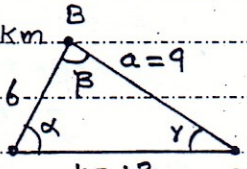
Q.10

$a = 9$ Km, $b = 13$ Km, $c = 6$ Km

$\alpha = ?$ $\beta = ?$ $\gamma = ?$



Using

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos\alpha = \frac{(13)^2+(6)^2-(9)^2}{2(13)(6)} = 0.7949$$

$$\alpha = \cos^{-1}(0.7949)$$

$$\boxed{\alpha = 37^\circ\ 2'}$$

Now $\cos\beta = \dfrac{c^2+a^2-b^2}{2ca}$

$$\cos\beta = \frac{(6)^2+(9)^2-(13)^2}{2(6)(9)} = -0.4815$$

$$\beta = \cos^{-1}(-0.4815)$$

$$\boxed{\beta = 118^\circ\ 47'}$$

$\because \alpha+\beta+\gamma = 180^\circ$

$\gamma = 180^\circ - \alpha - \beta$

$\gamma = 180^\circ - 37^\circ 2' - 118^\circ 47'$

$$\boxed{\gamma = 23^\circ\ 52'}$$

AREA OF TRIANGLE:-

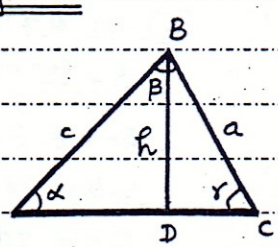
Case (1):-

When two sides and their included (3rd) angle is given:-

Consider $\triangle ABC$

Such that $\overline{BD} \perp \overline{AC}$

where $\overline{BD} = h$



We know that

Area of triangle $= \frac{1}{2}$ (Base)(Altitude)

Now from $\triangle ABD$ $\sin\alpha = \frac{p}{c}$

$\Rightarrow h = c\sin\alpha$

So

$\Delta = \frac{1}{2}(b)(h)$

$\Delta = \frac{1}{2}(b)(c\sin\alpha)$

$$\boxed{\Delta = \frac{1}{2}bc\sin\alpha}$$

Similarly, we can prove

$$\boxed{\Delta = \frac{1}{2}ab\sin\gamma}$$

$$\boxed{\Delta = \frac{1}{2}ca\sin\beta}$$

Case (2):-

When one side and two angles are given:-

We know that

$$\Delta = \frac{1}{2}ab\sin\gamma \quad \text{—①}$$

Using $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow b = \frac{a}{\sin\alpha} \times \sin\beta$$

putting in ①

$$\Delta = \frac{1}{2}a\left(\frac{a}{\sin\alpha} \times \sin\beta\right)\sin\gamma$$

$$\boxed{\Delta = \frac{1}{2}a^2\,\frac{\sin\beta\sin\gamma}{\sin\alpha}}$$

Similarly

$$\boxed{\Delta = \frac{1}{2}b^2\,\frac{\sin\gamma\sin\alpha}{\sin\beta}}$$

$$\boxed{\Delta = \frac{1}{2}c^2\,\frac{\sin\alpha\sin\beta}{\sin\gamma}}$$

Case (3):-

When Three sides are given:-

We know that

$\Delta = \frac{1}{2} bc \sin\alpha$

$\Delta = \frac{1}{2} bc \left(2 \sin\frac{\alpha}{2} \cos\frac{\alpha}{2}\right)$

$\Delta = bc \sqrt{\frac{(S-b)(S-c)}{bc}} \sqrt{\frac{S(S-a)}{bc}}$

$\because \sin\frac{\alpha}{2} = \sqrt{\frac{(S-b)(S-c)}{bc}}, \cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}}$

$\Delta = \frac{bc \sqrt{S(S-a)(S-b)(S-c)}}{bc}$

$\boxed{\Delta = \sqrt{S(S-a)(S-b)(S-c)}}$

This is called Hero's Formula for finding area of triangle ABC.

"EXERCISE : 12.7"

Q.1 Find area if two sides and one angle:

(i) $a = 200 \quad b = 120 \quad \gamma = 150°$

Using $\Delta = \frac{1}{2} ab \sin\gamma$

$\Delta = \frac{1}{2}(200)(120)\sin(150°)$

$\Delta = 6000$ Sq unit.

(ii) $b = 37 \quad c = 45 \quad \alpha = 30°50'$

Using $\Delta = \frac{1}{2} bc \sin\alpha$

$\Delta = \frac{1}{2}(37)(45)\sin(30°50')$

$\Delta = 426.69$ Sq unit.

(iii) $a = 4.33 \quad b = 9.25 \quad \gamma = 56°44'$

Using $\Delta = \frac{1}{2} ab \sin\gamma$

$\Delta = \frac{1}{2}(4.33)(9.25)\sin(56°44')$

$\Delta = 16.74$ Sq unit.

