in Circular measure.

Q.15

$\theta_1 = 45^\circ E$

$\theta_2 = 25^\circ W$

$\theta = \theta_1 + \theta_2$

$= 45 + 25$

$\theta = 70^\circ \times 0.01745$

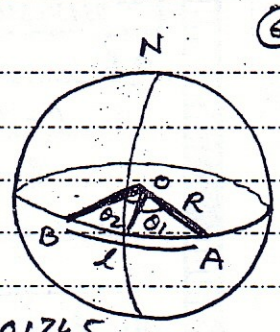
$\theta = 1.2215$ rad

$R = 6400$ Km

$\ell = R\theta$

$\ell = 6400 \times 1.2215$

$\ell = 7817.6$ Km



Q.16/ $\ell = ?$

$\theta = 0.5^\circ \times 0.01745$

$= 8.725 \times 10^{-3}$ rad.

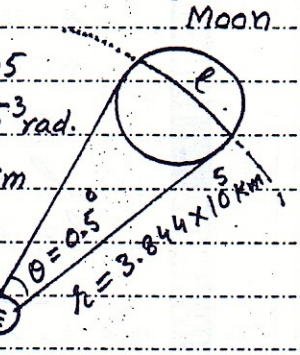
$r = 3.884 \times 10^5$ Km

$\because \ell = r\theta$

$\ell = 3.884 \times 10^5 \times 8.725 \times 10^{-3}$

$\ell = 33.5389 \times 10^2$ Km

$\ell = 3353.89$ Km



Q.17 Radius of Earth (R) = 6400 Km

$\ell = 2R = 2(6400)$ ....

$\ell = 12800$ Km

$\theta = 1°54' = (1 + \frac{54}{60})^\circ$

$\theta = 1.9 \times 0.01745$

$\theta = 0.0332$ rad.

$r = \ell/\theta$

$\Rightarrow r = \frac{12800}{0.0332}$

$r = 385542.17$ Km

which is the distance between earth and Moon.