Q.2 Find area if one side and two angles:

(i) $b = 25.4 \quad \gamma = 36°41' \quad \alpha = 45°17'$

$\because \alpha+\beta+\gamma = 180° \Rightarrow \beta = 180° - \alpha - \gamma$

$\beta = 180° - 45°17' - 36°41' = 98°2'$

Using $\Delta = \frac{1}{2} b^2 \frac{\sin\gamma \sin\alpha}{\sin\beta}$

$\Delta = \frac{1}{2}(25.4)^2 \frac{\sin(36°41')\sin(45°17')}{\sin 98°2'}$

$\Delta = 138.3$ Sq unit

(ii) $c = 32 \quad \alpha = 47°24' \quad \beta = 70°16'$

$\because \alpha+\beta+\gamma = 180° \Rightarrow \gamma = 180° - \alpha - \beta$

$\gamma = 180° - 47°24' - 70°16' = 62°20'$

Using $\Delta = \frac{1}{2} c^2 \frac{\sin\alpha \sin\beta}{\sin\gamma}$

$\Delta = \frac{1}{2}(32)^2 \frac{\sin(47°24')\sin(70°16')}{\sin(62°20')}$

$\Delta = 400.54$ Sq unit.

(iii) $a = 4.8 \quad \alpha = 83°42' \quad \gamma = 37°12'$

$\because \alpha+\beta+\gamma = 180° \Rightarrow \beta = 180 - \alpha - \gamma$

$\beta = 180 - 83°42' - 37°12' = 59°6'$

Using $\Delta = \frac{1}{2} a^2 \frac{\sin\beta \sin\gamma}{\sin\alpha}$

$\Delta = \frac{1}{2}(4.8)^2 \frac{\sin(59°6')\sin(37°12')}{\sin(83°42')}$

$\Delta = 6.013$ Square unit.

Q.3 (i) $a = 18 \quad b = 24 \quad c = 30$

$S = \frac{a+b+c}{2} = \frac{18+24+30}{2} = \frac{72}{2} = 36$

$S-a = 36-18 = 18$

$S-b = 36-24 = 12$

$S-c = 36-30 = 6$

$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$\Delta = \sqrt{36 \times 18 \times 12 \times 6} = \sqrt{46656}$

$\Delta = 216$ Sq unit.

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(ii) $a=524 \quad b=276 \quad c=315$

$s=\frac{a+b+c}{2}=\frac{524+276+315}{2}=\frac{1115}{2}=557.5$

$s-a=557.5-524=33.5$

$s-b=557.5-276=281.5$

$s-c=557.5-315=242.5$

$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

$\Delta=\sqrt{557.5\times 33.5\times 281.5\times 242.5}$

$\Delta=\sqrt{12749108860}$

$\Delta=35705.89$ Sq unit.

(iii) $a=32.65 \quad b=42.81 \quad c=64.92$

$s=\frac{a+b+c}{2}=\frac{32.65+42.81+64.92}{2}=\frac{140.38}{2}$

$s=70.19$

$s-a=70.19-32.65=37.54$

$s-b=70.19-42.81=27.38$

$s-c=70.19-64.92=5.27$

$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

$\Delta=\sqrt{70.19\times 37.54\times 27.38\times 5.27}$

$\Delta=\sqrt{380201.2757}$

$\Delta=616.605$ Sq unit.

Q.4 $\Delta=2437 \quad a=79 \quad c=97 \quad \beta=?$

Using $\Delta=\frac{1}{2}ac\sin\beta$

$\sin\beta=\frac{2\Delta}{ac}$

$\sin\beta=\frac{2(2437)}{79\times 97}$

$\sin\beta=0.6360$

$\beta=\sin^{-1}(0.6360)$

$\boxed{\beta=39^\circ 30'}$

Q.5 $\Delta=121.34 \quad \alpha=32^\circ 15' \quad \beta=65^\circ 37'$

$\gamma=? \qquad c=?$

$\because \alpha+\beta+\gamma=180^\circ$

$\gamma=180^\circ-\alpha-\beta$

$\gamma=180^\circ-32^\circ 15'-65^\circ 37'$

$\boxed{\gamma=82^\circ 8'}$

Using $\Delta=\frac{1}{2}c^2\frac{\sin\beta\sin\alpha}{\sin\gamma}$

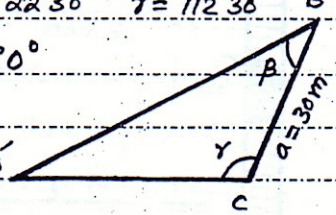
$c^2=\frac{2\Delta\sin\gamma}{\sin\alpha\sin\beta}$

$c^2=\frac{2(121.34)(\sin 82^\circ 8')}{(\sin 32^\circ 15')(\sin 65^\circ 37')}$

$c^2=494.6239$

$\boxed{c=22.24}$

Q.6 Let $a=30$ m $\quad \beta=22^\circ 30' \quad \gamma=112^\circ 30'$



$\because \alpha+\beta+\gamma=180^\circ$

$\gamma=180^\circ-\alpha-\beta$

$\gamma=180^\circ-22^\circ 30'-112^\circ 30'$

$\gamma=45^\circ$

Now $\Delta=\frac{1}{2}a^2\frac{\sin\beta\sin\gamma}{\sin\alpha}$

$\Delta=\frac{1}{2}(30)^2\frac{(\sin 22^\circ 30')(\sin 112^\circ 30')}{\sin 45^\circ}$

$\Delta=225\ m^2$

Now Cost for planting grass in $1\ m^2=5$ Rs.

Cost for planting grass in $225\ m^2=5\times 225$

$=1125$ rupees.

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